Optics Communications 376 (2016) 86–91

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Time-frequency representation measurement based on temporal Fourier transformation

Yifan Suen [a], Shaoqiu Xiao [a,*], Sumin Hao [a], Xiaoxiang Zhao [a], Yigao Xiong [a], Shenye Liu [b]

[a] School of Physical Electronics, University of Electronic Science and Technology of China, Chengdu 610054, China
[b] Research Center of Laser Fusion, China Academy of Engineering Physics, Mianyang 621900, China

ARTICLE INFO

Article history:
Received 3 December 2015
Received in revised form
24 March 2016
Accepted 9 May 2016
Available online 16 May 2016

Keywords:
Phase measurement
Time-lens
Four wave mixing
Chirping
Time-frequency spectrogram

ABSTRACT

We propose a new scheme to physically realize the short-time Fourier transform (STFT) of chirped optical pulse using time-lens array that enables us to get time-frequency representation without using FFT algorithm. The time-lens based upon the four-wave mixing is used to perform the process of temporal Fourier transformation. Pump pulse is used for both providing the quadratic phase and being the window function of STFT. The idea of STFT is physically realized in our scheme. Simulations have been done to investigate performance of the time-frequency representation scheme (TFRS) in comparison with STFT using FFT algorithm. Optimal measurement of resolution in time and frequency has been discussed.

1. Introduction

In recent years, all kinds of applications of space-time duality have been widely studied theoretically and experimentally [1–12]. The space-time duality connects the idea of light manipulation in the spatial domain and the idea of waveform manipulation in the temporal domain together. Impressive recent efforts have been done for all-optical ultrafast signal processing using time-lens, such as waveform compression [4] and stretch [5–8,10,12], time-to-frequency mapping [3,9,13–16], temporal cloaking [21]. Although previous research about time-lens is the signal processing that only manipulates the intensity waveform over the time and cannot detect the phase variation of the pulse.

To precisely represent the characteristics of an ultra-short laser pulse, the time-dependent phase is one of the important parameters, especially in the femtosecond regime. The phase and intensity of ultra-short pulse are easily influenced by dispersion and nonlinear effects. Electrical signals, which have low-bandwidth comparing with optical signals, are easier to be sampled and do sophisticated digital signal processing that can realize the time-frequency representation. Time-frequency representation is powerful tools for time-varying signal analysis, which presents the energy content of a signal as a function of both time and frequency [18]. Recently developed techniques for the phase measurements of ultra-short optical signal are frequency-resolved optical gating (FROG) [19] and spectral phase interferometry for direct electric-field reconstruction (SPIDER) [20]. But there are lots of sophisticated inversion algorithms based on iterative procedures that are time consuming and it has the potential to introduce errors in FROG, and single-shot operation can also be biased by spatial distortions of the beam. The SPIDER method also needs to transform spectrum into temporal signal by FFT algorithm.

In this paper, we propose the TFRS scheme to get the time-frequency representation of a detected pulse using time-lens array without using FFT algorithm. The core advantage of the technique is almost real-time representation, instantaneous transform, and high efficiency. The derivation process of the TFRS based on temporal Fourier transform using four-wave mixing is theoretically described in detail. Simulations have been done to investigate performance of the TFRS scheme in comparison with STFT using FFT algorithm and further system implementation of this scheme has been discussed. And the TFRS method is more suitable to analyze the picosecond signals due to limiting by the present level of sampling rate.

2. Theory of time-frequency representation based on temporal Fourier transformation

The space-time duality [1] is constructed by two straightforward approximations to the wave equation about the problems of

* Corresponding author.
E-mail address: xiaoshaoqiu@uestc.edu.cn (S. Xiao).

http://dx.doi.org/10.1016/j.optcom.2016.05.017
0030-4018/

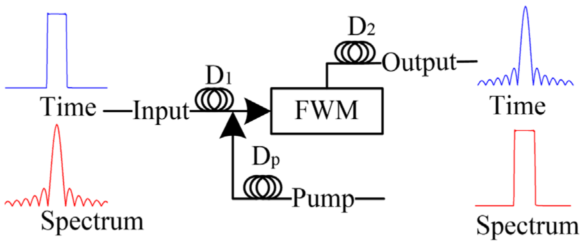

Fig. 1. The schematic of the time lens based on FWM.

paraxial diffraction and narrow-band dispersion. It is wise to apply these spatial-domain techniques to the temporal domain, which enables us to apply more sophisticated and powerful approaches to temporal processing and characterization of information. An object positioned at the front focal plane of a lens will produce a Fourier transformation of the object at the back focal plane. Time-lens is a device compared with the spatial lens that converts the temporal (spectral) profile of the input to the spectral (temporal) profile of the output at the focal plane.

The time lens based on the principle of four-wave mixing (FWM) [6] is presented in Fig. 1. After propagating through linear dispersion medium with group delay dispersion (GDD) of D_p, the pump pulse, which contains a quadratic phase $\exp(-jt^2/2D_p)$, imposes a quadratic temporal phase shift to the input optical signal in highly nonlinear materials, for example, silicon nano-waveguide or highly nonlinear fiber. The efficiency of FWM can be greatly enhanced if the phase match condition is satisfied. The idler electrical field is $E_i(t) \propto E_p^2(t)E_s^*(t)$ [6], where the electric field of input signal is $E_s(t)$ and the chirped pump is $E_p(t)$, and the center frequency of idler electrical field is $\omega_i = 2\omega_p - \omega_s$.

The theoretical derivation about time-frequency representation scheme (TFRS) based temporal Fourier transformation is presented as follows. The electric field of the pulse is $E(t) = A(t)\exp(j\omega_0 t)$, where $A(t)$ represents the complex envelope of the pulse, ω_0 is the angular frequency of light that could be ignored, the primary concern is the waveform of pulse. An input waveform propagates through a linear dispersive medium, a time-lens and another linear dispersive medium in sequence. It is always assumed that there is no loss in the dispersive medium. The output waveform from first dispersion medium with group delay dispersion (GDD) of D_1 is $A_1(t) = A_0(t)*h_1\exp(-jt^2/2D_1)$, where $h_1 = (j/2\pi D_1)^{1/2}$ and $A_0(0, t)$ is the input waveform. Four-wave mixing is the parametric process between input waveform and pump pulse in nonlinear medium, such as, silicon waveguide. Assuming that the phase match condition is satisfied, it has maximum efficiency. If the pump pulse transmits through the dispersion with GDD of D_p, the pump pulse contains a quadratic phase $\exp\left(jt^2/2D_p\right)$. Therefore the output complex waveform of time-lens can be written as

$$A_2(t) = \alpha A_1{*}(t)(exp(j\varphi_p))^2 = \alpha A_1{*}(t)exp\left(jt^2/D_p\right) \tag{1}$$

where α is the conversion factor of FWM and $\exp(j\varphi_p)$ is phase item introduced by pump pulse, D_p is GDD of dispersion medium for pump signal. Then it propagates through second dispersion medium with GDD of D_2, the output is the convolution between $A_2(t)$ and $(j/2\pi D_2)^{1/2}\exp(-jt^2/2D_2)$ in temporal domain, and it can be written as

$$\begin{aligned} A_3(t) &= A_2(t)*h_2\exp(-jt^2/2D_2) \\ &= \alpha h_1^* h_2 \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} A_0{*}(\eta)\exp\left(\frac{j(\tau-\eta)^2}{2D_1}\right)d\eta \\ &\cdot\exp\left(\frac{j\tau^2}{D_p} + \frac{-j(t-\tau)^2}{2D_2}\right)d\tau \end{aligned} \tag{2}$$

where $h_2 = (j/2\pi D_2)^{1/2}$, τ and η is the intermediate variables produced by the convolution. If $D_p = -2D_2$, the output can be simplified as

$$\begin{aligned} A_3(t) &= h_3 \int_{-\infty}^{+\infty} A_0^*(\eta)exp\left(-j\frac{t}{-D_2}\eta\right)d\eta \\ &= h_3\tilde{A}^*(-\omega')\Big|_{\omega'=-t/D_2} = h_3\tilde{A}^*\left(\frac{t}{D_2}\right) \end{aligned} \tag{3}$$

where $\tilde{A}(\omega)$ represents the spectrum of $A_0(t)$, $h_3 = \alpha h_2\exp(-j\psi t^2)$, $\psi = (1 + D_1/D_2)/2D_2$. It indicates that the time-lens converts the temporal (spectral) profile of the input waveform to the spectral (temporal) profile at the focal plane If the dispersion of pump is double of the output dispersion with opposite sign. Finally, the output intensity waveform is

$$\begin{aligned} I_3(t) &= |A_3(t)|^2 = \left|\tilde{A}^*(-\omega')\right|^2/2\pi D_2 \\ &= \left|\tilde{A}(t/D_2)\right|^2/2\pi D_2 \end{aligned} \tag{4}$$

$I_3(t)$ is exactly in direct proportion to the spectrogram of input waveform and scaling factor is

$$\Delta t/\Delta\omega = D_2 \tag{5}$$

From the Eq. (3), it is worth noting that the input dispersion has impact on the phase of envelope in temporal domain, but for intensity of output waveform, the input dispersion of D_1 has little effect on it. Therefore it is convenient to omit the input dispersion medium.

The short-time Fourier transform (STFT) is used to determine frequency content of local sections of a signal as it changes over time. The signal to be transformed is multiplied by a window function which is nonzero for only a short period of time. All parts of the signal are transformed into frequency domain, which is a one-dimensional processing, as the window function is slid along the time axis, resulting in a two-dimensional representation of the signal. Mathematically, it can be written as:

$$\mathrm{STFT}\left\{x(\tau)\right\}(t, \omega) = \int_{-\infty}^{+\infty} x(\tau)W(t-\tau)\exp(-j\omega\tau)d\tau \tag{6}$$

$x(\tau)$ is the complex signal to be transformed, $W(\tau - t)$ is the window function. And If drawing a time-frequency representation figure, what we need is $|\mathrm{STFT}\{x(\tau)\}(t, \omega)|$ to show the changes of energy density spectrum over time.

The schematic diagram about TFRS scheme is presented in Fig. 2. Pump pulse with FWHM of *Tw* is used for both providing the quadratic phase and being the window function of STFT. The detected optical signal with FWHM of *Ts* is divided to multiple duplicates that is delayed a period of time ΔT in turn. Different short period of time of multiple duplicates are separately transformed into the frequency domain by multiple time lens array. After transmitting through output dispersion of D_2, all of channels' outputs are the waveforms that are exactly in proportion to the Fourier transform of input waveforms.

However, being sampled from the outputs would use N independent data acquisition channels. it would increase the cost and practicality of system. Therefore the data from all parallel channels need to be serialized into one channel and be sent into one data acquisition unit. Some delay lines are added into each channel and the delay time increases T_d ($T_d \gg T_s$) in turn. The value of T_d can be estimated If we know signal bandwidth and pump pulse width, which is described in chapter 3.

After sampling serial data, we could get one-dimension intensity serial $I(M)$. Dividing $I(M)$ into N equal portions $I(i, P)$, where $i = 1, 2, \cdots, N$ and $M = N \times P$, N is the number of time-lens, M is the number of all sample points, P is the number of sample