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The propagation properties of the first-order and the second-order Airy vortex beams through strongly nonlocal nonlinear medium

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A B S T R A C T

By using the transfer matrix method, the propagation of the first-order and the second-order Airy vortex (AiV) beams through strongly nonlocal nonlinear medium is exhibited. Based on the Huygens diffraction integral formula, we derive the analytical expressions of the first-order and the second-order AiV beams propagate through the paraxial ABCD system and present corresponding characteristic parameters such as propagation path, intensity, phase distributions, beam centers, the Poynting vector and angular momentum (AM) density flow. The propagation trajectory is periodical and looks like a sine wave. The AiV beam focuses two times in one period. The phase, energy flow and AM density flow distribution show a reversal when the beam propagates near the focusing point. Additionally, as the order increased, the vortex of the second-order AiV beam is stronger.

1. Introduction

In 1979, the Airy beam was innovatively described as a solution of the force-free Schrödinger equation in the context of quantum mechanics by Berry and Balazs [1]. The Airy beam is one of the non-diffracting beams, which can recover itself with a self-healing property after passing through obstacles and also can transversely accelerate freely upon propagation. In 2007, the Airy beam was generated experimentally and its unique properties were demonstrated [2–4]. After years of research, the investigations of the characteristics and the applications of the Airy beam have made a lot of progress. It has been shown that Airy beam can be applied to the aspect of optical micromanipulation and optical switching [5].

Studies of an Airy beam propagating in various media such as a turbulence atmosphere [6], water [7], transparent particles [8] and uniaxial crystal [9] has been done. The propagation of Airy beam in the presence of a parabolic profile has been studied by several authors. Banders and Gutiërrez-Vega [10] have used the Huygens approach together with the ABCD matrix to study the transformation of a Gauss-Airy beam through a medium characterized by quadratic transverse profile. Zhang et al. [11] have carried out a detailed investigation of the periodic inversion and phase transition of finite-energy Airy beams in a medium with a parabolic potential. It is shown above that a lot of researches about the Airy beam propagating in various media have been carried out.

In recent years, the research on an optical vortex has been a highlight. Its special characteristics, such as phase singularity, autofocusing properties and vector structure have attracted a lot of scholars to carry on research. In addition, optical vortex has been used in optical communication, optical micromanipulation and other fields [12–14]. Lately, research on the fundamental Gaussian beams and the partially coherent beams with optical vortex has been carried out [15,16].

The study of the propagation of an Airy beam with an optical vortex is a new direction in optics. A number of researches have been done, such as the accelerating vortex nested in an Airy beam by imposing a spiral phase plate on a cubic phase pattern [17], the propagation dynamics of AiV beam by a phase spatial light modulator [18], the propagation of AiV beam in uniaxial crystals [19], and the propagation of AiV beam in chiral medium [20].

When the beam width is much shorter than the width of the material response function, the medium is referred to be the strongly nonlocal nonlinear medium [21]. The propagation of the Airy beam in the strongly nonlocal nonlinear medium has potential application in optical switching and optical micromanipulation [22]. The investigation of an Airy beam propagating in a strongly nonlocal nonlinear medium studied by Zhou et al. shows that the normalized intensity distribution and the aforementioned parameters versus the axial propagation distance are all periodic. It is very important to investigate the propagation of an AiV beam in the strongly nonlocal nonlinear medium because of its potential

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application in optical switching and optical micromanipulation. However, the related work has never been reported to the best of our knowledge.

In this paper, we are going to investigate the properties of the first-order and the second-order AiV beams in the strongly nonlocal nonlinear medium, and investigate their propagation laws. In addition, we will work out the intensity and phase distributions, the energy flow, AM density flow and beam centers of the first-order and the second-order AiV beams.

2. The first-order Airy vortex beam

We first consider the Airy beam with one vortex. As the z-axis is the propagation axis in the Cartesian coordinate system, we can describe the expression of the first-order AiV beam on the source plane $z=0$ as [23]:

$$E(X_0, Y_0, 0) = Ai(X_0)Ai(Y_0)\exp(aX_0 + aY_0)[(X_0 + X_d) + i(Y_0 + Y_d)] \quad (1)$$

where $X_0 = \frac{x_0}{w_0}$, $Y_0 = \frac{y_0}{w_0}$, w_0 is the transverse scale, a is a modulation parameter, X_d and Y_d are the X-offset and Y-offset respectively. Ai() is the Airy function whose integral definition is [24]:

$$Ai(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left[i\left(\frac{u^3}{3} + xu\right)\right]du \quad (2)$$

The propagation of an AiV beam in the nonlocal nonlinear medium is governed by the nonlocal nonlinear Schrodinger equation:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)E(x, y, z) + 2ik\frac{\partial}{\partial z}E(x, y, z) + \frac{2k^2\Delta n}{n_0}E(x, y, z) = 0 \quad (3)$$

where the nonlinear perturbation of refractive index Δn being given by:

$$\Delta n = n_2\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}R(r - r')|E(x', y', z)|^2dx'dy' \quad (4)$$

where k is the wave number in the medium without nonlinearity, and n_0 is the linear refractive index of the medium. n_2 is the nonlinear index coefficient. R is the normalized symmetrical real spatial response function of the medium. In the case of strong nonlocality, where the beam width is much shorter than the width of the response function, which, in turn, is the characteristic length of the material, the response function is expanded in a Taylor series and terms up to second order are retained. The Snyder–Mitchell model [25] corresponding to the nonlocal nonlinear Schrodinger equation above becomes:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)E(x, y, z) + 2ik\frac{\partial}{\partial z}E(x, y, z) - k^2\gamma^2P_0(x^2 + y^2)E(x, y, z) = 0 \quad (5)$$

here γ is the material constant related to the response function, and $P_0 = \iint|E(x, y, z)|^2dx\,dy$ is the input power on the source plane. Here we introduce the dimensionless quantities $X = \frac{x}{w_0}$, $Y = \frac{y}{w_0}$, $Z = \frac{z}{kw_0^2}$ and $\Omega = k(P_0)^{\frac{1}{2}}\gamma w_0^2$ so that the Snyder–Mitchell equation becomes

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}\right)E(X, Y, Z) + 2i\frac{\partial}{\partial Z}E(X, Y, Z) - \Omega^2(X^2 + Y^2)E(X, Y, Z) = 0 \quad (6)$$

The beam width of the AiV beam has been proved to be not large. Therefore, the condition that the beam width is much shorter than the width of the response function can be satisfied. The approximation that appears in the transition from Eq. (3) to Eq. (6) hardly brings any limitations to the AiV beam in a strongly nonlocal nonlinear medium. Here we discuss the situation where the first-order AiV beam propagates through a paraxial ABCD optical system. In a strongly nonlocal nonlinear medium, the propagation of the first-order AiV beam can be described by the following ABCD matrix [20,26,27]:

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cos(\Omega z) & -\frac{\sin(\Omega z)}{\Omega} \\ \Omega\sin(\Omega z) & \cos(\Omega z) \end{pmatrix} \quad (7)$$

The first-order AiV beam passing through an ABCD optical system obeys the well-known Collins integral formula [28]:

$$E(X, Y, Z) = \frac{\exp(iZ)}{i\lambda B}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}E(X_0, Y_0, 0)\exp\left\{\frac{i}{2B}[A(X_0^2 + Y_0^2) - 2(X_0X + YY_0) + D(X^2 + Y^2)]\right\}dX_0dY_0 \quad (8)$$

Substituting Eq. (1) into Eq. (8), we gain the ultimate output field distribution [29] as:

$$E(X, Y, Z) = \exp(Q(X, Y, Z))(P1 + P2 + P3) \quad (9)$$

$$Q(X, Y, Z) = iZ + \frac{iC(X^2 + Y^2)}{2A} + \frac{a(X + Y)}{A} - \frac{aB^2}{A^2} - \frac{iB^3}{6A^3} + \frac{ia^2B}{A} + \frac{i(X + Y)B}{2A^2} \quad (10)$$

$$P1 = \left[\frac{X + iY}{A^2} - \frac{B^2(i + 1)}{2A^3} + \frac{aB(i - 1)}{A^2} + \frac{x_d + iy_d}{A}\right]Ai(f(X))Ai(g(Y)) \quad (11)$$

$$P2 = \frac{1}{A}\left[\frac{\partial}{\partial a}Ai(f(X))\right]Ai(g(Y)) \quad (12)$$

$$P3 = \frac{i}{A}\left[\frac{\partial}{\partial a}Ai(g(Y))\right]Ai(f(X)) \quad (13)$$

$$f(X) = \frac{X}{A} - \frac{B^2}{4A^2} + \frac{iaB}{A} \quad (14)$$

$$g(Y) = \frac{Y}{A} - \frac{B^2}{4A^2} + \frac{iaB}{A} \quad (15)$$

3. Numerical results for the first-order Airy vortex beam

We have carried out numerical calculations in order to examine the propagation properties of the first-order AiV beam through the strongly nonlocal nonlinear medium and obtained some interesting results. Using the discussion above, we know that Eq. (9) is the general analytical expression of the field distribution of the AiV beam propagating through the optical ABCD system. We assume $a=1$, $w_0 = 0.15$ mm, $\lambda = 632.8$ nm, $X_d = Y_d = 0$. The Rayleigh distance of the beam is $z_0 = (kw_0^2)/2 = 11.1703$ cm.

We plot Fig. 1 to analyze the intensity and phase distributions of the first-order AiV beam. We find that the first-order AiV beam has a main lobe and a lot of side lobes. In addition, the first-order AiV beam focuses two times in one period and bends itself during propagation.