Optics Communications 367 (2016) 351–355

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Sequential formation of multiple lattice gap solitons in defocusing photovoltaic–photorefractive crystals

Juanli Hui, Keqing Lu*, Chong Zhao, Lixu Gao, Weijun Chen

School of Electronics and Information Engineering, Tianjin Polytechnic University, Tianjin 300387, China

ARTICLE INFO

Article history:
Received 2 November 2015
Received in revised form
19 December 2015
Accepted 19 January 2016
Available online 4 February 2016

Keywords:
Gap solitons
Optical lattices
Photorefractive nonlinearity
Nonlinear optics

ABSTRACT

We show that an odd- or even-numbered sequence of multiple gap solitons (MGSs) supported by optical lattices imprinted in defocusing photovoltaic–photorefractive (PP) crystals is possible. The side lobes of MGSs are the small intensity, which can be controlled by changing the propagation constant. The lobes of MGSs are the same intensity except for the side lobes. The existence domain of MGSs decreases with decreasing the lattice depth and vanishes when the lattice depth decreases to below a certain threshold. The stability of MGSs has been investigated numerically and it has been found that they are stable.

1. Introduction

Study of light propagation in optical periodic structures such as waveguide arrays imprinted in semiconductors [1], photonic lattices [2–4], and arrays made in photovoltaic–photorefractive (PP) crystals [5,6] has attracted growing interest due to both fundamental physics and applications. Such periodic structures support gap solitons [7–17], and even and twisted solitons [18], or higher-order soliton trains [19,20]. Of particular interest are the arrays made in PP crystals that exhibit saturable nonlinearities impacting lattice soliton properties. In the arrays made in PP crystals, higher-order solitons have been predicted [21] and observed experimentally in defocusing lattices [22]. However, it would be of interest to explore whether sequential formation of multiple solitons [23] can be realized in optical lattices imprinted in defocusing PP crystals.

In this paper, we report that an odd- or even-numbered sequence of multiple gap solitons (MGSs) is possible in optical lattices imprinted in defocusing PP crystals. The lobes of MGSs are the same intensity except for the side lobes. The intensities of the side lobes of MGSs are small and increase with the propagation constant. The existence domain of MGSs decreases with decreasing the lattice depth, whereas MGSs do not exist when the lattice depth is equal to a certain threshold value. The stability of MGSs has been investigated numerically and we have found that they are stable.

* Corresponding author.
E-mail address: kqlutj@126.com (K. Lu).

http://dx.doi.org/10.1016/j.optcom.2016.01.054

2. The model

To start, let us consider the propagation of light beam along the z-axis of a defocusing PP crystal with a periodic modulation of the linear refractive index in the x-direction. The dynamics of propagation is described by the nonlinear Schrödinger equation for the dimensionless complex light field amplitude q [22]

$$i\frac{\partial q}{\partial \xi} = -\frac{1}{2}\frac{\partial^2 q}{\partial \eta^2} + \frac{q\,|q|^2}{1+S\,|q|^2} - pRq. \tag{1}$$

In the above equation, we have used the following normalized coordinates: $\xi = z/(kx_0^2)$ and $\eta = x/x_0$, where x_0 is an arbitrary transverse scale. $S = 2/(k^2 n_e^2 r_{33}|E_p|x_0^2)$, $k = 2\pi n_e/\lambda$ is the optical wave number in the PP crystal, n_e is the unperturbed extraordinary index of refraction, λ is the wavelength, r_{33} is the electro-optic coefficient, E_p is the photovoltaic field constant. $p = k^2\sigma x_0^2/2$ is the lattice depth, where σ is the depth of the refractive index modulation.The function $R = \cos^2(\Omega\eta)$ describes the transverse profile of the refractive index, where Ω is the modulation frequency. In this paper, let us consider a $LiNbO_3$ crystal with defocusing saturable nonlinearity, in which a waveguide array is imprinted. We take the values of parameters in Eq. (1) as $\Omega = 2.067$, $p=16$, and $S=0.5$, which are the experimental conditions in Ref. [22]. Notice that in Ref. [22], the experimental observation of higher-order solitons was reported in a waveguide array imprinted in a $LiNbO_3$ crystal with defocusing saturable nonlinearity. Eq. (1) conserves the energy flow $U = \int_{-\infty}^{\infty} |q|^2\, d\eta$.

In order to show the existent conditions for MGSs, it is necessary to first understand the dispersion relation and bandgap

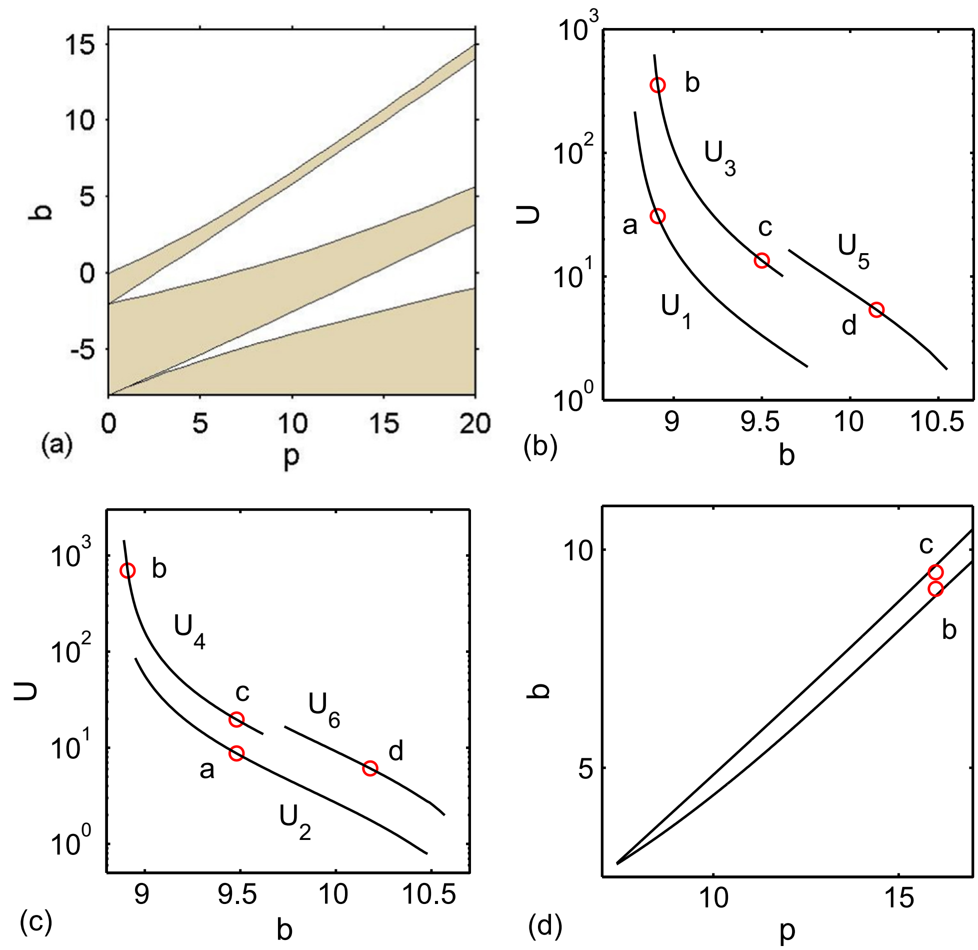

Fig. 1. (Color online) (a) Bandgap structure of periodic lattices; the shaded regions are Bloch bands. (b) and (c) Energy flow versus propagation constant for (b) odd and (c) even gap solitons of first three orders at $\Omega = 2.067$, $p=16$, and $S=0.5$. (d) Domain of existence for the triple gap soliton on the (p, b) plane at $S=0.5$. Profiles of MGSs at the circled points in (b)–(d) are displayed in Figs. 2 and 3.

structure of the linear version of Eq. (1). According to the Bloch theorem, eigenfunctions of the linear version of Eq. (1) can be sought in the form $q = u(\eta)\exp(ik_\eta\eta - ib\xi)$, where $u(\eta)$ is the complex periodic function with the same periodicity as the lattices, k_η is the wave number in the first Brillouin zone, and b is the Bloch-wave propagation constant. After substituting this latter form of $q(\eta, \xi)$ into the linear version of Eq. (1), an eigenvalue equation is obtained

$$\frac{1}{2}\frac{d^2q}{d\eta^2} + ik_\eta\frac{du}{d\eta} - \frac{1}{2}k_\eta^2 u + pRq = -bu, \tag{2}$$

from which the bandgap diagram can be obtained by the plane wave expansion method. The bandgap structure of the periodic photonic lattices is shown in Fig. 1(a). It reveals that there exist a single semi-infinite gap and an infinite number of finite gaps. In this study, we focus only on the same first finite gap as Ref. [22], in which higher-order gap solitons are observed in optical lattices imprinted in defocusing PP crystals.

To find the stationary soliton solutions in Eq. (1), let us express the dimensionless complex light field amplitude in the usual fashion: $q = u(\eta)\exp(-ib\xi)$, where $u(\eta)$ is the real-valued function. Direct substitution of this form of $q(\eta, \xi)$ into Eq. (1) leads to the following nonlinear equation:

$$\frac{1}{2}\frac{d^2u}{d\eta^2} - \frac{u^2}{1 + Su^2} + pRq + bu = 0, \tag{3}$$

from which MGSs can be determined by expanding functions $u(\eta)$ into discrete Fourier series and then converting Eq. (3) into a matrix eigenvalue problem with b being the eigenvalue [24].

3. Numerical results

We consider first an odd-numbered sequence of MGSs in Eq. (3). Fig. 1(b) shows the energy flow diagram of odd gap solitons for the fundamental, triple, and quintuple gap solitons when $\Omega = 2.067$, $p=16$, and $S=0.5$. This figure demonstrates that the energy flow of odd gap solitons decreases with an increase in the propagation constant b and increases with the order of odd gap solitons when b is fixed. The fundamental, triple, and quintuple gap solitons cannot exist at $9.76 < b < 8.78$, $9.62 < b < 8.89$, and $10.55 < b < 9.66$, respectively. Fig. 2(a) shows the profile of a fundamental gap soliton at $b=8.91$ [point a in Fig. 1(b)]. The fundamental gap soliton acquires its intensity maximum in the lattice minimum [Fig. 2(a)]. The triple-gap-soliton structure consists of two side lobes and a central lobe. The intensity of the two side lobes is smaller than that of the central lobe and increases with b [Fig. 2(b) and (c)]. When $b=8.91$ and 9.50 [points b, c in Fig. 1(b) and (d)], the profiles of the triple gap solitons are displayed in Fig. 2(b) and (c). The width of the existence domain for the triple gap soliton decreases with decreasing the lattice depth p and vanishes when p decreases to below a certain threshold, as