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Invited Paper

Disentanglement in a two-qubit system subjected to dissipative environment: Exact analysis

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ABSTRACT

We investigate the time evolution of entanglement of various entangled states of two-qubit atomic system in vacuum environment using exact analysis. Compared to our earlier work under Markov approximation [M. Ikram, F.-L. Li, M.S. Zubairy, Phys. Rev. A 75 (2007) 062336] we show that disentanglement rate is slower and sudden death times are higher than the earlier study in each set of entangled state.

1. Introduction

The open system dynamics of composite systems, being initially in entangled states, is well explored in recent years. It is well known that the individual quantum systems obey half-life and decay exponentially. However, for a composite system–environment scenario, the coupling makes correlated dynamics quite complex. Yu and Eberly [1] were the first to study the behavior of a composite two-qubit mixed atomic system in the dissipative environment. They investigated that although local systems may decay asymptotically but, in contrast, the composite entangled systems may decay in finite times depending upon the mixing of doubly excited component. Their work was extended to a class of initially mixed and pure states for non-interacting [2–5] and interacting [6–8] qubits. All of this work is done exploiting the Markov approximation for weak system–environment coupling. This assumption ensures short memory in the sense that correlation time is very short and there is no feedback from the environment to the system. However, when the system–environment coupling is not weak, Markov approximation is no more valid [9]. In such a scenario, systems do have feedback from their environment and retain memory of interaction as implied by Jaynes–Cumming model. In such cases, memory effects are important and interesting from many points of views. During the time span when the memory effects are not negligible, the flow of energy and information from the system to the environment can be momentarily reversed. The reversal of these processes causes recoherence and restoration of previously lost superpositions [10]. These systems are treated as non-Markovian [11]. Non-Markovian systems appear in many branches of physics, such as quantum optics [12,13], solid sate physics [14], quantum chemistry [15] and quantum information processing [16]. Memory effects are usually characterized by a structured spectral density implying that the quantum system interacts more strongly with some modes of the reservoir than with others. Leaky optical cavities and photonic band-gap materials, for example, have such spectral densities [10,13].

The entanglement dynamics in strong coupling regime has been recently investigated under different theoretical models [17–20]. Particularly, the role of spectral width of system–environment coupling and mixing of the initial state is investigated for two-qubit systems [22]. In this paper, we investigate the entanglement dynamics of a two qubit system, with qubits as two-level atoms trapped in two leaky cavities, thus having structured vacuum reservoir inside the cavities. Due to structured reservoir–system interaction, Markov approximation cannot be applied here. Knowing that doubly excited component in the entangled state is the main source of disentanglement, we consider a set of atomic system having mixing of doubly excited component and study the entanglement evolution of these states in non-Markovian system–reservoir interaction. On contrary to the previous study [2], non-Markovian effect or the exact treatment not only suggests the

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postponement of the death of the entanglement but it exhibit an evident enhancement in the entanglement. The Wootters concurrence formula [21] is used as a quantitative measure of two-qubit entanglement.

The paper is organized as follows. In Section 2, we present the theoretical model employed to investigate the non-Markovian effects on the two-qubit entanglement dynamics. Further analytical and numerical results are presented for different cases of initial mixed states. Section 3 finally concludes the paper with a brief discussion.

2. Model

We consider the similar model as in [2], i.e., two two-level atoms representing a bipartite system trapped in two separate cavities containing structured vacuum acquired through the interaction of cavity fields with the outside vacuum as shown in Fig. 1. However, the correlation between the atoms depends only on the initial quantum entanglement between them. We also consider that the cavities are far apart with no direct cross-mutual interaction between the atoms or the cavity fields. The total Hamiltonian can be written as

$$H = H_o + H_I, \tag{1}$$

where H_o and H_I are the free and interaction parts, respectively, of the Hamiltonian, and are given by

$$H_o = \hbar\omega_o \sum_{i=1}^{2} \sigma_+^i \sigma_-^i + \hbar \sum_{k=1}^{N} \omega_k b_k^\dagger b_k, \tag{2}$$

$$H_I = \hbar \sum_{k=1}^{N} \sum_{i=1}^{2} \left(g_k \sigma_+^i b_k + g_k^* \sigma_-^i b_k^\dagger \right). \tag{3}$$

Here, in these equations, ω_o is the transition frequency of the two-level atom, σ_+^i (σ_-^i) is the raising (lowering) operator for the atom i and index k labels the different field modes of the reservoir with frequencies ω_k with $b_k(b_k^\dagger)$ being the field annihilation(creation) operator. Using the rotating-wave approximation, the interaction Hamiltonian between an atom and N-mode reservoir takes the form [23]

$$H_I = \hbar \sum_{k=1}^{N} \sum_{i=1}^{2} g_k^i (\sigma_-^i b_k^\dagger e^{-i(\omega_0-\omega_k)t} + \sigma_+^i b_k e^{i(\omega_0-\omega_k)t}), \tag{4}$$

where $g_k{}^i$ is the coupling constant between the atom i and the vacuum reservoir. We focus on the case for which the structured reservoir is the electromagnetic field inside the lossy cavity. It means that the discrete cavity modes can be effectively replaced with the spectral density function. We consider a case where the atom is interacting resonantly with the cavity field reservoir having Lorentzian spectral density that characterizes the coupling strength of the reservoir to the qubit as follows:

$$J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda^2}{(\omega_c - \omega)^2 + (\lambda)^2}. \tag{5}$$

This corresponds to a cavity supporting a single mode of frequency ω_c which can be leaked out through the non-ideal cavity walls with a probability proportional to λ^2, where λ is the spectral width of the field distribution inside the lossy cavity. It is connected to the reservoir correlation time τ_B by the relation $\tau_B = \lambda^{-1}$ and the time scale τ_R on which the state of the system changes is given by $\tau_R = \gamma_0^{-1}$. Here the parameter γ_0 is proportional to the strength of the atom–cavity coupling. Typically, in weak coupling regime ($\lambda > 2\gamma_0$), the qubit–reservoir system is Markovian and in strong coupling regime ($\lambda < 2\gamma_0$), non-Markovian dynamics occurs accompanied by a reversible decay.

In this paper, we are interested in two-qubit entanglement dynamics in strong coupling regime. To incorporate the parameters that control the atomic dynamics under strong coupling, we need to study the decay of a single two-level atom. We, therefore, consider a single two-level atom initially in excited state $|a\rangle$ trapped in a cavity containing vacuum modes, then time dependent wave function of the system and the environment can be written as

$$\left|\psi(t)\right\rangle = A(t)\left|a, 0\right\rangle + \sum_k B_k(t)\left|b, 1_k\right\rangle, \tag{6}$$

where $A(t)$ and $B_k(t)$ are the probability amplitudes of atom in excited state $|a\rangle$ with vacuum in cavity and atom in ground state $|b\rangle$ with cavity in single photon in kth mode $|1_k\rangle$, respectively. From Schrodinger equation we get the integro-differential equation

$$\dot{A}(t) = -\int_0^t d\acute{t} f\left(t - \acute{t}\right) A\left(\acute{t}\right), \tag{7}$$

where $f(t - \acute{t})$ is a correlation function defined in terms of continuous limits of the environment frequency as

$$f(t - \acute{t}) = \int_{-\infty}^{\infty} d\omega\, J(\omega) \exp\left[i(\omega_0 - \omega_k)\left(t - \acute{t}\right)\right]. \tag{8}$$

Considering the frequency distribution inside the cavity as defined in Eq. (5) for $\lambda > 0$ and $t - \acute{t}$ real, we get

$$f(t - \acute{t}) = \tfrac{1}{2}\gamma_0 \lambda e^{-\lambda(t-\acute{t})}, \tag{9}$$

where it is assumed that atomic transition frequency ω_0 is resonant with the cavity's central frequency mode ω_c. Now, we can solve the integro-differential equation (Eq. (7)) using initial condition $A(0) = 1$ i.e., atom is initially in excited state and vacuum in the cavity, as

$$A(t) = e^{-(1/2)t\lambda}\left(\cosh\left(\frac{dt}{2}\right) + \frac{\lambda}{d}\sinh\left(\frac{dt}{2}\right)\right), \tag{10}$$

where $d = \sqrt{\lambda^2 - 2\gamma_0\lambda}$ and is defined in the strong coupling regime $\gamma_0 > \lambda/2$ or $\tau_R < 2\tau_B$. The single atom dynamics exhibits an exponential decay by the oscillatory function $p(t) = \cosh(dt/2) + \frac{\lambda}{d}\sinh(dt/2)$. Thus we can easily calculate the modified decay rate using $\Gamma(t) = -2\,\mathrm{Re}[\dot{A}(t)/A(t)]$, as

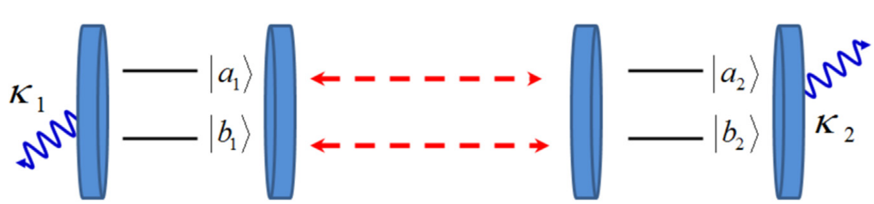

Fig. 1. Two two-level atoms, initially prepared in an entangled state, trapped in two cavities having structured vacuum reservoir surrounded by vacuum environment.