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Three-dimensional phase transformation by impedance-matched dielectric slabs and generation of hollow beams based on transformation optics

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ABSTRACT

We propose a three-dimensional (3D) phase transformation method by an impedance-matched dielectric slab and apply it to generating hollow beams. We first employ transformation optics to establish a method for the transformation between two arbitrary 3D wavefronts through a flat dielectric and impedance-matched material. Then the method is used to convert a solid beam into a hollow beam with desired wavefront. By tuning the transformation surface, different hollow beams can be produced. The results are further validated by 3D finite-difference time-domain simulations.

1. Introduction

The past decade has witnessed the flourishing of a new science area, transformation optics (TO) [1]. TO provides a powerful recipe to manipulate the direction, amplitude, phase, and polarization of electromagnetic waves as desired by using artificial materials [2–6]. It has given birth to unprecedented applications in which the most well-known is the invisibility cloak [7]. In controlling phase by TO, the conversion between two wavefronts can be realized, while leading to transformation media with irregular shapes [8,9]. On this issue, we proposed a method to achieve any phase transformation (PT) by a flat material [10] and used it to generate Laguerre–Gaussian beams [11]. On the other hand, TO usually chooses linear coordinate transformations (CTs) for simplicity, which leads to impedance mismatch and thus to scattering on the boundary, and the materials prescribed by TO are both electrically and magnetically anisotropic, rendering practical implementation often infeasible. For the two reasons, we suggested to realize PT [12] using the techniques proposed in cloaking to eliminate impedance mismatch by adopting a high-order CT [13–17] and to simplify material parameters [7] to render the transformation medium dielectric.

In recent years, hollow beams with zero central intensities, such as Laguerre–Gaussian beams [18], Bessel–Gaussian beams [19–21] and hollow Gaussian beams [22], have been extensively studied [23–25]. Hollow beams manifest annular intensity distributions and have unique phase fronts. They can possess spin and orbital angular momenta [18,26] and propagate without diffraction [19]. Owing to their unique characteristics, hollow beams have been used in manipulating particles [27,28] or atoms [29–31], optical communications [32,33], imaging [34], laser processing [35], biophysics [36], nonlinear optics [37] and so on. So far, a few techniques have been put forward to produce hollow beams [23–25]. Among them, the geometrical optical method using spiral phase plates or axicons is very simple, but the beam quality relies on precise alignment and is affected by the boundary reflection [38–41]; computer-generated holograms have limited conversion efficiencies [42–44]; spatial light modulators are able to generate almost all the hollow beams flexibly but can not endure high power [45–48]; and optical fibers require small incidence angle and the intensity distribution is susceptible to the fiber bending [21,49].

By now, the phase devices based on TO are mostly two-dimensional (2D) [3,50]. Such 2D devices are only effective to one polarization, e.g. transverse-electric polarized incident waves. Recently, three-dimensional (3D) TO lenses, e.g. Luneburg lens and zone plate lens, were realized and demonstrated a control on the phase for different polarizations in 3D space [51,52]. In this work, we establish a 3D PT method to realize the conversion between any two 3D wavefronts that results in a dielectric and impedance-matched transformation material and use it to generate hollow beams. First, we extend the previous method [12] to the 3D case to

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realize the conversion of an arbitrary 3D curved wavefront to another one by a compact impedance-matched dielectric slab. Then, we apply this method to convert a solid beam into a hollow beam with desired wavefront and intensity distribution. Through changing the transformation surface, different hollow beams, e.g. single- or multi-ringed beams, hollow beams with or without angular orbital momentum, and optical bottle beams or optical cages [53–55], can be realized. Finally, we verify the theoretical results using 3D finite-difference time-domain (FDTD) simulations [56].

2. Method of 3D PT by impedance-matched dielectric slab

Our object is to realize the PT between two arbitrary 3D wavefronts by an impedance-matched dielectric slab. Without loss of generality, suppose that the incident beam propagates along z-axis, the incident wavefront is characterized by $z = f_i(x, y)$, while the output one $z = f_o(x, y)$, as shown in Fig. 1. If using previous PT methods based on TO [8,9], the above task can be fulfilled by transforming $z = f_i(x, y)$ to $z = f_o(x, y)$ directly [Fig. 1(a)]. Therein, the optical path length is $f_o - f_i$. However, the resulting transformation material has an indefinite profile and then the aberration is inevitable [58]. More seriously, it is dielectric-magnetic, which makes itself difficult to be constructed, and the impedance is not matched on the exit boundary, which results in unwanted scattering. To overcome these shortcomings in this direct transformation method, we take an alternative way.

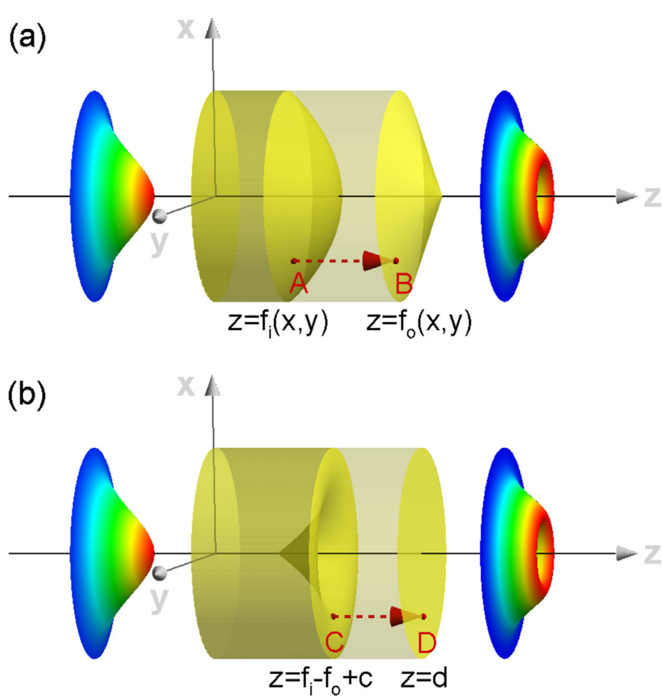

Fig. 1. Schematics of the 3D phase transformation between two arbitrarily curved wavefronts by (a) the direct transformation method in the literature and (b) the indirect method in the present work. In (a), the CT is performed by converting the incident wavefront $z = f_i(x, y)$ to the output one $z = f_o(x, y)$ directly, i.e. $A \to B$. The virtual space (from $z=0$ to $z = f_i$) is transformed into the physical space (from $z=0$ to $z = f_o$) that has the same irregular shape as the exit wavefront. In (b) the CT is carried out by flattening the transformation surface $z = f_i(x, y) - f_o(x, y) + c$, a shifted version of the incident wavefront $z = f_i(x, y)$ minus the desired one $z = f_o(x, y)$, to the exit plane $z=d$, i.e. $C \to D$. Meanwhile, the virtual space (from $z=0$ to $z = f_i - f_o + c$) is stretched into the physical space (from $z=0$ to $z=d$), i.e. the planar slab. The constant c does not influence the slab's function, except changing the range of the material parameter. Here shown is an example where an incident solid beam is converted into a hollow beam by the slab which reduces to a cylinder due to the symmetry of the beams.

Firstly, we use an indirect transformation method to obtain a flat figuration. We choose a reformed surface $z = f_i - f_o + c$ to transform into the plane surface $z=d$ [Fig. 1(b)]. The optical path length is $f_o - f_i + d - c$, which is equivalent to that of the direct transformation method in addition to a constant that does not influence the resulting phase. According to the principle of equal optical path length [59], the two method can implement exactly the same function. This method can be understood intuitively: Imagine that in Fig. 1(a), the surface $z = f_o(x, y)$ is stretched or compressed to a plane $z=c$ so that the resultant device has a planar exit surface. To preserve the optical path length, the input surface $z = f_i(x, y)$ has to undergo the same distortion and be reformed as $z = f_i + (c - f_o)$. In a word, we can choose a profile of the original wavefront minus the desired one to be transformed to a plane in order to render the device planar [10,12].

As shown in Fig. 1(b), the virtual space from $z=0$ to $z = f_i - f_o + c$ is stretched into the physical space from $z=0$ to $z=d$. Let (x, y, z) denote an arbitrary point in the virtual space while (x', y', z') in the physical space [57]. The corresponding CT can be expressed as

$$x' = x, \quad y' = y, \quad z' = g(x, y, z), \tag{1}$$

which satisfy two conditions

$$z' = 0 \quad \text{for } z = 0, \quad z' = d \quad \text{for } z = \Delta, \tag{2}$$

on the incident and exit boundaries, respectively. Here d is the width of the slab and

$$\Delta(x, y) = c + f_i(x, y) - f_o(x, y) \tag{3}$$

denotes the profile of spatial separation between the two wavefronts. Following TO, the permittivity tensor $\boldsymbol{\varepsilon}$ and the permeability tensor $\boldsymbol{\mu}$ of the transformation material are respectively related to the original $\boldsymbol{\varepsilon}_0$ and $\boldsymbol{\mu}_0$ by $\boldsymbol{\varepsilon} = J\boldsymbol{\varepsilon_0}J^T/\det(J)$ and $\boldsymbol{\mu} = J\boldsymbol{\mu_0}J^T/\det(J)$, where J is the Jacobian matrix of transformation between the transformed and the original coordinates [60]. In order to simplify material parameters, we only consider the transformation in the z' direction that is set independent of x and y (i.e., $\partial z'/\partial x = 0$ and $\partial z'/\partial y = 0$ [61,57]). Substitution of Eq. (1) leads to a diagonal Jacobian matrix, $J = \mathrm{diag}[1, 1, \partial g/\partial z]$. It then yields a general result of the relative material parameters for the conversion between two arbitrary 3D wavefronts:

$$\varepsilon(\mu) = \mathrm{diag}\left[(\partial g/\partial z)^{-1}, (\partial g/\partial z)^{-1}, \partial g/\partial z\right]. \tag{4}$$

Obviously, the material is anisotropic and dielectric-magnetic, which is difficult to fabricate in practice. In order to realize the same function by a dielectric medium, it was proposed to reform the material parameters by keeping the dispersion relation invariant [13]. Consider a TM wave incident on the slab with the magnetic field along the x' direction. We set $\mu_{x'x'} = 1$ and multiply $\varepsilon_{y'y'}$ and $\varepsilon_{z'z'}$ with $\mu_{x'x'}$ in Eq. (4) to preserve the dispersion relationship, thereby attaining

$$\varepsilon_{x'x'} = 1, \quad \varepsilon_{y'y'} = (\partial g/\partial z)^{-2}, \quad \varepsilon_{z'z'} = 1. \tag{5}$$

It needs to point out that the impedance of the resultant material is not matched between the exit boundary and the air. For this reason, we impose additional requirements on the impedance. That is, the y' component $Z_{y'}|_{z'=d} = \sqrt{\mu_{x'x'}/\varepsilon_{z'z'}}|_{z'=d} = 1$ which is satisfied automatically and the z' component:

$$Z_{z'}|_{z'=d} = \sqrt{\mu_{x'x'}/\varepsilon_{y'y'}}|_{z'=d} = |\partial g/\partial z|_{z'=d} = 1. \tag{6}$$

As well known, there are lots of functions satisfying the above conditions [13–17]. For simplicity, we choose a quadratic function $g(z) = pz^2 + qz + b$. Applying the conditions Eqs. (2) and (6) yields $p = (\Delta - d)/\Delta^2$, $q = (2d - \Delta)/\Delta$ and $b=0$. Thus