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Dynamics and nonclassical properties of an opto-mechanical system prepared in four-headed cat state and number state

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ABSTRACT

The nonlinear interaction between an optical field and a mechanical resonator of an opto-mechanical system plays an important role in quantum optics and quantum information. This work applies the nonlinearity of opto-mechanical system to generate a nonclassical state for an initial state composed of four-headed cat state of photonic mode and number state of mechanical mode. It is interesting to find that the Wigner function of a mechanical mode is composed of finite Wigner functions of time-dependent displaced number states, which is very useful in quantum information process. Furthermore, nonclassical properties of the photonic and mechanical modes are investigated by using Wigner function. An interesting result is that the negative volume of Wigner function for the photonic (or mechanical) mode increases with parameter α (or k), which means that the larger initial value of photonic (or mechanical) mode will improve the nonclassicality of the photonic (or mechanical) mode. We also investigate the influence of different initial photonic states on the nonclassicality of mechanical and photonic modes.

1. Introduction

It is well known that nonclassical states play an important role in quantum optics [1], quantum computation [1,2], quantum information [3,4], quantum metrology [5] and investigation of fundamental quantum theory [6–9]. Over the years, much attention has been paid to generate nonclassical states and increase nonclassicality of quantum states theoretically and experimentally. For example, a non-Gaussian operation (photon subtraction or photon addition) on classical states may produce nonclassical states [10–13]; a linear superpositions of coherent states may exhibit strong nonclassical properties [4,14,15]; a movable mirror in an opto-mechanical system can entangle two cavity fields and generate multicomponent cat states [16–18]. As specific multicomponent cat states, the four-head cat states (4HCS), a superposition of four different coherent state (4HCS) with different phases prepared in the opto-mechanical system [16] or a cavity quantum electrodynamics system [17], is "more nonclassical than even/odd coherent states" [3].

Early applications of opto-mechanical systems are for high precision displacement measurement [19] and gravitational wave detection [20]. Due to the fast developed cooling technique, opto-mechanical systems enter quantum region when the mechanical resonator is cooled to near ground state. Opto-mechanical systems exhibit quantum phenomena of macroscopic objects [21,22] and generate new quantum states for both photonic mode and mechanical mode. As a nonlinear quantum dynamical system, opto-mechanical systems are very suitable to generate nonclassical states. Knight et al. [16] found that when both the initial state of the photonic mode and the mechanical mode are coherent states, an opto-mechanical system can produce multicomponent cat states and near-number states for photonic mode and cat-like state for mechanical mode.

It is well known that, for a nonlinear quantum dynamical system, the final states of the system are critically dependent on the initial states. The final state may show dramatically different quantum properties for different initial states. Hence, it is interesting to try to prepare interesting quantum states by injecting different initial states into a common nonlinear quantum dynamical system. In this work we assume that initial states of the photonic mode and the mechanical mode of the opto-mechanical system are 4HCS state and number state, respectively. We study non-classical properties of photonic/mechanical mode by using the density matrix method. Furthermore, the influence of system parameters on the probability distribution and the nonclassicality

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of photonic/mechanical mode is also investigated.

This paper is structured as follows. In Section 2, we introduce the dynamics of the opto-mechanical system and obtain the system state at time t based on the time evolution operator. In Section 3, we investigate nonclassicalities of photonic mode and mechanical mode via the Wigner function method. In Section 4, we study the fidelity between final system state at time t and initial system state. Section 5 is the summary of our main results.

2. Dynamics of the undamped system

Here we consider an opto-mechanical system composed of an optical resonator (the optical mode) with a movable mirror (the mechanical mode). The Hamiltonian of the system is given by [16,23]

$$H = \hbar\omega_m b^\dagger b - \hbar g_0 a^\dagger a(b^\dagger + b), \tag{1}$$

where $b^\dagger(b)$ are creation (annihilation) operators for the mechanical mode with angular frequency ω_m, interacting with the optical mode, denoted by creation (annihilation) operators $a(a^\dagger)$, with frequency ω_0. The coupling coefficient g_0 is

$$g_0 = \frac{\omega_0}{L}\sqrt{\frac{\hbar}{2M\omega_m}}, \tag{2}$$

where L is the cavity length, and M is the effective mass of the mirror. Then we obtain the time evolution operator for the Hamiltonian of system (details see the Appendix):

$$U(t) = \exp\{ig^2(a^\dagger a)^2(t - \sin t)\}\exp\{ga^\dagger a(b^\dagger\eta - b\eta^*)\} \exp\{-itb^\dagger b\}, \tag{3}$$

with $\eta = 1 - e^{-it}$, $g = g_0/\omega_m$ is the scaled coupling strength, and t represents a scaled time which is equal to ω_m multiplying the actual time t'. Considering experimental feasibility [24], the values of some related parameters can be adopted as: $\omega_0/2\pi \sim 10^{15}$ Hz, $\omega_m/2\pi \sim 10^8$ Hz, $M \sim 10$ pg, $L \sim 1$ μm.

As seen in Eq. (3), there are generalized Kerr term [16,25] and generalized displacement operator in $U(t)$. This implies that the opto-mechanical system may exhibit some properties of non-linearity and displacement, which can be seen from the Wigner function of the opto-mechanical system in Section 3.

We assume that the initial state of the system is

$$\left|\Psi(0)\right\rangle = \left|C_4(\alpha)\right\rangle_p \otimes |k\rangle_m, \tag{4}$$

where $|k\rangle$ is the initial number state of the mechanical mode, and $|C_4(\alpha)\rangle$ [3] is the initial state of the photonic mode, which is called four-headed cat state (4HCS) and is the superposition of coherent states with four different phases, and it is defined as

$$|C_4(\alpha)\rangle = \frac{1}{\sqrt{M_4}}\left(|\alpha\rangle + \left|\alpha e^{i\frac{\pi}{2}}\right\rangle + \left|\alpha e^{i\pi}\right\rangle + \left|\alpha e^{i\frac{3\pi}{2}}\right\rangle\right) = \frac{4e^{\frac{-|\alpha|^2}{2}}}{\sqrt{M_4}}\sum_{n=0}\frac{\alpha^{4n}}{\sqrt{(4n)!}}|4n\rangle, \tag{5}$$

where

$$M_4 = 4\left[1 + e^{-2|\alpha|^2} + 2e^{-|\alpha|^2}\cos\left(|\alpha|^2\right)\right].$$

The state of the system at time t is given by

$$|\Psi(t)\rangle = U(t)|\Psi(0)\rangle = \frac{4e^{\frac{-|\alpha|^2}{2}}}{\sqrt{M_4}}\sum_{n=0}^{\infty}\frac{\alpha^{4n}}{\sqrt{(4n)!}}\exp\{i16g^2n^2(t - \sin t)\}|4n\rangle_p \otimes \exp\{-ikt\}D(4gn\eta)|k\rangle_m, \tag{6}$$

which is obtained by using the following displacement operator and relevant relations [26]:

$$D_b(\alpha) = \exp\{\alpha b^\dagger - \alpha^* b\}. \tag{7}$$

From Eq. (6), we find that when g satisfies the relation $16g^2 = N$ (positive integer), the minimum evolution period of the opto-mechanical system is 2π. This means that the photonic mode will return to 4HCS and the mechanical mode will return to the number state after scaled time 2π.

Naturally we can obtain the density matrix of system $\rho = |\Psi(t)\rangle\langle\Psi(t)|$, and obtain the density matrix of photonic/mechanical mode through tracing on the mechanical/photonic mode, which are given by

$$\rho_p = \mathrm{Tr}_m(\rho) = \frac{16e^{-|\alpha|^2}}{M_4}\sum_{n,n_1=0}^{\infty}\frac{\alpha^{4n}}{\sqrt{(4n)!}}\frac{\alpha^{*4n_1}}{\sqrt{(4n_1)!}}L_k\left(\left|4g\eta(n - n_1)\right|^2\right) \times \exp\left\{-\frac{\left|4g\eta(n - n_1)\right|^2}{2}\right\} \times \exp\{i16g^2(n^2 - n_1^2)(t - \sin t)|4n\rangle_{pp}\langle 4n_1|, \tag{8}$$

and

$$\rho_m = \mathrm{Tr}_p(\rho) = \frac{16e^{-|\alpha|^2}}{M_4}\sum_{n=0}^{\infty}\frac{|\alpha|^{8n}}{(4n)!}D(4gn\eta)|k\rangle_{mm}\langle k|D^\dagger(4gn\eta). \tag{9}$$

where we have used the generating function of Laguerre polynomial [27] to obtain Eq. (8),

$$L_n(xy) = \frac{(-1)^n}{n!}\frac{\partial^{2n}}{\partial t^n\partial s^n}\exp\{xt + ys - ts\}|_{t=s=0}. \tag{10}$$

The correlation between the photonic mode and the mechanical mode can be expressed by their linear entropy [28]:

$$S = 1 - \mathrm{Tr}[\rho^2]. \tag{11}$$

Hence we can obtain S of the opto-mechanical system and $S_p(S_m)$ of the photonic (mechanical) mode:

$$S = 0, \tag{12}$$

$$S_p = S_m = 1 - \left(\frac{16e^{-|\alpha|^2}}{M_4}\right)^2\sum_{n,n_1=0}^{\infty}\frac{|\alpha|^{8(n+n_1)}}{(4n)!(4n_1)!}\left[L_k\left(\left|4g\eta(n - n_1)\right|^2\right)\right]^2 \times \exp\{-\left|4g\eta(n - n_1)\right|^2\}. \tag{13}$$

The evolution of the linear entropy S_p (S_m) reflects the evolution character of the correlation degree between the photonic mode and the mechanical mode. We can see from Eq. (13) and Fig. 1(a) that linear entropy of the opto-mechanical system S is always 0, while linear entropies of photonic mode S_p and mechanical state S_m are the same and change periodically with period 2π during time evolution. That means the opto-mechanical system is in pure state for all time, but the subsystems (photonic mode and mechanical mode) evolve between pure state ($S_{p(m)} = 0$) and mixed state ($S_{p(m)} \neq 0$). Furthermore, the larger value of linear entropy S_p (S_m) corresponds to the stronger correlation and the deeper entanglement degree. From Fig. 1(a) we find that the photonic mode is entangled with the mechanical mode in the