Optics Communications 367 (2016) 50–58

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Invited Paper

Zero-distance phase front of an isoplanar optical system

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A R T I C L E I N F O

Article history:
Received 27 September 2015
Received in revised form
7 January 2016
Accepted 9 January 2016
Available online 4 February 2016

OCIS Codes:
080.1010 Aberrations (global)
080.7343Wave dressing of rays
110.2990 Image formation theory
220.1010 Aberrations (global)

Keywords:
Zero-distance phase front
Isoplanar optical system
Eikonals
Wave aberration
Transverse aberrations
Fourier optics

A B S T R A C T

The concept of “the zero-distance phase front” of an isoplanar optical system is used to describe its aberration. It is shown that Walther's wave interpretation of eikonals allows treating “the zero-distance phase front” as the wave aberration function of the optical system and calculating its transverse aberrations.

1. Introduction

An ideal optical system transforms a family of spherical wave fronts diverging from an object point into a family of spherical wave fronts converging to an image point. A real optical system has aberrations and transforms the family of spherical wave fronts into a family of changing wave fronts converging in a neighborhood of the image point. In the image space of the real optical system the shape of the wave fronts depends on the distance from the image point. The wave aberrations of a real optical system are traditionally defined as the difference between the real wave front and the spherical wave front (the so-called “reference sphere”) in the plane of the exit pupil of the optical system [1–5].

It is known that from the family of wave fronts can be extracted one in the simplest shape which is a representative of the entire family nevertheless [6]. This wave front is the so-called wave front with a zero phase (or the zero-distance phase front) [6–14]. For example, in optics of ultrafast laser pulses the zero distance phase front of an optical system helps me to calculate the dispersion of a stretcher [15].

The goal of this work is to reformulate the traditional theory of the wave aberration of an optical system for the zero distance phase front by using Walther's wave treatment of eikonals [5,16–21].

2. Geometrical description of light wave propagation

Monochromatic wave propagation can be described by a family of phase fronts which is a locus of points of the wave with the same phase of oscillation [5,14,22]. An alternative way to describe wave propagation is given by orthogonal trajectories to the phase fronts, the so-called light rays. The shape of phase fronts (and light rays) is usually determined by the geometry of the source and the properties of the medium. In a homogeneous medium a point source S generates a family of concentric spherical (circular) phase fronts with a common center at S (or a homocentric beam of light rays with a vertex at S) (Fig. 1a), and a plane source generates plane phase fronts (or parallel rays) (Fig. 1b). All of these wave fronts move with the same speed at a certain distance from one another. This distance is the so-called wavelength of a monochromatic wave

$$\lambda = \lambda_o/n, \tag{1}$$

where λ_o is the wavelength of the monochromatic wave in vacuum.

In an optically inhomogeneous medium with the refraction index distribution $n(x,y,z)$, the wavelength of a monochromatic wave varies from point to point:

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http://dx.doi.org/10.1016/j.optcom.2016.01.023
0030-4018/

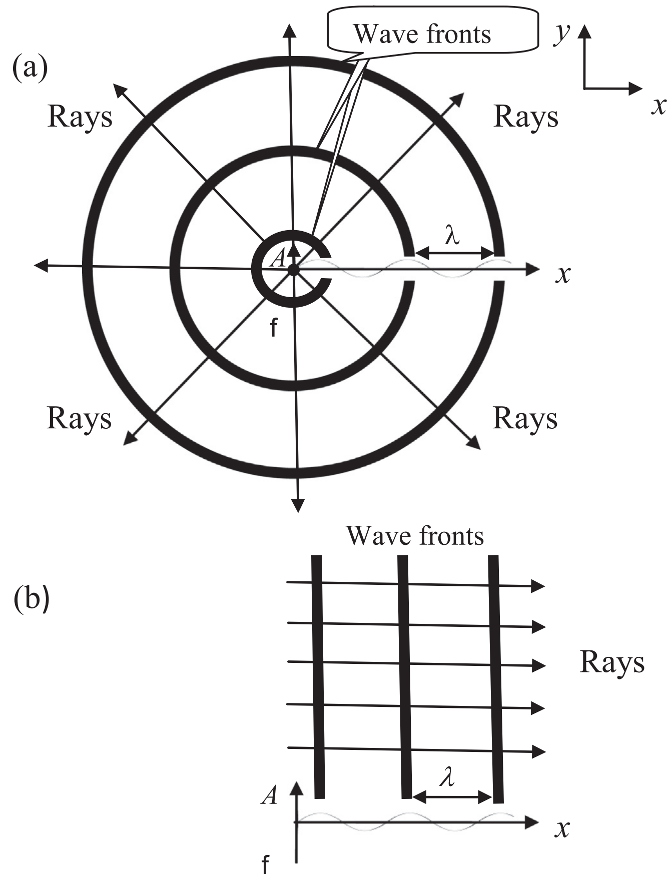

Fig. 1. (a) Spherical wave front and (b) plane wave front in a homogeneous and isotropic medium.

$$\lambda(x, y, z) = \lambda_o / n(x, y, z). \tag{2}$$

Thus, in this case wave fronts are not spheres, and the light rays propagate curvedly. But, according to the theorem of Malus and Dupin [22], the normal rectilinear congruence between light rays and wave fronts remains normal after passing through any optical system. The tautochronism principle is carried out as well [14,15,23,24]: in an optical system consisting of refracting and reflecting elements the phase delay T between any two wave fronts is identically the same for all rays (Fig. 2a). The tautochronism principle may be restarted in term of the optical path length which is defined by the line integral along any ray path γ from point A to point B:

$$\|AB\| \equiv \int_A^B n(x, y, z) ds, \tag{3}$$

where $n(x,y,z)$ is the refraction index at each point along the ray path γ, and $ds^2 = dx^2 + dy^2 + dz^2$. Thus, the tautochronism principle is the same as the principle of equal optical paths [22]: in an optical system consisting of refracting and reflecting elements the optical path length between any two wave fronts is the same for all rays. It implies that the wave fronts are “optically parallel” to each other.

An ideal optical system transforms spherical wave fronts emitted from an object point P into the spherical wave fronts gathering at point P' called the image point (Fig. 2b). As these optical conjugate points P and P' lie on the wave fronts, the necessary condition for the existence of a perfect optical system is a remarkable corollary of the tautochronism principle [14,15,23,24]: light takes identically the same time T_o to travel from the object point P to the image point P' along all rays traversing an ideal

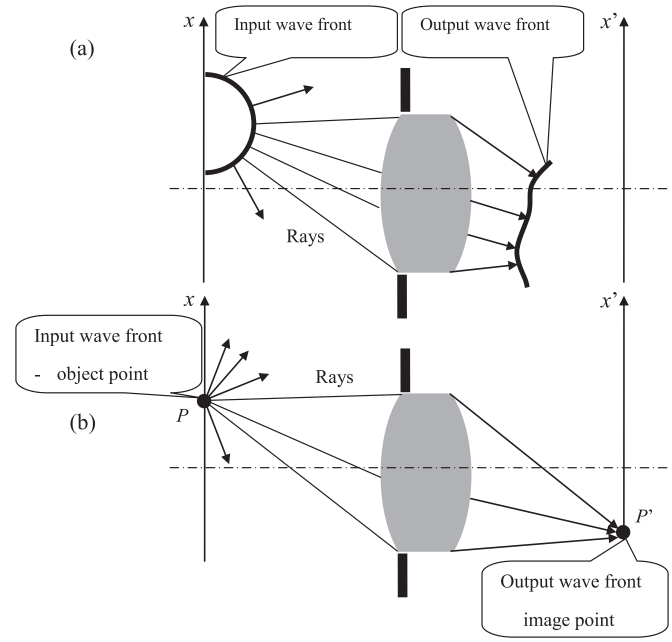

Fig. 2. Tautochronism principle: (a) in a general case, (b) in an ideal optical system.

optical system:

$$\|PP'\| = cT_o = R = const. \tag{4}$$

3. k_o-Fourier transformations reproduce Legendre transformations

In wave optics the k_o-Fourier transformations and the Legendre transformations are widely used [20,21,25,26]. The direct k_o-Fourier transformation of the complex-valued function $U(x,y)$ of real variables (x,y) to the complex-valued function $\tilde{U}(p, q)$ of real variables (p,q):

$$\tilde{U}(p, q) = {}^{k_o}F_{\substack{x\to p\\ y\to q}}\{U(x, y)\} \equiv \left(\frac{k_o}{2\pi}\right) \iint_{R^2} U(x, y)\cdot exp\left[-ik_o(x\cdot p + y\cdot q)\right]dxdy, \tag{5a}$$

and the inverse k_o-Fourier transformation of the complex-valued function $\tilde{U}(p, q)$ of real variables (p,q) to the complex-valued function $U(x,y)$ of real variables (x,y):

$$U(x, y) = {}^{k_o}F^{-1}_{\substack{p\to x\\ q\to y}}\left\{\tilde{U}(p, q)\right\} \equiv \left(\frac{k_o}{2\pi}\right) \iint_{R^2} \tilde{U}(p, q)\cdot exp\left[ik_o(x\cdot p + y\cdot q)\right]dpdq, \tag{5b}$$

are widely employed. The parameters p and q have the meaning of dimensionless spatial frequencies. The composition of the direct k_o-Fourier transformation and the inverse k_o-Fourier transformation (or the composition of the inverse k_o-Fourier transformation and the direct k_o-Fourier transformation) is the identity transformation I:

$${}^{k_o}F_{\substack{x\to p\\ y\to q}} \circ {}^{k_o}F^{-1}_{\substack{p\to x\\ q\to y}} = I, \;\ldots\ldots\; {}^{k_o}F^{-1}_{\substack{p\to x\\ q\to y}} \circ {}^{k_o}F_{\substack{x\to p\\ y\to q}} = I. \tag{6}$$

If the complex functions $U(x,y)$ and $\tilde{U}(p, q)$ are factorized into amplitude and phase factors: