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Laser frequency stabilization using folded cavity and mirror reflectivity tuning

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ABSTRACT

A new method of laser frequency stabilization using polarization property of an optical cavity is proposed. In a standard Fabry–Perot cavity, the coating layers thickness of cavity mirrors is calculated to obtain the same phase shift for s- and p-wave but a slight detuning from the nominal thickness can produce s- and p-wave phase detuning. As a result, each wave accumulates a different round-trip phase shift and resonates at a different frequency. Using this polarization property, an error signal is generated by a simple setup consisting of a quarter wave-plate rotated at 45°, a polarizing beam splitter and two photodiodes. This method exhibits similar error signal as the Pound–Drever–Hall technique but without need for any frequency modulation. Lock theory and experimental results are presented in this paper.

1. Introduction

Fabry–Perot cavities have a variety of applications, such as high X/γ ray flux production [1–3], high-harmonic generation [4,5], polarized positron generation [6,7], laser-wire and e-beam polarization measurement [8,9].

Frequency locking is needed to maintain a cavity in resonance. An error signal that is proportional to the difference between the laser and the cavity resonance frequencies [10] is thus required. This error signal is used in a feedback system to actuate the laser repetition frequency to compensate the fluctuation between it and the cavity free spectral range. A common method for laser frequency stabilization is the Pound–Drever–Hall (PDH) method [11,12] where the laser frequency is modulated to create non-resonant sidebands in the laser spectrum to produce an error signal. In a similar way, tilt locking uses non-resonant spatial modes produced by a misalignment of the laser beam with respect to the cavity as sidebands [10,13]. In addition, Hansch–Couillaud method introduces a polarizer inside the cavity to change the polarization of the laser beam to produce an error signal [14]. A variation of this method has been realized in Ref. [15] without the addition of a polarizer, exploiting the unique property of non-planar cavities to have a different round-trip phase shift for left and right circular-polarization waves. Finally, the intrinsic birefringence of the mirrors coating of an optical cavity can also be used to produce an error signal [16].

A tiny change of the coating layer thickness of the cavity mirrors leads to s- and p-wave phase detuning. In the same way, if the incidence angle of cavity mirror is different from the nominal one for which the coating has been optimized, s- and p-wave also exhibit different phase shifts although the cavity is made of an even number of mirrors [17]. As a result, s- and p-wave accumulate a different phase after one round-trip in the cavity, i.e. they resonate at different frequencies. In this paper, a new method is proposed to implement a simple error signal by using this polarization property. To the best of our knowledge, this lock method using the mirror reflection phase shift for different polarization has never been implemented before. Contrary to the method of Ref. [16], the method presented in this paper is not usable for two-mirror cavities since it uses a phase shift between the s and p waves reflection coefficient. However, the error signal obtained with the method of Ref. [16] depends on the azimuth orientation of the cavity mirrors [18], while it only depends on the incident angles which are fixed by the cavity geometry in the method described in this paper. The implementation of the method of Ref.

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[16] in the case of four-mirrors cavity may therefore be more complex.

This paper is organized as follows. Phase detuning of the mirror coating is presented in Section 2. The scheme of locking method is described in Section 3. The experimental results are in Section 4.

2. Phase detuning of the mirror coating

The phase shift difference between s- and p-wave can be controlled by adjusting the last layer of a quarter-wave layers stack like $(HL)^N HL_\alpha$, where H is a high index quarter-wave layer (e.g. Ta_2O_5), L is a low index quarter-wave layer (e.g. SiO_2), N is the number of periods, and L_α is the special tuning layer where α is the tuning coefficient of the last coating thickness. The matrix theory developed in Ref. [19] allows us to compute the normalized electric and magnetic fields **B** and **C** at the front surface of the multilayer and parallel to it. All the optical properties can be deduced from these two quantities.

The characteristic matrix of a non-absorbing layer with refraction index n and a thickness e is defined by

$$R(\delta, \eta) = \begin{pmatrix} \cos\delta & i\sin\delta/\eta \\ i\eta\sin\delta & \cos\delta \end{pmatrix} \tag{1}$$

where $\delta = 2\pi ne\cos\theta/\lambda$, $\theta = \arcsin(\sin\theta_0/n)$, θ_0 is the angle of incidence in vacuum, η is the optical admittance given by $\eta = \eta_0 n\cos\theta$ or $\eta = \eta_0 n/\cos\theta$ for s- and p-wave respectively, and $\eta_0 = \sqrt{\varepsilon_0/\mu_0}$ is the vacuum admittance. The dependence of η with polarization comes from different boundary conditions for **B** and **C** at the interface in both polarization planes.

A multilayer stack can be represented by an equivalent surface with an admittance Y given by

$$Y = \frac{\| \mathbf{C} \|}{\| \mathbf{B} \|} \tag{2}$$

$$\begin{pmatrix} B \\ C \end{pmatrix} = R(\delta_L', \eta_L) R\left(\frac{\pi}{2}, \eta_H\right)\left[R\left(\frac{\pi}{2}, \eta_L\right) R\left(\frac{\pi}{2}, \eta_H\right)\right]^N \begin{pmatrix} 1 \\ \eta_{sub} \end{pmatrix}. \tag{3}$$

The subscripts H, L and sub stand for high index material, low index material and the substrate respectively. The first matrix is the characteristic matrix of the tuned layer, for which $\delta_L' = \alpha\pi/2$, the second matrix is for the last high index quarter-wave layer and the last one is for N periods of quarter-wave layers (HL).

The amplitude reflection coefficient is calculated from $r = (\eta_0 - Y)/(\eta_0 + Y)$. Thus, the phase shift on reflection can be deduced from

$$\tan\varphi = \frac{\mathrm{Im}(r)}{\mathrm{Re}(r)} \tag{4}$$

where

$$\mathrm{Im}(r) = -2\cos\delta_L' \sin\delta_L' \eta_0 [\eta_{sub}^2 \eta_L/\eta_H^2 (\eta_L/\eta_H)^{2N} - \eta_H^2/\eta_L (\eta_H/\eta_L)^{2N}] \tag{5}$$

and

$$\mathrm{Re}(r) = \eta_0^2 \Big[\sin^2\delta_L' \eta_H^2/\eta_L^2 (\eta_H/\eta_L)^{2N} + \cos^2\delta_L' \eta_{sub}^2/\eta_H^2 (\eta_L/\eta_H)^{2N}\Big] - \cos^2\delta_L' \eta_H^2 (\eta_H/\eta_L)^{2N} - \sin^2\delta_L' \eta_{sub}^2 (\eta_H/\eta_L)^{2(N+1)} \tag{6}$$

For high reflective coating with a large number of periods N (i.e. $N > 10$), Eq. (4) reads

$$\varphi \simeq \arctan\left(\frac{2\eta_{air}/\eta_L \cos\delta_L' \sin\delta_L'}{\eta_{air}^2/\eta_L^2 \sin^2\delta_L' - \cos^2\delta_L'}\right) \tag{7}$$

In this case, the phase shift depends only on the tuned layer phase factor δ_L' and optical admittance η_L. Unlike the former, the latter depends on the polarization plane, thus inducing different phase shifts on reflection for both polarizations. The phase shift difference is given as a function of the tuning parameter α in Fig. 1. Phase shift differences up to ± 3.5 mrad can be achieved with α=0.62 or 1.38. The effect of the birefringence was simulated using the model of Ref. [20] and found that it did not change significantly the curve of Fig. 1. Influence of thermal increase of the coatings on the phase shift difference was simulated and found to be of the order of 7.5×10^{-6} rad/K. This effect can thus be neglected.

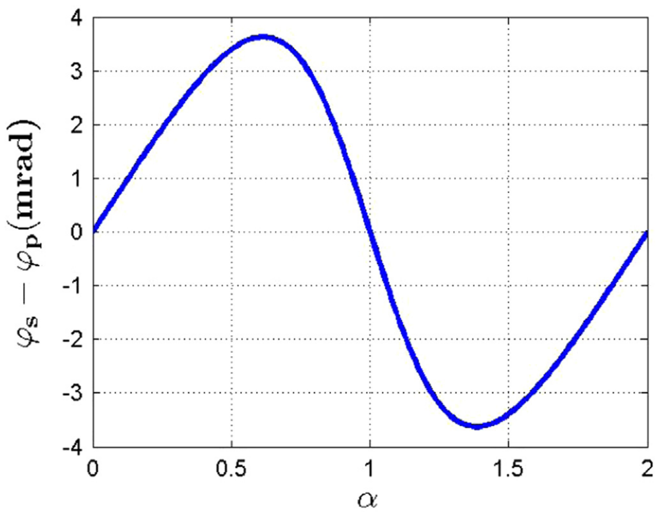

Fig. 1. Difference between the s- and p-wave reflection phase shifts $(\varphi_s - \varphi_p)$ as a function of the tuning coefficient α of the first layer.

This technique has been used to design the coating of the four-mirror planar bow-tie cavity in Section 3.

3. Stabilization method

This section concentrates on four-mirror planar cavities, but the method can be extended for other topologies of cavities. The complex reflection coefficient for s- and p-wave in the basis attached to the incidence plane of mirror i reads [17] $r_{is,p} = \rho_{is,p}\exp(i\varphi_{is,p})$ where $\rho_{is,p}$ and $\varphi_{is,p}$ $(i \in [1, 4])$ are respectively the amplitude and phase shift of the reflection coefficient for s- and p-waves of the ith mirror. The Jones vector circulating in the cavity is [17]

$$\mathbf{V}_{circ} = \begin{pmatrix} \dfrac{t_{1s}}{1 - \xi_s\exp(i\Psi_s)} & 0 \\ 0 & \dfrac{t_{1p}}{1 - \xi_p\exp(i\Psi_p)} \end{pmatrix} \mathbf{V}_0 \tag{8}$$

where $t_{1s,p}$ are the transmission factors of the input mirror for both s- and p-wave; $\xi_{s,p} = \rho_{1s,p}\rho_{2s,p}\rho_{3s,p}\rho_{4s,p}$, and $\Psi_{s,p} = \varphi_{1s,p} + \varphi_{2s,p} + \varphi_{3s,p} + \varphi_{4s,p} + \varphi_{space}$, with φ_{space} being the spatial propagation phase shift, $\mathbf{V}_0^T = (V_{0s}, V_{0p})$ normalized to 1 is the incident Jones vector of polarization. The intensity circulating in the cavity is

$$\frac{I_{circ}}{I_{in}} = \left|\frac{t_{1s}}{1 - \xi_s\exp(i\Psi_s)} V_{0s}\right|^2 + \left|\frac{t_{1p}}{1 - \xi_p\exp(i\Psi_p)} V_{0p}\right|^2. \tag{9}$$

Ignoring the losses in the cavity, the complex amplitude of the reflected wave is [14,21]