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Entanglement propagation of a quantum optical vortex state

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ABSTRACT

We study the entanglement evolution of a quantum optical vortex state propagating through coupled lossless waveguides. We consider states generated by coupling two squeezed modes using a sequence of beam splitters and also by subtracting photons from one of the output modes in spontaneous parametric down conversion. We use the Wigner function to study the variation in the structure of the vortex state with distance and quantify the entanglement after propagation using *logarithmic negativity*.

1. Introduction

Recently a lot of work has been done to study states with non-Gaussian quadrature space distribution, which finds numerous applications in quantum information processing. Experiments have been conducted to produce non-Gaussian states using techniques like homodyne detection from a single mode squeezed state of light [1,2]. Photon subtraction/addition from a single mode squeezed state also gives rise to non-Gaussian states with associated Wigner function showing negative regions [3]. These states are important since they find a variety of useful applications in quantum computation [4], entanglement distillation [5,6] and loophole free tests of Bells inequality [7].

Quantum optical vortex states, considered by us in this article, belong to such a class of non-Gaussian states with interesting nonclassical features and a negative Wigner function. These states were introduced in [8] and studied in some detail in [9–14]. These are states with topological defects in the phase space and exhibit a vortex structure in the quadrature space. Such states can be generated from two mode squeezed vacuum under a linear transformation belonging to the SU(2) group with certain restrictions [9]. These states have been realized in the laboratory using photon subtraction [15]. It has been pointed out that photon subtraction/addition leads to enhancement of entanglement [16]. In this article we deal with vortex states arising from photon subtraction from one of the modes of a two mode squeezed vacuum. The order of the vortex is determined from the number of photons subtracted. Interestingly, a vortex state of order m carries OAM given by $m\hbar$.

We also consider vortex states produced by mixing two squeezed modes using a beam spliiter (BS) or a dual channel directional coupler (DCDC) [11]. It should be mentioned that a similar state can be generated by using a Λ type three level atom with counter rotating photons having circular polarization and performing a conditional measurement [8]. Entanglement being a fundamental resource in quantum information processing, it is interesting to study states with enhanced entanglement from a task oriented point of view. It has been shown that vortex states carry more entanglement compared to the Gaussian states from which they are generated and the entanglement carried can be controlled by altering the squeezing parameter or the ratio of mixing of the two input modes in a beam splitter [13].

In this article we study the propagation of entanglement of quantum optical vortex states using coupled lossless waveguides. The importance of coupled waveguides lie in their efficiency to manipulate the flow of light [17–24]. They have been used extensively to implement quantum random walk [25] which finds important applications in quantum computation and quantum algorithms. Coupled waveguides have been successfully used to implement a CNOT gate on a silica chip [26]. Given all these developments, it is important to study how such a system affects the physical structure, nonclassical nature and entanglement present in the light moving through it. We use the quadrature distribution

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and the Wigner function to study the difference in the physical characteristics of the state after propagating through a modest distance. The negativity of the Wigner function has already been used to study nonclassicality of states [27] and violation of Bell's inequality [28]. We use the negativity of the Wigner function as a sufficient indicator of the non-Gaussian nature of the generalized vortex state as well as its nonclassicality. Although the Glauber-Sudarshan P distribution is better suited for differentiating between the so called "classical" and "nonclassical" states of light, however, the negativity of the Wigner function is a sufficient criterion for drawing a similar conclusion though not a necessary criterion. An added advantage of the Wigner function is its usefulness in state tomography [29,30] which renders its study in evolution problems even more important. However, time evolution of the Wigner function has been a difficult problem due to its negative values [31]. Therefore, we use a numerical approach to evolve the Wigner function as a function of the phase space coordinates and use it to study the quantum correlations between the two modes at a later time. *Logarithmic negativity*, first introduced in [32] and later proven to be a proper entanglement monotone [33] is a commonly used measure of entanglement. We use this as a quantitative measure to study the behavior of entanglement on propagation through the coupled waveguides.

The article is organized as follows. In the next section we briefly introduce the model used for vortex evolution. In Section 2.1, we solve for the time evolution of the photon subtracted vortex states. We present the equation and an interpretation and explanation for the results in the same section. In Section 2.2 we discuss the time evolution of the generalized vortex state generated by coupling two squeezed modes using a series of beam splitters and explain the results obtained therein. We also construct the Wigner functions for the respective states at a later time. In Section 3, we present an explicit approach for studying the entanglement and its variation with time for both the states using *logarithmic negativity*.

We conclude the article in Section 4 with a brief review of the important results.

2. Quantum optical vortex under time evolution

The model that we consider here consists of two single mode coupled waveguides. The Hamiltonian for this system [34] can be written as follows

$$H = \hbar\omega\left(a^\dagger a + b^\dagger b\right) + \hbar C(a^\dagger b + b^\dagger a) \tag{1}$$

where a and b are the regular bosonic mode annihilation operators for the two single mode waveguides. The first two terms correspond to the free energy while the next two terms take into account the evanescent coupling between the two waveguides with C as the coupling strength. Typical values of the coupling strength range between $\sim 10^{10}\,\mathrm{s}^{-1}$ for waveguides like Lithium Niobate to $\sim 10^{11}\,\mathrm{s}^{-1}$ for Silica waveguides [35]. So we have used $C = 2\times 10^{10}\,\mathrm{s}^{-1}$ in our article. We study the systems at the initial time and after an interval of 10^{-6} s. Such an interval was chosen for a typical distance of 300 m to check the variation in the structure of the state after a fair distance of travel. Since we consider lossless propagation over such short time intervals, the Heisenberg equations of motion can be used to study the time evolution of the bosonic field operators, a and b for the two modes [35]. The time dependence of these operators are then given as

$$a(t) = a(0)\cos(Ct) - ib(0)\sin(Ct)$$
$$b(t) = b(0)\cos(Ct) - ia(0)\sin(Ct) \tag{2}$$

We consider two different input states. One of them is generated by subtracting k photons from one of the modes of a two mode squeezed vacuum which we study in Section 2.1. The other one can be generated by using k beam splitters to couple two squeezed mode vacuum states which we study in Section 2.2. The difference between these two states is that the former is already entangled before the process of photon subtraction while the latter gets correlated after being coupled by the beam splitters.

2.1. Photon Subtraction

In this section we study the effect of propagation through coupled waveguides on photon subtracted two mode squeezed vacuum states also referred to as the two mode squeezed vortex states. Subtraction of photons creates topological defects in the phase space resulting in a vortex structure in the quadrature space [12,14]. It would be worthwhile to mention that these states also possess orbital angular momentum $k\hbar$ if k photons are subtracted. It would also be interesting to study how the vortex structure is affected due to the propagation.

A two mode squeezed vacuum state can be written as,

$$|\xi\rangle = \exp\left(\xi a^\dagger b^\dagger - \xi^* ab\right)|0, 0\rangle, \quad \xi = re^{i\phi} \tag{3}$$

where ξ is a complex parameter, r is the squeezing amplitude and a and b are the regular bosonic mode operators. If k photons are subtracted from one of the modes (see Appendix A for details), Eq. (3) can be simplified to

$$|\xi\rangle_k = \frac{e^{ik\phi}}{\cosh^2 r}\sum_{m=0}^{\infty} e^{im\phi}\tanh^m r\sqrt{m+k}\,|m+k, m\rangle = \frac{e^{ik\phi}}{\cosh^2 r}a^{\dagger k}|\xi\rangle \tag{4}$$

The state at time t can be found out by operating it with the time evolution operator $\mathcal{U}(t)$ and solving the equation in the Schrodinger picture.

$$|\xi\rangle_k^t = \mathcal{U}(t)|\xi\rangle_k = \exp\left[-\frac{iHt}{\hbar}\right]|\xi\rangle_k \tag{5}$$

where the Hamiltonian H is same as defined in Eq. (1). We present time evolution of the state defined by Eq. (4) in Fig. 1. The rotation produced is evident from the contour plot of the intensity. There is also a visible distortion. The order remains constant which means the orbital angular momentum is conserved.

The Wigner function associated with Eq. (4) is derived as,

$$W(\tilde{\alpha}, \tilde{\beta}) = \frac{4}{\pi^2}(-1)^k\mathcal{L}_k\left[4|\tilde{\alpha}|^2\right]\exp\left[-2\left(|\tilde{\alpha}|^2 + |\tilde{\beta}|^2\right)\right] \tag{6}$$

where $\mathcal{L}_k$ is the Laguerre polynomial of order k, corresponding to the number of photons subtracted. $\tilde{\alpha}$ and $\tilde{\beta}$ are related to the coherent state parameters $\alpha = x - ip_x$ and $\beta = y - ip_y$ by a simple transformation given by,

$$\begin{pmatrix}\tilde{\alpha}\\ \tilde{\beta}^*\end{pmatrix} = \begin{pmatrix}\cosh r & -\sinh r e^{i\phi}\\ -\sinh r e^{i\phi} & \cosh r\end{pmatrix}\begin{pmatrix}\alpha\\ \beta^*\end{pmatrix} \tag{7}$$

To study the dynamics of the Wigner function analytically, one needs to solve the equation of motion for the Wigner function which is as follows

$$\frac{\partial W\left(\vec{r}, \vec{p}, t\right)}{\partial t} = -\left\{\left\{W\left(\vec{r}, \vec{p}, t\right), H\right\}\right\} \tag{8}$$

where $\{\{.,\ .\}\}$ is the Moyal bracket. But this is a difficult problem for most Hamiltonians and a perfect solution is known only for a few cases. In this article we study the time evolution of the Wigner function, Eq. (6), numerically. We follow the process outlined in [31]. Given the Wigner function $W(\tilde{\alpha}, \tilde{\beta})$ at time $t = t_0$, we wish to