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Propagation of Gaussian beam through a uniaxial anisotropic slab

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ABSTRACT

An exact analytical solution to the reflection and transmission of an incident Gaussian beam by a uniaxial anisotropic slab is obtained in terms of the rectangular vector wave function expansion form. In the uniaxial anisotropic slab, the optical axis is parallel to the interface. For a localized beam model, numerical results of the normalized field intensity distributions are presented, and the propagation characteristics are analyzed concisely.

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1. Introduction

The interaction between electromagnetic (EM) waves and anisotropic media has been studied extensively over the years, for a variety of applications in optical signal processing, optimum design of optical fibers, radar cross section controlling, and microwave device fabrication, etc. It is of fundamental importance to analyze the reflection and transmission of EM waves at a plane interface separating an isotropic and anisotropic medium. With the wave splitting technique, the reflection and transmission properties have been discussed for an EM plane wave normally incident on a stratified bianisotropic slab [1]. Graham et al. investigated the reflection and transmission of an EM plane wave striking a biaxially anisotropic–isotropic interface [2]. For an incident shaped beam, Stamnes et al. presented the formulations and numerical results of focused paraxial field intensities inside a uniaxial and biaxial crystal [3–6].

In one previous paper, we have studied the reflection and transmission of an incident Gaussian beam (focused TEM_{00} mode laser beam) by a uniaxial anisotropic slab with the optical axis perpendicular to the interface by using the cylindrical vector wave functions (CVWFs) [7]. This paper, based on the rectangular vector wave function (RVWF) expansion form, is devoted to the presentation of an exact analytical solution to the case of the optical axis parallel to the interface.

The body of this paper is organized as follows. Section 2 provides the theoretical procedure for the determination of the reflected, internal and transmitted fields for a Gaussian beam incident on a uniaxial anisotropic slab. Numerical results of the normalized field intensity distributions are given in Section 3. The work is summarized in Section 4.

2. Formulation

2.1. Expansions of Gaussian beam, reflected beam, transmitted and internal beams in terms of the rectangular vector waves

As shown in Fig. 1, an incident Gaussian beam propagates from free space to an infinite uniaxial anisotropic slab of thickness d, with its propagation direction $O'z'$ having the polar coordinates ζ, η with respect to the Cartesian coordinate system $Oxyz$ and its beam waist middle located at origin λ on the axis $O'z'$. Origin O has a coordinate z_0 on the axis $O'z'$, and the planes $z = 0$ and $z = d$ are the interfaces between free space and the uniaxial anisotropic slab. In this paper, a time dependence of the form $\exp(-i\omega t)$ is assumed and suppressed for the EM fields.

In Appendix A, an expansion for the EM fields of an incident Gaussian beam (focused TEM_{00} mode laser beam) in terms of the RVWFs with respect to the system $Oxyz$ is obtained, as follows:

$$\mathbf{E}^i = \mathbf{E}_1^i + \mathbf{E}_2^i \tag{1}$$

where the electric field $\mathbf{E}_1^i$ is described by

$$\mathbf{E}_1^i = E_0 \int_0^{2\pi} d\beta \int_0^{\frac{\pi}{2}} [I_{TE}\mathbf{m}_{k_0}(\alpha, \beta) + I_{TM}\mathbf{n}_{k_0}(\alpha, \beta)] d\alpha \tag{2}$$

and $\mathbf{E}_2^i$ by the same expression as $\mathbf{E}_1^i$ but integrated over α from $\pi/2$ to π.

For a TE-polarized mode, the Gaussian beam shape coefficients I_{TE} and I_{TM} are

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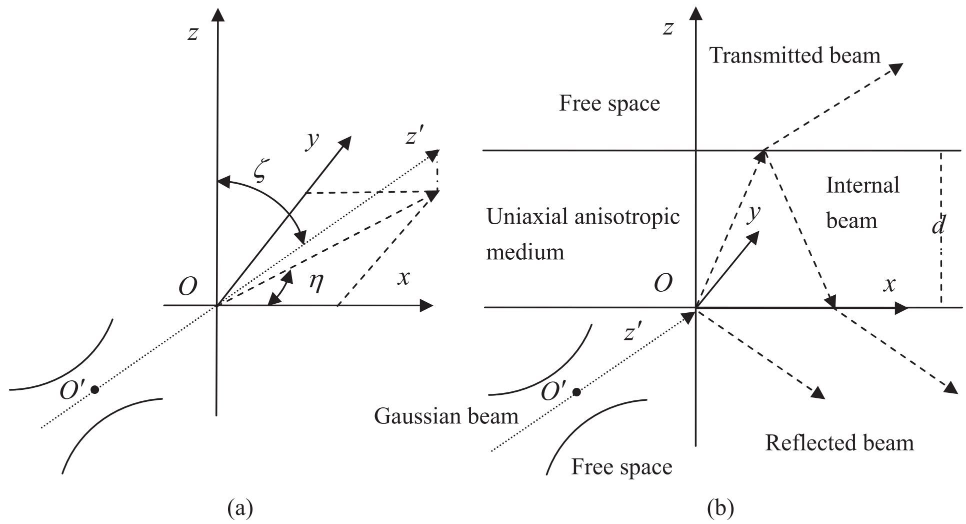

Fig. 1. A Gaussian beam striking a uniaxial anisotropic slab. (a) The coordinate system, (b) the configuration.

$$I_{TE} = \frac{i}{4\pi k_0} \sum_{n=1}^{\infty}\sum_{m=-n}^{n} g_n \frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!} e^{im(\beta-\eta)}\left[\frac{dP_n^m(\cos\zeta)}{d\zeta}\frac{dP_n^m(\cos\alpha)}{d\alpha} + m^2\frac{P_n^m(\cos\zeta)}{\sin\zeta}\frac{P_n^m(\cos\alpha)}{\sin\alpha}\right] \tag{3}$$

$$I_{TM} = \frac{i}{4\pi k_0} \sum_{n=1}^{\infty}\sum_{m=-n}^{n} g_n \frac{2n+1}{n(n+1)}\frac{(n-m)!}{(n+m)!} e^{im(\beta-\eta)} m\left[\frac{dP_n^m(\cos\zeta)}{d\zeta}\frac{P_n^m(\cos\alpha)}{\sin\alpha} + \frac{P_n^m(\cos\zeta)}{\sin\zeta}\frac{dP_n^m(\cos\alpha)}{d\alpha}\right] \tag{4}$$

When the Davis–Barton model of the Gaussian beam is used [8], μ_r can be computed by the localized approximation as [9,10]

$$g_n = \frac{1}{1+2isz_0/w_0}\exp(ik_0z_0)\exp\left[\frac{-s^2(n+1/2)^2}{1+2isz_0/w_0}\right] \tag{5}$$

where $s = 1/(k_0w_0)$, and w_0 is the beam waist radius.

The corresponding expansions for a TM-polarized Gaussian beam can also be obtained by replacing I_{TE} in Eq. (2) with iI_{TM}, and I_{TM} with iI_{TE}.

Eq. (1) can be interpreted that an incident Gaussian beam is expanded into a continuous spectrum of rectangular vector waves, with each rectangular vector wave having a propagation vector $\mathbf{k}_0 = k_0(\sin\alpha\cos\beta\hat{x} + \sin\alpha\sin\beta\hat{y} + \cos\alpha\hat{z})$ defined by the polar coordinates α and β in the system $Oxyz$. Then, from the configuration in Fig. 1 we can see that only $\mathbf{E}_1^i$ represents those rectangular vector waves that are incident on the interface $z = 0$, due to the fact that α, made by the propagation vector $\mathbf{k}_0$ and the axis Oz, is from 0 to $\pi/2$ for $\mathbf{E}_1^i$.

Following Eq. (2), the reflected beam and transmitted beam can be expanded as

$$\mathbf{E}^r = E_0\int_0^{2\pi} d\beta \int_0^{\frac{\pi}{2}} [a(\alpha,\beta)\mathbf{m}_{k_0}(\pi-\alpha,\beta) + b(\alpha,\beta)\mathbf{n}_{k_0}(\pi-\alpha,\beta)]\, d\alpha \tag{6}$$

$$\mathbf{E}^t = E_0\int_0^{2\pi} d\beta \int_0^{\frac{\pi}{2}} [c(\alpha,\beta)\mathbf{m}_{k_0}(\alpha,\beta) + d(\alpha,\beta)\mathbf{n}_{k_0}(\alpha,\beta)]d\alpha \tag{7}$$

We consider that the uniaxial anisotropic medium of the slab has an optical axis parallel to the interfaces $z = 0$ and $z = d$, and that its constitutive relations are expressed by a permittivity tensor $\bar{\varepsilon} = \hat{x}\hat{x}\varepsilon_1 + \hat{y}\hat{y}\varepsilon_2 + \hat{z}\hat{z}\varepsilon_2$ in the system $Oxyz$ and a scalar permeability μ_0.

From Appendix B, we obtain the eigen plane wave spectrum representations of the internal beams that propagate towards the interfaces $z = d$ and $z = 0$, respectively described by $\mathbf{E}_1^w$ and $\mathbf{E}_2^w$ as

$$\mathbf{E}_1^w = E_0 \sum_{q=1}^{2}\int_0^{2\pi} d\beta \int_0^{\frac{\pi}{2}} f_q(\alpha,\beta)\mathbf{F}_q^e(\alpha,\beta) \exp[i(k_0x\sin\alpha\cos\beta + k_0y\sin\alpha\sin\beta + k_{qz}z)]d\alpha \tag{8}$$

$$\mathbf{E}_2^w = E_0 \sum_{q=1}^{2}\int_0^{2\pi} d\beta \int_0^{\frac{\pi}{2}} g_q(\alpha,\beta)\mathbf{G}_q^e(\alpha,\beta) \exp[i(k_0x\sin\alpha\cos\beta + k_0y\sin\alpha\sin\beta - k_{qz}z)]d\alpha \tag{9}$$

For the sake of brevity, only the expansions of the electric fields are presented, and the magnetic fields can be expanded correspondingly with the following relations

$$\mathbf{H} = \frac{1}{i\omega\mu_0}\nabla\times\mathbf{E},\ \left[\mathbf{m}_{k_0}(\alpha,\beta)\quad \mathbf{n}_{k_0}(\alpha,\beta)\right] = \frac{1}{k_0}\nabla\times\left[\mathbf{n}_{k_0}(\alpha,\beta)\quad \mathbf{m}_{k_0}(\alpha,\beta)\right] \tag{10}$$

2.2. Gaussian beam propagation through a uniaxial anisotropic slab

The unknown expansion coefficients $a(\alpha,\beta)$, $b(\alpha,\beta)$ in Eq. (6), $c(\alpha,\beta)$, $d(\alpha,\beta)$ in Eq. (7), as well as $f_q(\alpha,\beta)$, $g_q(\alpha,\beta)$ $(q = 1, 2)$ in Eqs. (8) and (9) can be determined by using the boundary conditions respectively at $z = 0$ and $z = d$, as follows:

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