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Perfect tunneling of obliquely-incident wave through a structure with a double-negative layer

S.A. Afanas'ev*, D.I. Sementsov, Y.V. Yakimov

Department of Radiophysics and Electronics, Ulyanovsk State University, 432017 Ulyanovsk, Russian Federation

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ABSTRACT

The oblique incidence of TE-polarized plane electromagnetic wave on a three-layered lossless structure containing the layer of double-negative medium is discussed. The resonant values of the angle of incidence are obtained, for which the perfect tunneling of electromagnetic power through the structure can be achieved. The results of exact numerical analysis are compared with approximate solution based on the model of symmetrical slab waveguide.

1. Introduction

The media transparent for electromagnetic (EM) radiation may be either double-positive (DPS) or double-negative (DNG) depending on the sign of its permittivity ε and permeability μ. Conventional DPS (or "right-handed") media with $\varepsilon,\ \mu > 0$ have positive refractive index $n > 0$. DNG media with simultaneously negative ε and μ and $n < 0$ were introduced hypothetically by Veselago [1], who called them "left-handed". The artificial composite DNG media called "metamaterials" were first demonstrated in 2000 [2,3]. They exhibit unusual electromagnetic properties if compared with DPS media. Among them, the remarkable transmission properties of layered structures containing DNG layers are of great interest [3–5].

Also there are single-negative (SNG) media, in which only one of the two material constants is negative. They can be epsilon-negative (ENG) with $\varepsilon < 0,\ \mu > 0$ or my-negative (MNG) when $\varepsilon > 0$, $\mu < 0$. In contrary to DPS and DNG media, the SNG media are opaque even if they are lossless. The matter is that they support only evanescent waves because of purely imaginary wavevector. However, evanescent waves can transfer EM power through a slab of SNG material of a finite thickness. The EM field in the slab is the result of the interference of two evanescent waves decaying at opposite directions. As it is known, these two waves give rise to the non-decaying energy flux, providing the power transmission through the slab, i.e. the EM tunneling. Moreover, the transmission may be total, i.e. the transmittance coefficient may reach unity. This effect is called the "perfect" or "complete" tunneling. It was predicted in [6] for a system of alternating ENG and MNG layers with matching parameters. In general, the perfect tunneling may be reached for any type of waves in various structures supporting evanescent waves [7–9]. Commonly, the total transmission is connected with resonance effects such as, for example, the excitation of surface waves. Up to date, a lot of various lossless layered structures containing SNG layers have been demonstrated to realize the perfect tunneling [10–15]. Yet in [6], the possibility of complete tunneling has been shown also for multilayer structures with alternating DPS and DNG layers. Since then a few works have appeared considering the use of DNG metamaterials in tunneling systems [5,16–20].

In [5,16,17] the perfect tunneling has been considered in the system based on a hollow rectangular waveguide that included three sections. The central section, operating below cutoff, was empty. The input and output sections filled with DPS dielectric having $\varepsilon > 1,\ \mu = 1$ were above cutoff. The propagative fundamental TE_{10} mode existing in the first section transformed into the evanescent mode in the second section of the waveguide. Due to the presence of reflected evanescent mode decaying at opposite direction, some power could tunnel to the third section. A multilayer structure that included m layers of DNG metamaterial spaced by air gaps was placed in the central section. In [5,16,17], only the case of "perfect" DNG metamaterial with $\varepsilon = \mu = -1$ ($n = -1$) has been discussed. These waveguide structures were called in [17] "the mth-order structures", where m is the number of DNG layers. Under certain conditions, the tunneling of power through these waveguide structures could be perfect. As it has been shown in

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* Corresponding author.
E-mail address: asa_rpe@mail.ru (S.A. Afanas'ev).

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[5,16,17], the perfect tunneling is accompanied by the resonant amplification of evanescent waves at the metamaterial – air interfaces.

Another way to obtain evanescent waves in layered structures is provided by the phenomenon of total internal reflection. If an EM wave is incident on a slab from optically dense medium at the angle of incidence which exceeds the critical angle of total reflection, the evanescent waves decaying in opposite directions are excited in the slab and the tunneling of EM power is observed. So, there is a possibility to obtain the evanescent waves in air gap sandwiched between two media with $|n| > 1$.

In [21], the anomalous Goos–Hänchen effect has been studied in a five-layer structure containing a slab of DNG metamaterial. The middle DNG layer was separated by two gap slabs from optically dense bounding medium. The oblique incidence of a Gaussian beam of transverse-electric (TE) polarization at the angle larger than critical angle of total reflection was considered. At these circumstances, the waves in gap slabs were evanescent, resulting in the excitation of either surface waves at the surfaces of DNG layer or leaky guided waves supported by DNG slab. If the tangential component of beam wavevector was the same as the propagation constant of one of the guided modes of DNG slab, the large values of Goos–Hänchen effect were observed.

The arrangement of layers in the five-layered structure studied in [21] is similar to the waveguide structure of first order ($m = 1$) from [5,16,17]. In this connection, we may suppose the possibility of perfect tunneling for such structures under oblique incidence of EM wave in the case of total reflection. Also, one may expect that the condition of total transmission may be associated with the guidance conditions of a DNG slab waveguide. Indeed, if the guidance conditions are matched for a symmetric slab waveguide, the distribution of EM field amplitude across the slab has a symmetrical shape, what is the distinctive feature of perfect tunneling.

In this paper we consider a lossless layered structure DPS–air – DNG–air–DPS which is analogous to the structure from [21]. In contrast to [21], our purpose is to determine the conditions of total transmission under the oblique incidence of a plane TE-polarized EM wave from the first semi-indefinite DPS medium. We vary in a wide enough range the angle of incidence, the refractive index of the middle DNG layer as well as the ratio of thicknesses of DNG layer and air gaps. The results obtained by exact numerical solution of EM boundary problem are compared with the known solution [22–24] for guided modes of a symmetric DNG-slab waveguide.

2. Basic relationships

The layered structure which is under our investigation is depicted schematically in Fig. 1 with the axes of attached Cartesian coordinate system. We imply that the layers are infinite in the directions of x- and y-axes. In the direction of z-axis, the structure is bounded by two semi-infinite dielectric DPS media (regions 1 ($z < 0$) and 5 ($z > z_3$)), having equal material parameters $\varepsilon_1 > 0$, $\mu_1 > 0$ and positive refractive index $n_1 = (\varepsilon_1\mu_1)^{1/2}$. The central layer (region 3, $z_1 < z < z_2$) of thickness L_3 is DNG medium with $\varepsilon_3 < 0$, $\mu_3 < 0$ and negative refractive index $n_3 = -(\varepsilon_3\mu_3)^{1/2}$. The regions 2 ($0 < z < z_1$) and 4 ($z_2 < z < z_3$) are two air gaps of equal thickness $L_2 = L_4$ having parameters $\varepsilon_2 = \mu_2 = 1$ and $n_2 = 1$.

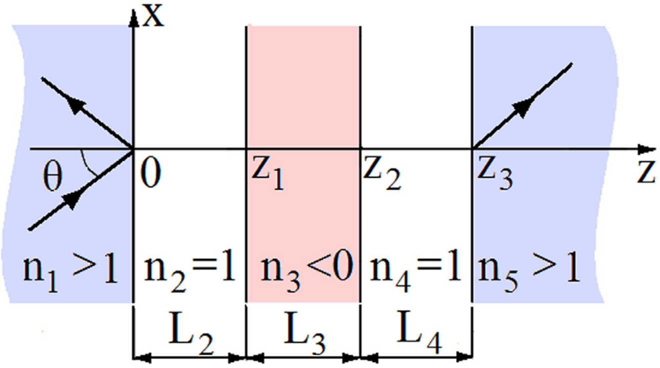

Fig. 1. Geometry of the problem.

A plane monochromatic wave of frequency ω with the electric vector polarized along y-axis is incident on the structure from region 1 at the angle of incidence θ. The x-component h of its wavevector is

$$h = k_0 n_1 \sin\theta, \tag{1}$$

where $k_0 = \omega/c$ – wavenumber for vacuum (air) and c is the light velocity for vacuum. The quantity h has the same value in each region, while the transversal z-component of the wavevector depends on the material parameters of a medium and is, in general, complex:

$$\gamma_j = \pm\sqrt{k_0^2\varepsilon_j\mu_j - h^2} = \beta_j + i\alpha_j, \tag{2}$$

where $j = 1 \ldots 5$ is a number of the region.

The electric field of incident wave in the region 1 may be written as

$$E_{1y}^{+}(z) = A\exp(i\gamma_1 z), \tag{3}$$

where A is the field amplitude and $\gamma_1 = k_0 n_1 \cos\theta$ has a positive real value (in the case of lossless DPS medium). Besides the incident wave, there is the reflected wave in the region 1, with the field determined as

$$E_{1y}^{-}(z) = rA\exp(-i\gamma_1 z), \tag{4}$$

where r is the complex amplitude coefficient of reflection. Let us note that the fields (3) and (4) have the phase factor $\exp(ihx - i\omega t)$, common to them and to all fields in each region of the structure. In what follows we shall omit this factor.

The electric fields in regions 2–4 are also the superposition of two waves propagating in opposite directions along z-axis:

$$E_{jy}^{\pm}(z) = a_j^{\pm}A\exp(\pm i\gamma_j z),\ j = 2,\ 3,\ 4, \tag{5}$$

where $a_j^{\pm}$ are complex amplitude coefficients. The upper signs in (5) and below correspond to the waves propagating forward and backward to z-axis. For lossless media in the regions 2–4 these waves may be non-decaying (γ_j is real) or evanescent (γ_j is imaginary). The first case is realized under the condition $h^2 < k_0^2\varepsilon_j\mu_j$ when $\gamma_j = \beta_j$ and the plus sign of the radical in (2) should be chosen for air in the regions 2 and 4 and minus sign for the DNG medium in the region 3. The condition for evanescent waves is $h^2 > k_0^2\varepsilon_j\mu_j$, when the quantity of γ_j is determined by $\gamma_j = i\alpha_j = i\sqrt{h^2 - k_0^2\varepsilon_j\mu_j}$ for $j = 2,\ 3,\ 4$. Here, the sign of the radical is always plus, corresponding to decaying of field amplitudes along the direction of wave propagation.

There is only a single transmitted wave in the region 5, which electric field differs from the field of the incident wave (1) by the amplitude coefficient of transmission t:

$$E_{5y}^{+}(z) = tA\exp(i\gamma_1 z). \tag{6}$$

Accordingly to Maxwell equations, the magnetic field components may be obtained from the electric fields (3)–(6) with the help of expressions

$$H_{jx}^{\pm} = \pm\zeta_j E_{jy}^{\pm}; \qquad H_{jz}^{\pm} = \xi_j E_{jy}^{\pm}, \tag{7}$$