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Random phase-shifting interferometry based on independent component analysis

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ABSTRACT

In random phase-shifting interferometry, a novel phase retrieval algorithm is proposed based on the independent component analysis (ICA). By performing the recombination of pixel position, a sequence of phase-shifting interferograms with random phase shifts are decomposed into a group of mutual independent components, and then the background and the measured phase of interferogram can obtained with a simple arctangent operation. Compared with the conventional advanced iterative algorithm (AIA) with high accuracy, both the simulation and the experimental results demonstrate that the proposed ICA algorithm reveals high accuracy, rapid convergence, and good noise-tolerance in random phase-shifting interferometry.

1. Introduction

Phase-shifting interferometry (PSI), revealing rapid speed and high accuracy in optical phase measurement, has been widely used in optical surface-profiling, three-dimensional shape measurement, biological cell imaging and digital holography and other fields [1–4]. In recent years, a lot of phase retrieval algorithms of PSI have been proposed, which are usually classified into two types according to the known phase shifts and the random phase shifts. In the PSI with the known phase shifts, at least three-frame phase-shifting interferograms need to be captured [5]. Moreover, the accuracy of phase retrieval is greatly related with the accuracy of phase shifts, so it is required to calibrate the phase shifts error in advance, and this will greatly limit the application of PSI.

In random PSI [6–16], based on the spatial Fourier transform (SFT), the phase and phase-shifts can be retrieved from one-frame interferogram [6,7], but this method requires the interferogram with the additional carrier frequency to achieve the spectral separation of the zero order and the first order. Second, by searching for the intensity maximum and the intensity minimum of each pixel from a sequence of phase-shifting interferograms within one phase-shifting period, the corresponding background and the modulation amplitude of interferogram can be determined, and then the measured phase with high accuracy can be obtained from an arccosine function [8]. However, this method is time-consuming due to the background and the modulation amplitude of interferogram are calculated by the pixel to pixel. Third, based on the principle component analysis (PCA) [9–12], Vargas et.al proposes a fast and high accuracy algorithm to retrieve the measured phase and the phase shifts. However, in this PCA algorithm, since the background elimination is achieved by the temporal averaging operation, it is required that the phase shifts of interferograms should be distributed uniformly in an integer period, and the number of phase-shifting interferograms should be large enough. Fourth, by analyzing the characteristic of interferogram, a fast and simple phase shifts extraction algorithm based on Euclidean matrix norm (EMN) is presented [13]. In addition, based on the intensity difference between the first interferogram and the other interferograms to calculate the 1-norm, this algorithm can work well even if the fringe number of interferogram is less than one [14]. However, to search for the extreme value of interferogram, these norm algorithms require a number of phase-shifting interferograms. Fifth, based on the least-square error estimation and the spatial alternative iteration, an advanced iterative algorithm (AIA) of phase retrieval with high accuracy is proposed, but it is greatly time-consuming [15,16].

Collectively, there are various requirements in above random PSI methods, including the spatial carrier frequency in interferogram [6,7], or the background elimination of interferogram in advance [9–12], the uniform distribution of phase shifts and the number of phase-shifting interferograms [13,14], the complex iterative calculation [15,16]. Usually, these requirements will increase the processing time or the complexity of phase retrieval. To

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address this, we introduce the independent component analysis (ICA) [17,18] into the random PSI. Moreover, by using a linear transformation of a multi-dimensional distribution, ICA method can minimize the statistical dependence between the components, which is named as ICA-based fast fixed-point (FastICA) algorithm. In recent years, some research results about introducing ICA algorithm into digital holography have been reported [19,20]. In Ref. [19], ICA algorithm is employed to perform the computational holographic three-dimensional imaging and the automated object recognition; in Ref. [20], ICA algorithm is successful in lowering the background speckle of digital holograms. Following, we will introduce the principle of using ICA algorithm into the random PSI, and then present the simulation and the experimental results.

2. Principle

In PSI, the intensity distribution of the nth interferogram can be described as

$$I_n(x, y) = A(x, y) + B(x, y)\cos[\varphi(x, y) + \theta_n] \tag{1}$$

where $n=0, 1, 2,\ldots,N$ and (x, y) respectively represent the sequence number of phase-shifting interferograms and the pixel position, $A(x, y)$ and $B(x, y)$ are the background and the modulation amplitude, respectively; $\varphi(x, y)$ and θ_n denote the measured phase and the phase shifts, respectively. If the pixel number of interferogram in x and y direction are M_x and M_y, the total pixel number of interferogram is equal to $M=M_x \times M_y$. By performing the recombination of pixel position, one two-dimensional interferogram can be transformed into a column with the length of M pixels, which can be expressed by a matrix of $M \times 1$

$$I_n = [i_{1n} i_{2n} \cdots i_{mn} \cdots i_{Mn}]^T \tag{2}$$

where i_{mn} denotes the intensity of interferogram in the mth pixel, $m = 1, 2,\ldots,M$ represents the pixel position; $[\cdot]^T$ represents the transposing operation. Thus, the intensity distribution of the nth phase-shifting interferogram in the mth pixel can be described as

$$i_{mn} = a_m + b_m \cos(\varphi_m + \theta_n) \tag{3}$$

where a_m and b_m respectively represent the background and the modulation amplitude; φ_m denotes the measured phase.

Moreover, Eq. (3) can be further expanded as

$$i_{mn} = a_m + i_{cm} \cos \theta_n + i_{sm} \sin \theta_n \tag{4}$$

and

$$i'_{mn} = a_m + i'_{cn} \cos \varphi_m + i'_{sn} \sin \varphi_m \tag{5}$$

where $i_{cm} = b_m \cos \varphi_m$, $i_{sm} = -b_m \sin \varphi_m$, $i'_{cn} = b_m \cos \theta_n$, $i'_{sn} = -b_m \sin \theta_n$. From (Eqs. (4) and 5), we can see that one interferogram can be decomposed into the background and two mutually orthogonal signals.

Following, by employing a_m, i_{cm}, i_{sm}, i'_{cn}, i'_{sn} as the elements, we can constitute the corresponding matrices of A, I_c, I_s, I'_c and I'_s, respectively. That is to say, once these matrices can be determined, the measured phase, the phase shifts and the background of interferogram can be calculated simultaneously. And then we rewrite N-frame phase-shifting interferograms as a matrix of $N \times M$

$$I = [I_1 I_2 \cdots I_n \cdots I_N]^T \tag{6}$$

Next, we introduce the ICA method to determine matrix I, in which the implement is performing through using Fast ICA algorithm. Firstly, we choose the whitening processing to reduce the complexity of matrix I, and Eq. (6) is carried out by the linear transform.

$$U = WI \tag{7}$$

where W, a whitening square matrix of N order, can be obtained by

$$W = E_V^{-1/2} E_M^T \tag{8}$$

where E_V and E_M denote the eigenvalue matrix and the eigenvector matrix of the covariance matrix of I, respectively; superscript "$-1/2$" represents the reciprocal of square root operation. And then we perform the optimization for each column of W through using the FastICA iterative calculation. If W is chosen as the initial value of optimization, and the element of the nth column is employed as the initial vector $W_{n,0}$, the column vector $W_{n,k}$, obtaining by k iterations through Newton formula-simplified FastICA algorithm [18], can be expressed as

$$\tilde{W}_{n,k} = \left\langle Ug(\tilde{W}_{n,k-1}^T U) \right\rangle - \left\langle g'(\tilde{W}_{n,k-1}^T U) \right\rangle \tilde{W}_{n,k-1} \tag{9}$$

In Eq.(9), $\langle \cdot \rangle$ represents the averaging operation; $g(u)$ is a nonlinear function using the elements of matrix u as the variable; $g'(u)$denotes the derivative of $g(u)$; superscript "$\sim$" represents the normalized processing, namely

$$\tilde{W}_{n,k} = W_{n,k} / \| W_{n,k} \| \tag{10}$$

where $\|\cdot\|$ denotes the module of vector. In this case, if we set $g(u)=u^3$, Eq. (10) can be simplified as

$$\tilde{W}_{n,k} = \left\langle U(\tilde{W}_{n,k-1}^T U)^3 \right\rangle - 3\tilde{W}_{n,k-1} \tag{11}$$

where superscript "3" denotes the cubic operation.

Following, using Eq. (11), we perform the iteration optimization for each column of W until the iteration result satisfies the convergence condition

$$\left\| \left| \tilde{W}_{n,k} \right| - \left| \tilde{W}_{n,k-1} \right| \right\| < \varepsilon \tag{12}$$

where $|\cdot|$ denotes the summation of absolute value; ε and k respectively denote the convergence threshold and the iteration number. When the iteration is performed for all columns of W, we can separate the optimization matrix S from I, and then the independent component matrix Z can be obtained by

$$Z = SI \tag{13}$$

And then

$$C = S^{-1} \tag{14}$$

where superscript "-1" represents the inverse operation of matrix. In PSI, by selecting the first three independent components (the first three columns of matrix C), corresponding to the constant 1, I'_c and I'_s, we can obtain the phase shifts as

$$\theta_n = \arctan(-I'_s / I'_c) \tag{15}$$

Similarly, by selecting the first three independent components of matrix Z, corresponding to A, I_c and I_s, we can obtain the wrapped phase as

$$\varphi = \arctan(-I_s / I_c) \tag{16}$$

3. Simulation

Numerical simulation is carried out to verify the effectiveness of the proposed ICA method, in which the laser with wavelength of $\lambda=632.8$ nm is employed. The simulated interference pattern is generated according to Eq. (1) by setting the parameters as