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Accelerating Airy beams in the presence of inhomogeneities

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ABSTRACT

Studies have already been made of accelerating Airy beams in the presence of deterministic inhomogeneities, illustrating, in particular, that the inherent self-healing properties of such beams are preserved. The cases of a range-dependent linear transverse potential and a converging GRIN structure (harmonic oscillator) have been examined thoroughly. Examples will be given in this article of novel accelerating Airy beams in the presence of five other types of potential functions. Three of the resulting exact analytical solutions have a common salient characteristic property: they are constructed using the free-space accelerating Airy beam solution as a seed.

1. Introduction

The basic finite-energy *accelerating* (nonlinearly laterally bending) Airy beam solution

$$\psi_f(x,z) = Ai\left[x - \left(\frac{z}{2}\right)^2 + iaz\right]\exp\left[ax - \frac{az^2}{2}\right] \exp\left[-i\left(\frac{z^3}{12} - \frac{a^2 z}{2} - \frac{xz}{2}\right)\right]; \quad a > 0 \tag{1}$$

is governed by the paraxial equation

$$i\frac{\partial}{\partial z}\psi_f(x,z) + \frac{1}{2}\frac{\partial^2}{\partial x^2}\psi_f(x,z) = 0 \tag{2}$$

in free space. Here, $x = X/X_0$ and $z = Z/(kX_0^2)$ are, respectively, dimensionless transverse and longitudinal variables, defined in terms of the original variables X and Z, the wavenumber k and a scaling factor X_0 with units of meters. The positive parameter α entering into the Airy beam solution in Eq. (1) ensures the square integrability (finite energy) of the input function $\psi_f(x, 0)$ and, hence, of $\psi_f(x, z)$ for $z > 0$. The solution given in Eq. (1) was first formulated analytically by Siviloglou and Christodoulides [1] and subsequently demonstrated experimentally by Siviloglou, Broky, Dogariu and Christodoulides [2]. Their work was motivated by the infinite-energy (nonspreading) *accelerating* Airy solution to the Schrödinger equation introduced by Berry and Balazs [3]; see also [4] and [5] in the context of quantum mechanics. A full wave theoretical analysis of the Airy beam has been undertaken by Kaganovsky and Heyman [6]. An Airy beam is slowly diffracting; it can retain its intensity over several diffraction lengths while bending laterally along a parabolic path despite the fact that its centroid is constant. Another feature, which has been demonstrated both analytically and experimentally, is that an Airy beam propagating in free space can perform ballistic dynamics akin to those of projectiles moving under the action of gravity [7]. Ultraintense Airy beams have been investigated in the nonlinear regime [8–10]. Accelerating spatiotemporal Airy wave packets can defy effectively both dispersion and diffraction [11–13].

Both bending Airy beams and accelerating Airy wavepackets are characterized by self-healing properties; they tend to reform in spite of the severity of imposed perturbations [14–16]. The robustness of such beams in scattering and turbulent environments has been studied analytically, numerically and experimentally in the optical regime [17–19]. These exotic properties suggest various physical applications, such as Airy beam-mediated particle cleaning and vacuum electron acceleration [20–22]. A recent review of the theory, generation and applications of Airy beams has been published by Hu et al. [23].

The specific purpose in this paper is to examine extended bending Airy beam solutions to the paraxial equation in the presence of a variety of deterministic inhomogeneties. The free-space paraxial equation is augmented by a "potential" function $V(x, z)$,

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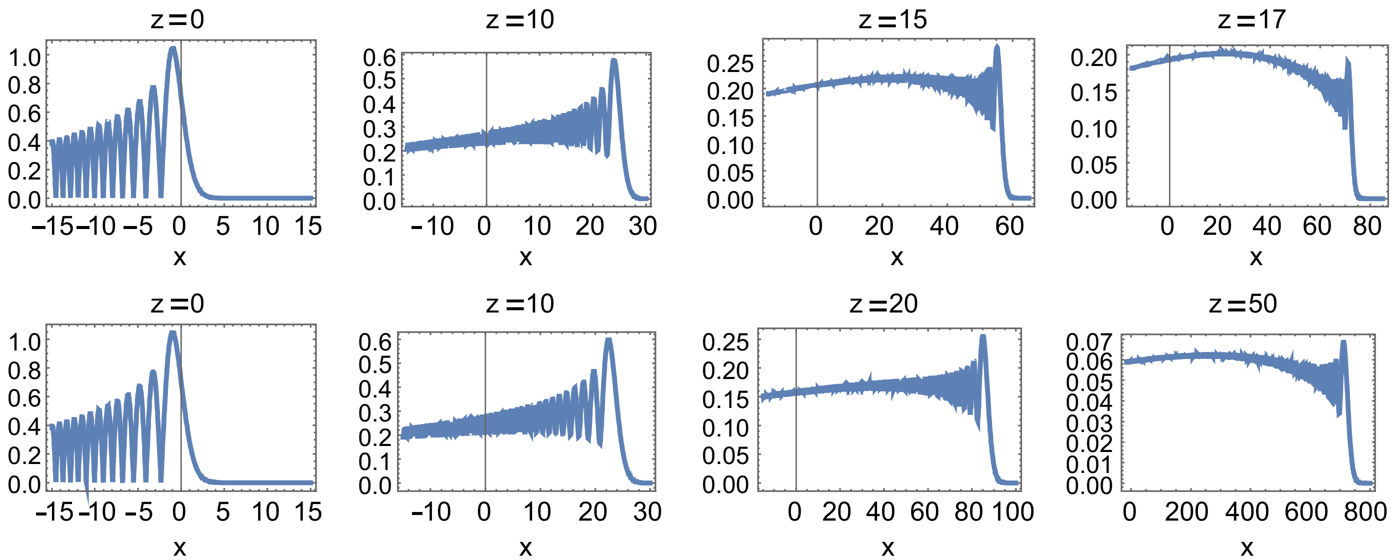

Fig. 1. Comparison of the intensity of the free-space Airy beam (upper raw) to that of the inverted harmonic oscillator (lower raw) versus x for different values of the range z. The parameter values are chosen as follows: $a = 2.5 \times 10^{-2}$ and $\omega = 1/16$. Even for such a relatively small value of ω, the inverted oscillator Airy beam outperforms that for free space. The free-space Airy beam loses its effectiveness as an almost nondiffracting beam at range $z = 17$, whereas the inverted oscillator maintains its effectiveness up to the much larger range $z = 50$ despite the relatively small value of the parameter ω. Thus, the former diffracts more quickly.

viz.,

$$i\frac{\partial}{\partial z}\psi(x, z) + \frac{1}{2}\frac{\partial^2}{\partial x^2}\psi(x, z) + V(x, z)\psi(x, z) = 0. \quad (3)$$

($V(x, z)$ differs by a negative sign from the conventional quantum mechanical potential function. Within the context of electromagnetic applications, the "potential function" should more appropriately be replaced by a permittivity depending on the dimensionless transverse and longitudinal variables x and z, respectively.) Accelerating Airy beams in the presence of special potential functions have already been studied. The case of a range-dependent linear transverse potential $V(x, z) = F(z)x$ was first examined by Berry and Balazs [3] in connection with the Schrödinger equation, where z is replaced by t. It has also been examined thoroughly by Efremidis et al. [24] in connection with the paraxial equation. Depending on the choice of the function $F(z)$, the lateral bending of the beam can be different from parabolic. The presence of a linear transverse optical potential has proven important for the efficient manipulation of plasmonic Airy beams [25,26]. Finally, Chávez-Cerda et al. [27] have demonstrated, both theoretically and experimentally, that one can reduce the acceleration and obtain a solitary wave with an Airy profile by modifying the strength of the linear gradient index.

Another situation that has already been examined is that of the "harmonic oscillator" potential $V(x, z)=-\frac{1}{2}\omega^2x^2$. Bandres and Gutiérrez-Vega [28] have used the Huygens approach together with the ABCD matrix to study the transformation of a Gauss-Airy beam through a medium characterized by a quadratic transverse profile. A family of paraxial hypergeometric laser beams propagating in a parabolic index fiber has been studied by Kotlyar et al. [29]. Zhang et al. [30] have carried out a detailed investigation of the periodic inversion and phase transition of finite-energy Airy beams in a medium with a parabolic potential. Finally, although not quite pertinent for the present article, it should be mentioned that an exact solution for Fresnel diffraction of a parabolic beam in a converging GRIN structure has been obtained by Dela Valle, Gatti and Longhi [31] using the Friedman–Robertson–Walker transformation.

Examples will be given below of novel accelerating Airy beams in the presence of five potential functions that are distinct from those mentioned earlier. A number of the resulting exact analytical solutions have a common salient characteristic property: they are constructed using the free-space Airy beam solution in Eq. (1) as a seed.

2. Accelerating Airy beams for the inverted harmonic oscillator

The inverted harmonic oscillator, with potential $V(x, z)=\frac{1}{2}\omega^2x^2$, has attracted great attention not only because it is one of the exactly solvable potentials in quantum mechanics but also for having a wide range of applications in several branches of physics, e.g., high energy physics and solid state theory. In the framework of optics, the negative parabolic index of refraction is a special case of the "antiguide" potentials discussed by Chremmos and Giamalaki recently [32].

Given a solution $\psi_f(x, z)$ of the free-space paraxial equation in Eq. (2), a solution for the inverted harmonic oscillator corresponding to the potential $V(x, z)=\frac{1}{2}\omega^2x^2$ is given by a variant of the Niederer transformation [33]; specifically,

$$\psi(x, z) = \frac{1}{\sqrt{\cosh(\omega z)}}e^{i\frac{\omega}{2}x^2\tanh(\omega z)}\psi_f\left[\frac{x}{\cosh(\omega z)}, \frac{\tanh(\omega z)}{\omega}\right]. \quad (4)$$

If $\psi_f(x, z)$ is replaced by the basic finite-energy accelerating Airy beam solution given in Eq. (1), one obtains the following accelerating Airy beam solution to the paraxial Eq. (3) with an inverted parabolic index profile:

$$\psi(x, z) = \frac{1}{\sqrt{\cosh(\omega z)}}\exp\left[i\frac{\omega}{2}x^2\tanh(\omega z)\right]$$
$$\times\exp\left[-\frac{1}{24}\left(2a + i\frac{\tanh(\omega z)}{2}\right)\left(4a^2 - i8a\frac{\tanh(\omega z)}{\omega} + 2\left(-6x\operatorname{sech}(\omega z) + \frac{\tanh^2(\omega z)}{\omega^2}\right)\right)\right]$$
$$\times Ai\left[x\operatorname{sech}(\omega z) + \frac{\tanh(\omega z)}{4\omega}\left(4ia - \frac{\tanh(\omega z)}{4\omega}\right)\right]. \quad (5)$$