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A novel synthesis approach for birefringent filters having arbitrarily amplitude transmittances

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ABSTRACT

In this paper, we present a novel procedure for the synthesis of a filter having an arbitrarily specified amplitude transmittance. The filter configuration consists of *N* birefringent stages placed between a polarizer and an analyzer, with each stage containing an identical section and a variable section. An additional variable section is placed in front of the analyzer. The synthesis procedure is based on the resolution of a generalized nonlinear equation system directly deducted from the Jones matrix formalism to determine the angles of each stage, the angle of the analyzer and the phase shifts of the variable sections. A typical example of a 6-stage birefringent filter having an arbitrarily non-symmetric amplitude transmittance is shown and the opto-geometrical parameters are given to demonstrate the efficiency of the proposed synthesis procedure. The results obtained show an excellent agreement with those developed in the literature.

1. Introduction

Optical filters whose filtering characteristics can be predefined are highly desirable for a large number of applications [1–8]. Among these filters, birefringent filters have attracted particular interest since the early 20th century [9–12] and several synthesis algorithms have been developed in order to design filters having a specified amplitude transmittance. Harris et al. [13] developed an algorithm based on equations relating the input impulses to the output impulses at each equal length birefringent crystal. N birefringent crystals are then required to realize a symmetric amplitude transmittance approximated with a Fourier series containing $(N+1)$ terms. The algorithm determines the rotation angle of each crystal and the rotation angle of the output polarizer. Amman et al. [14] extended the Harris' method to a non-symmetric amplitude transmittance by using birefringent crystals associated to compensators introducing variable phase shifts. Chu et al. [15] proposed a method based on the Z-transform. The two orthogonal components of the electric field deducted from the Jones formalism are expressed as z-polynomial series. The digital filter design techniques are then used to calculate the parameters of each birefringent element. Nevertheless, these algorithms are relatively cumbersome since an inverse transform technique is usually needed to obtain the specified spectral response. Recently, Vitanov et al. [16,17] designed N-stacked composite plates which, when rotated at specific angles, act as either broadband half-wave "retarders or tunable narrowband filters. The *N* rotation angles are found by solving *N* nonlinear equations.

In this paper, we present a new method for the synthesis of birefringent filters having an arbitrary-shape amplitude transmittance. It is based on the resolution of a generalized nonlinear equation system deducted from the Jones matrix formalism. The transmittance is approximated by a Fourier series containing $(N+1)$ complex coefficients C_k, where N is the number of the filter stages. On the other hand, the electric field at the output of the filter is expressed as an exponential series with $(N+1)$ complex coefficients E_k which are derived from the Jones matrix formalism. Equating the imaginary and real parts of C_k and E_k leads to a generalized nonlinear equation system which can be resolved by optimization methods.

The paper is organized as follows. In Section 2, the mathematical formulation and the setting of the equations deducted from the Jones matrix formalism are exposed. Once the generalized system of nonlinear equations is established, we proceed to its resolution to determine the filter's opto-geometrical parameters. In Section 3, demonstration results followed by discussions are exposed. We expose, as a proof of principle test, the synthesis of filters whose amplitude transmittances are non-symmetric. Finally, some conclusion remarks are given in Section 4.

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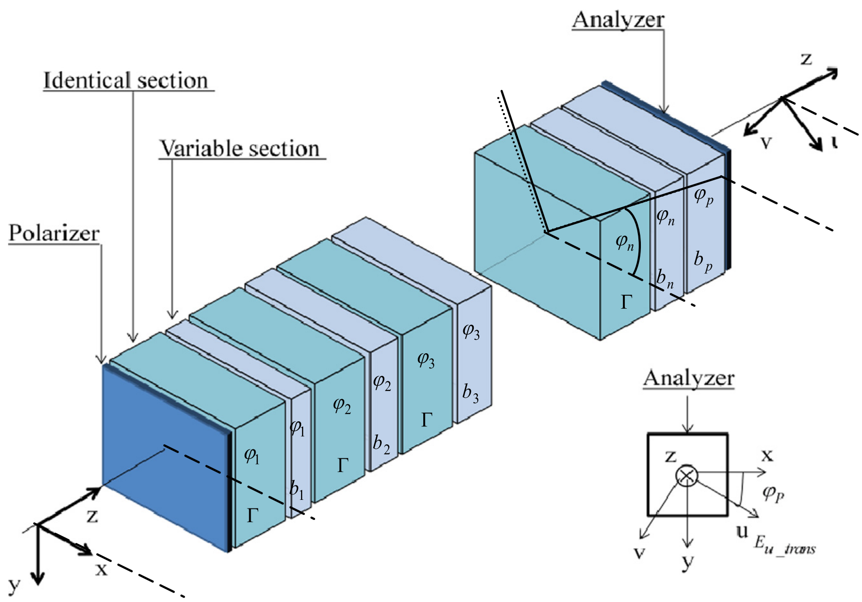

Fig. 1. The generic diagram of the birefringent filter having arbitrarily amplitude transmittances. Γ: Phase shift introduced by the identical section, b_k : Phase shift introduced by the variable section of the kth stage, φ_k : Rotation angle of the kth stage.

2. Synthesis method and filter structure

Fig. 1 depicts the physical structure of the birefringent filter whose amplitude transmittance has an arbitrary shape. The filter consists of a stack of N stages. Each stage is composed of an identical section associated to a variable section whose slow and fast axes are parallel to each other. An additional variable section introducing a phase shift b_p is placed in front of the output analyzer with its slow axis parallel to the transmission axis of this analyzer. The identical sections, oriented each at an angle φ_k (k=1, 2,…N) with respect to the x axis, have the same geometrical thickness and introduce the same phase shift Γ. The variable sections introduce variable phase shifts b_k.

For the sake of simplicity, we assume that the transmission axis of the polarizer P is chosen parallel to the laboratory x axis while the transmission axis of the analyzer A is parallel to the u axis as shown in the inset of Fig. 1. We also assume that the input electric vector has an amplitude unity and is totally polarized parallel to the transmission axis of the polarizer. The synthesis method adopted is then based on the Jones formalism to express the electric field vector at the output of the filter as a function of the opto-geometrical parameters of the constituent sections.

Referring to Fig. 1, the output electric vector E_{u_trans} emerging parallel to the transmission axis of the analyzer is found by multiplying the input electric vector by the overall Jones matrix representing the birefringent filter as,

$$\begin{pmatrix} E_{u\_trans} \\ 0 \end{pmatrix} = P_A.\ R(\varphi_P).\ \{R(-\varphi_P).\ M_P.\ (b_P).\ R(\varphi_P)\}.\ \{R(-\varphi_N).\ M_N(\Gamma_N).\ R(\varphi_N)\}......\ \{R(-\varphi_2).\ M_2(\Gamma_2).\ R(\varphi_2)\}.\ \{R(-\varphi_1).\ M_1(\Gamma_1).\ R(\varphi_1)\}.\ P_p.\ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (1)$$

where φ_k represents the rotation angle of the kth stage, with respect to the x axis,

and

$\Gamma_k = \Gamma + b_k$ is the phase shift introduced by the stage, where Γ is the phase shift introduced by the identical section, and b_k represents the phase shift introduced by the variable section of the stage.

$M_k = \begin{pmatrix} e^{-i\Gamma_k} & 0 \\ 0 & 1 \end{pmatrix}$ $k = 1, 2, ... N$, is the matrix of the kth section expressed in its own reference frame,

$M_P(b_P) = \begin{pmatrix} e^{-ib_P} & 0 \\ 0 & 1 \end{pmatrix}$ is the Jones matrix of the last variable section expressed in its own reference frame,

$P_p = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, is the Jones matrix of the polarizer, expressed in the x–y laboratory reference frame,

$P_A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, is the Jones matrix of the analyzer, expressed in its own u–v reference frame.

This field can also be written as:

$$\begin{pmatrix} E_{u\_trans} \\ 0 \end{pmatrix} = P_A.\ M_P(b_P).\ \{R(\theta_P).\ M_N(\Gamma_N)\}......\ \{R(\theta_2).\ M_1(\Gamma_1)\}.\ R(\theta_1).\ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (2)$$

where

$R(\delta) = \begin{pmatrix} \cos\delta & \sin\delta \\ -\sin\delta & \cos\delta \end{pmatrix}$ is the rotation matrix expressed in the x–y laboratory reference frame with

$\delta = \varphi_p, \theta_p, \varphi_k, \theta_k$ where θ_k represents the relative angle such as

$\theta_1 = \varphi_1$

$\theta_2 = \varphi_2 - \varphi_1$

.....

$\theta_N = \varphi_N - \varphi_{N-1}$

$\theta_P = \varphi_P - \varphi_N$

Eq. (2) can be written in the form,

$$\begin{pmatrix} E_{u\_trans} \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} . \begin{pmatrix} e^{-ibP} & 0 \\ 0 & 1 \end{pmatrix} . . \begin{pmatrix} \cos\theta_P e^{-i\Gamma_N} & \sin\theta_P \\ -\sin\theta_P e^{-i\Gamma_N} & \cos\theta_P \end{pmatrix} . \prod_{k=2}^{N} . \begin{pmatrix} \cos\theta_k e^{-i\Gamma_{k-1}} & \sin\theta_k \\ -\sin\theta_k e^{-i\Gamma_{k-1}} & \cos\theta_k \end{pmatrix} . \begin{pmatrix} \cos\theta_1 \\ -\sin\theta_1 \end{pmatrix} \quad (3)$$

Carrying out the matrix product in Eq. (3) leads to the expression of the electric field E_{u_transm} in the form of an exponential series such as,

$$E_{u\_transm} = \sum_{k=0}^{N} E_k e^{-ik\Gamma} = E_0 + E_1 e^{-i\Gamma} + \dots\dots + E_N e^{-iN\Gamma} \quad (4)$$