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Probing a single dipolar interaction between a pair of two-level quantum system by scatterings of single photons in an aside waveguide

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ABSTRACT

Weak dipolar interactions exist widely in various atomic, nuclear and molecular systems, and could be utilized to implement the desired quantum information processings. However, these interactions are relatively weak and hard to be measured precisely. Here, we propose an approach to detect such a weak interaction by probing the transport of a single waveguide-photon scattered by two aside qubits with a single dipolar exchange-interaction. By a full quantum theory of photon transports in optical waveguide, we show that the dipolar interaction between the aside two qubits significantly influence the transmitted spectra of the photon traveling along the one-dimensional waveguide. Thus, probing the relevant changes in the transmitted spectra and the transmission probability distribution specifically for the resonant photons, compared with those scattered by the two individual qubits, the information of the single dipolar interaction between the qubits could be extracted. The feasibility of the proposal is also discussed.

1. Introduction

It is well-known that, due to the wave function overlap, mediated by quantum tunneling, and through other processes, dipolar interactions widely exist in various nuclear, atomic, and molecular systems [1–3]. Given these interactions are related to many physical phenomena, e.g., quantum magnetism [4], many-body quantum correlations [5], and quantum computation [6,7], exactly characterizing them are particularly important. Traditionally, this is achieved by detecting the absorption spectra of dipolar-interaction matter radiated by stronger electromagnetic driving [8]. For example, by observing the coherent spin dynamics of the lattice-confined polar molecules or quantum gas, the dipolar spin-exchange interactions between them were successfully measured [9–13]. Note that in these demonstrations the collection information of many dipolar interactions, rather than the exact signal of a single dipolar interaction, are extracted. A question is, how to probe a significantly weak dipolar interaction among few atoms/molecules. This is not trivial as it requires the implementation of very weak electromagnetic signal detections at few-photon level.

In recent years, the development of integrated optoelectronics the detection [14,15] and manipulation of single photons on a chip [16–20] has been achieved. Quantum interface between the ions and single photons in optical cavity also has been realized in experiments [21,22]. These provide a robust way to realize the high quantum efficiency photon detection [14,23], since the photon is now confined in a waveguide. Based on these developments, we propose here an approach to realize the weak dipolar interaction between two two-level atoms (i.e., qubits) by probing the single-photon transports along a one-dimensional waveguide [24]. In fact, controllable single-photon transports along one-dimensional waveguide have been extensively studied [16–20]. One of the main results in these works is the atom–photon interaction can be utilized to control the transmission and reflection spectra of the photon transporting along the waveguide. Naturally, the scatter from the dipolar-interaction matter of the photon in the waveguide changes the spectra of the transmitted photons. As a consequence, a single dipolar interaction between the two scatters could be detected by probing the relevant spectral changes. Specifically, the existence of the scatters should modify the transmission probability of the incident resonant photons for different phase shifts.

The paper is organized as follows. In Section 2 we introduce our model, in the framework of a full quantum scattering theory, to treat the transports of single photons along a one-dimensional waveguide scattered by a pair of two-level atoms. The transmission

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and reflection probabilities of the single photons are specifically analyzed in Section 3. We investigate analytically and numerically how the dipolar interaction between the qubits influences the transmission spectra of the photons and also the phase-shift-dependent transmission probability distributions of the incident resonant photons. Compared with those for two individual qubits as the scatters, the existing dipolar interaction between the two qubits can be extracted. Finally, we summarize our results in Section 4 and discuss the feasibility of the present proposal.

2. Scatterings of single photons by a pair of qubits aside a one-dimensional waveguide

We consider the configuration shown in Fig. 1, wherein a single photon propagates along a one-dimensional optical waveguide scattered by two two-level atoms encoded as the qubits. Generically, the dipolar interaction between the atoms can be written as

$$H_{dip} = -\frac{\mu_0}{4\pi r_{12}^3}\left[3(\mathbf{m}_1\cdot\mathbf{e}_{12})(\mathbf{m}_2\cdot\mathbf{e}_{12}) - \mathbf{m}_1\cdot\mathbf{m}_2\right], \tag{1}$$

with μ_0 being the magnetic constant and $\mathbf{m}_j$ being the magnetic moment of the j-th atom. r_{12} is the distance between the two atoms with $\mathbf{e}_{12}$ being the unit vector parallel to the line joining the centers of the two atoms. Under the usual rotating-wave approximation the above dipolar interaction can be simplified as

$$H_{dip} = g(\sigma_1^+\sigma_2^- + \sigma_2^+\sigma_1^-), \tag{2}$$

with the dipolar interaction strength $g = -\mu_0\hbar^2\gamma_1\gamma_2(1 - 3\cos^2\Theta)/(16\pi r_{12}^3)$. Here, γ_j is the gyromagnetic ratios of the j-th atom, Θ is the angle between the quantization axis of the both atoms and the direction of the spins, and σ_j^+ and σ_j^- are the j-th atomic raising and lowering ladder operators, respectively. Above, the usual rotating-wave approximation has been used. Therefore, the single photon propagating in the waveguide scattered by the two atoms can be described by the following Hamiltonian ($\hbar = 1$) [24]:

$$\begin{aligned} H = &\sum_{j=1,2}\left(\Omega_j - \frac{i\Gamma_j}{2}\right)\sigma_j^+\sigma_j^- + g(\sigma_1^+\sigma_2^- + \sigma_2^+\sigma_1^-) \\ &+ \int\frac{c}{i}dx\left[a_R^\dagger(x)\frac{\partial}{\partial x}a_R(x) - a_L^\dagger(x)\frac{\partial}{\partial x}a_L(x)\right] \\ &+ \sum_{j=1,2}\int dx\, V_j\delta(x - l_j)\left[a_R^\dagger(x)\sigma_j^- + a_L^\dagger(x)\sigma_j^- + H.c.\right]. \end{aligned} \tag{3}$$

Here, Ω_j and Γ_j are the j-th atomic transition frequency and dissipation rate, respectively. l_j is the position of the j-th atom and $L = l_2 - l_1$ is the distance between the atoms. Also, $a_R^\dagger(x)$ ($a_R(x)$) and $a_L^\dagger(x)$ ($a_L(x)$) are the creation (annihilation) operators of the photon propagating right and left, respectively. c is the group velocity of the photon traveling along the waveguide. V_j is the coupling strength between the photon and the j-th atom.

Suppose a single photon is incident from the left direction. Then, the stationary solution of the system can be obtained by solving the time-independent Schrödinger equation

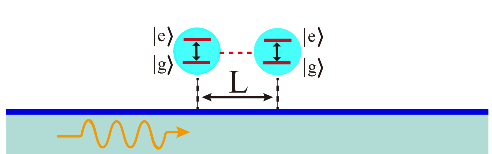

Fig. 1. Single photon transports along a one-dimensional waveguide scattered by two aside two-level atoms (separated by L).

$$H|\Phi\rangle = E|\Phi\rangle, \tag{4}$$

where E is the eigenvalue, and

$$|\Phi\rangle = \int dx\left[\phi_R(x)a_R^\dagger(x) + \phi_L(x)a_L^\dagger(x)\right]|\varnothing\rangle + (e_1\sigma_1^+ + e_2\sigma_2^+)|\varnothing\rangle, \tag{5}$$

is the generic stationary wave function [19] of the system. Here, $|\varnothing\rangle$ denotes the vacuum state without any photon in the waveguide and all the atoms are in their ground states. $\phi_{R/L}$ and e_j stand for the probabilistic amplitudes of the photon propagating along the R/L direction and the excitation of the j-th atom, respectively.

Substituting Eq. (5) into Eq. (4), the coefficients in Eq. (4) are determined by

$$E\phi_R(x) = \frac{c}{i}\frac{\partial}{\partial x}\phi_R(x) + V_1\delta(x - l_1)e_1 + V_2\delta(x - l_2)e_2, \tag{6}$$

$$E\phi_L(x) = \frac{-c}{i}\frac{\partial}{\partial x}\phi_L(x) + V_1\delta(x - l_1)e_1 + V_2\delta(x - l_2)e_2, \tag{7}$$

$$Ee_1 = V_1\phi_R(l_1) + V_1\phi_L(l_1) + \left(\Omega_1 - \frac{i\Gamma_1}{2}\right)e_1 + ge_2, \tag{8}$$

$$Ee_2 = V_2\phi_R(l_2) + V_2\phi_L(l_2) + \left(\Omega_2 - \frac{i\Gamma_2}{2}\right)e_2 + ge_1. \tag{9}$$

Furthermore, assume that the wave vector of the photon is k and the j-th atom is located at l_j aside the waveguide. Then, the coefficients $\phi_R(x)$ and $\phi_L(x)$ can be rewritten as [19]

$$\phi_R(x) = \frac{e^{ikx}}{\sqrt{2\pi}}\left[\theta(l_1 - x) + t_{12}\theta(x - l_1)\theta(l_2 - x) + t\theta(x - l_2)\right], \tag{10}$$

$$\phi_L(x) = \frac{e^{-ikx}}{\sqrt{2x}}\left[r\theta(l_1 - x) + r_{12}\theta(x - l_1)\theta(l_2 - x)\right], \tag{11}$$

where $\theta(x)$ is the step function, $t_{12}(r)$ and $t(r_{12})$ are the transmission (reflection) amplitudes at l_1 and l_2, respectively. Substituting Eqs. (10)–(11) into Eqs. (6)–(9) and setting $\theta(0) = 1/2$ for simplicity, the reflection and transmission amplitudes of the traveling photon are obtained as

$$r = \frac{-\frac{iV_2^2}{c}e^{2ikl_2}\left(E - \Omega_1 + i\frac{\Gamma_1}{2} - i\frac{V_1^2}{c}\right) - i\frac{V_1^2}{c}e^{2ikl_1}\left(E - \Omega_2 + i\frac{\Gamma_2}{2} + i\frac{V_2^2}{c}\right) - 2ig\frac{V_1V_2}{c}e^{ik(l_1+l_2)}}{\Pi_{j=1,2}\left(E - \Omega_j + i\frac{\Gamma_j}{2} + i\frac{V_j^2}{c}\right) - \left[i\frac{V_2^2}{c}e^{2ik(l_2-l_1)} - \frac{gV_2}{V_1}e^{ik(l_2-l_1)}\right]\left[i\frac{V_1^2}{c} - \frac{gV_1}{V_2}e^{-ik(l_2-l_1)}\right]}, \tag{12}$$

$$t = \frac{\left(E - \Omega_1 + i\frac{\Gamma_1}{2}\right)\left(E - \Omega_2 + i\frac{\Gamma_2}{2}\right) + ig\frac{V_1V_2}{c}e^{ik(l_2-l_1)} - ig\frac{V_1V_2}{c}e^{-ik(l_2-l_1)} - g^2}{\Pi_{j=1,2}\left(E - \Omega_j + i\frac{\Gamma_j}{2} + i\frac{V_j^2}{c}\right) - \left[i\frac{V_2^2}{c}e^{2ik(l_2-l_1)} - \frac{gV_2}{V_1}e^{ik(l_2-l_1)}\right]\left[i\frac{V_1^2}{c} - \frac{gV_1}{V_2}e^{-ik(l_2-l_1)}\right]}, \tag{13}$$

with $E{=}kc$. Specifically, if the atoms are identical and also ideal (without any dissipation), i.e., $\Gamma_1 = \Gamma_2 = \Gamma \sim 0$, $\Omega_1 = \Omega_2 = \Omega$ and $V_1 = V_2 = V$, then the above equations reduce to

$$t = \frac{\Delta^2 + ig\frac{\gamma}{2}e^{i\theta} - ig\frac{\gamma}{2}e^{-i\theta} - g^2}{\left(\Delta + i\frac{\gamma}{2}\right)^2 - \left(i\frac{\gamma}{2}e^{2i\theta} - ge^{i\theta}\right)\left(i\frac{\gamma}{2} - ge^{-i\theta}\right)}, \tag{14}$$

$$r = \frac{-i\frac{\gamma}{2}e^{i2\theta}\left(\Delta - i\frac{\gamma}{2}\right) - i\frac{\gamma}{2}\left(\Delta + i\frac{\gamma}{2}\right) - ig\gamma e^{i\theta}}{\left(\Delta + i\frac{\gamma}{2}\right)^2 - \left(i\frac{\gamma}{2}e^{2i\theta} - ge^{i\theta}\right)\left(i\frac{\gamma}{2} - ge^{-i\theta}\right)}. \tag{15}$$