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Invited Paper

Generation of hybrid four-qubit entangled decoherence-free states assisted by the cavity-QED system

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ABSTRACT

We propose three effective protocols to generate four-qubit entangled decoherence-free states assisted by the cavity-QED system. These schemes are based on optical selection rules realized with a single electron charged self-assembled GaAs/InAs quantum dot in a micropillar resonator. Compared with previous photonic protocols, the first scheme is to replace the entangled-state resources with much simpler single-photon resources and has a deterministic success probability. Moreover, the cavity-QED system may be used to generate four-spin entangled decoherence-free states and hybrid four-qubit of spin-photon entangled decoherence-free states. These states may be applied up to different requirements because of different superiorities of photons and spins. All schemes may be implemented with current physical technologies.

1. Introduction

Quantum entanglement occurs when different particles are generated or interact in ways such that each particle cannot be described independently. Bipartite or multipartite entanglements as essential ingredients for testing local hidden variable have become most important resources for quantum teleportation [1,2], quantum computation [3–5], quantum key distribution [6–8], quantum dense coding [9–11], etc. Most of these protocols require maximal entanglements or noiseless quantum channels [1–11]. However, in practice, they are easily degraded because of the coupling between the quantum system and the environment or equipments [12], which may greatly reduce the application fidelity. Different ways have been explored to deal with these quantum decoherences. One way is to concentrate the decoherenced quantum entanglements. Entanglement concentration is used to get the maximal entanglement from partially entangled pure states. Bennett et al. [13] introduced the first entanglement concentration protocol (ECP) using the Schmidt projection method and collective measurements for two-photon systems. After that, many interesting ECPs have been proposed for photon systems [14–23]. The other typical scheme uses the quantum error-correction and dynamical decoupling techniques. Another useful way is to encode information into the symmetry state in the decoherence-free subspace to avoid the system–environment interaction. Thus the decoherence-free subspace is inherently immune to quantum decoherence and robust to perturbing error processes. Therefore, the decoherence-free states are very useful for long-distance quantum communication and quantum computation and applied in quantum error correction codes [12].

The N-qubit decoherence-free states were originally proposed by Kempe et al. [24], and are invariant under any identical unitary transformation on each of the qubits. One decoherence-free singlet state is special EPR state

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)_{12} \tag{1}$$

Another nontrivial example is the four-qubit entangled decoherence-free state

$$|\Phi\rangle = \alpha|\Psi_0\rangle + \beta|\Psi_1\rangle \tag{2}$$

with

$$|\Psi_0\rangle = |\psi^-\rangle_{12}|\psi^-\rangle_{34},$$

$$|\Psi_1\rangle = \frac{1}{\sqrt{3}}|0011\rangle + \frac{1}{\sqrt{3}}|1100\rangle - \frac{1}{\sqrt{3}}|\psi^+\rangle_{12}|\psi^+\rangle_{34}, \tag{3}$$

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and $|\psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$. This four-qubit state is sufficient to fully protect an arbitrary logical qubit against collective decoherence in contrast to the two-qubit state. With its interesting applications, Bourennane et al. [25] have generated a four-photon polarization-entangled decoherence-free states via a spontaneous parametric down-conversion source. They coherently overlapped two four-photon sources $|\Psi_1\rangle$ and $|\Psi_2\rangle$. Recently, Zou et al. [26] and Gong et al. [27] proposed schemes to generate four-photon polarization-entangled decoherence-free states based on linear optical elements and post-selection strategy. However, those schemes mentioned in Refs. [25-27] work in the destructive way because the generated four-photon polarization-entangled decoherence-free states cannot be used for further quantum information processing and quantum computation when a four-photon coincidence measurement was made on the photonic states. Wang et al. [28] propose a probabilistic linear-optics-based scheme for local conversion of four Einstein–Podolsky–Rosen photon pairs into four-photon polarization-entangled decoherence-free states.

In this paper, we propose deterministic schemes to generate the four-photon entangled decoherence-free states, four-spin entangled decoherence-free states, hybrid four-qubit entangled decoherence-free states assisted by the cavity-QED system. Hybrid systems (photon-spin) [29,30] have been explored to effectively enable strong nonlinear interactions between single photons [31] in the weak-coupling regime. The optical selection rules realized with a single electron charged self-assembled GaAs/InAs quantum dot in a micropillar resonator [32,33] may be applied to construct qubit gates on photon systems [31,34–38]. We first present a theoretical preparation scheme of four-qubit entangled decoherence-free states with CNOT gates and one-qubit rotations. And then, we generate four-photon entangled decoherence-free states by constructing the CNOT gate on a two-photon system with the help of the cavity-QED system. Moreover, by constructing the CNOT gate on a two-spin system, we can generate four-spin entangled decoherence-free states with the help of the cavity-QED system. Furthermore, with the hybrid CNOT gate on a photon-spin or spin-photon system, hybrid four-qubit entangled decoherence-free states may be generated. These schemes may be experimentally realized with present technology.

2. Generations of four-qubit entangled decoherence-free states

In order to generate four-qubit entangled decoherence-free states deterministically, we consider its theoretical decomposition circuits using the elementary gates of the CNOT gate and single-qubit rotations. Notice that $|\Phi\rangle$ may be rewritten as $|\Phi\rangle = \frac{1}{\sqrt{3}}\beta|00\rangle|11\rangle - \frac{1}{\sqrt{3}}\beta|\overline{01}\rangle|\overline{01}\rangle + \alpha|\overline{10}\rangle|\overline{10}\rangle + \frac{1}{\sqrt{3}}\beta|11\rangle|00\rangle$ with $|\overline{01}\rangle = |\psi^+\rangle$ and $|\overline{10}\rangle = |\psi^-\rangle$. Using the two-qubit logic gate $T_{11}\cdot T_{12}\cdot T_{13}$, the initial state $|0011\rangle$ is changed into

$$\left(\frac{1}{\sqrt{3}}\beta|00\rangle - \frac{1}{\sqrt{3}}\beta|01\rangle - \alpha|10\rangle + \frac{1}{\sqrt{3}}\beta|11\rangle\right)_{12}|11\rangle_{34} \tag{4}$$

where

$$T_{11} = \begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad T_{12} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 & 0 \\ -\frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$
$$T_{13} = \begin{pmatrix} \beta & 0 & \alpha & 0 \\ 0 & \beta & 0 & \alpha \\ -\alpha & 0 & \beta & 0 \\ 0 & -\alpha & 0 & \beta \end{pmatrix} \tag{5}$$

And then using two CNOT gates on qubits (1,3) and qubits (2,4), we can get

$$\left(\frac{1}{\sqrt{3}}\beta|0011\rangle - \frac{1}{\sqrt{3}}\beta|0110\rangle - \alpha|1001\rangle + \frac{1}{\sqrt{3}}\beta|1100\rangle\right)_{1234} \tag{6}$$

which may be changed into $|\Phi\rangle$ using two two-qubit logic gates $T_2 = CNOT1\cdot T_{14}\cdot CNOT1$ and $T_3 = CNOT1\cdot T_{14}^{-1}\cdot CNOT1$ on the first two-qubit (1,2) and the last two-qubit (3,4) respectively. Here,

$$T_{14} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{pmatrix} \tag{7}$$

Now, using the CNOT gate and qubit rotations [39,40], each controlled-U can be realized with $(I_2 \otimes A)\cdot CNOT1\cdot(I_2 \otimes B)\cdot CNOT1\cdot(I_2 \otimes C)$, where $U = R_z(\alpha)R_y(\theta)R_z(\beta)$, $A = R_z(\alpha)R_y(\theta/2)$, $B = R_y(-\theta/2)R_z(-(\alpha+\beta)/2)$, and $C = R_z((\beta-\alpha)/2)$. Thus we can get the following decompositions:

$$\begin{aligned} T_{11} &= (X \otimes I_2)\cdot CNOT1\cdot(A_1^{-1} \otimes I_2)\cdot CNOT2\cdot(A_1 \otimes I_2)\cdot CNOT2\cdot CNOT1\cdot(X \otimes I_2), \\ T_{12} &= (X \otimes A_2^{-1})\cdot CNOT1\cdot(I_2 \otimes A_2)\cdot CNOT1\cdot(X \otimes I_2), \\ T_{13} &= A_3 \otimes I_2, \\ T_{14} &= (A_1 \otimes I_2)\cdot CNOT2\cdot(A_1^{-1} \otimes I_2)\cdot CNOT2 \end{aligned} \tag{8}$$

where X denote the Pauli flip,

$$A_1 = R_y\left(\frac{\pi}{4}\right), \quad A_2 = R_y\left(\frac{\theta_1}{2}\right), \quad A_3 = R_y(-\theta_2) \tag{9}$$

with $\theta_1 = \arctan(1/\sqrt{2})$, $\theta_2 = 2\arctan(\alpha/\beta)$, and CNOT1 denotes the controlled NOT gate with the first qubit is the controlling qubit while CNOT2 denotes the controlled NOT gate with the second qubit as the controlling qubit. $R_y(\theta) = \cos(\theta/2)|0\rangle\langle 0| - \sin(\theta/2)|0\rangle\langle 1| + \sin(\theta/2)|1\rangle\langle 0| + \cos(\theta/2)|1\rangle\langle 1|$ and $R_z(\varphi) = \exp(i\varphi/2)|0\rangle\langle 0| + \exp(-i\varphi/2)|1\rangle\langle 1|$ denote the qubit rotations along the y-axis or z-axis in Pauli sphere respectively. X denotes the Pauli flip.

2.1. A singly charged quantum dot in an one-side optical microcavity

The cavity-QED system used in our proposal is constructed by a singly charged In(Ga)As quantum dot located in the center of a one-side optical cavity [41–43], as shown in Fig. 1. The single electron states have $J_z = \pm 1/2$ spin ($|\uparrow\rangle, |\downarrow\rangle$) and the holes have $J_z = \pm 3/2$ ($|\Uparrow\rangle, |\Downarrow\rangle$). The two electrons form a singlet state and therefore have total spin zero, which prevents electron spin interactions with the hole spin [41]. Photon polarization is commonly defined with respect to the direction of propagation, i.e. z-axis, where the absolute rotation direction of its electro-magnetic fields does not change. Label the optical states by their circular polarization ($|L\rangle$ and $|R\rangle$ for left and right circular polarization respectively). A negatively charged exciton $|\uparrow\downarrow\Uparrow\rangle$ or $|\downarrow\uparrow\Downarrow\rangle$ may be created by resonantly absorbing $|L\rangle$ or $|R\rangle$, respectively.

The input–output relation of this one-side cavity system can be calculated from the Heisenberg equation [41] of motions for the cavity field operator and dipole operator

$$\begin{aligned} \frac{d\hat{a}}{dt} &= -\left(i\Delta\omega_c + \frac{\kappa}{2} + \frac{\kappa_s}{2}\right)\hat{a} - g\hat{\sigma}_- - \sqrt{\kappa}\,\hat{a}_{in}, \\ \frac{d\hat{\sigma}_-}{dt} &= -\left(i\Delta\omega_x + \frac{\varsigma}{2}\right)\hat{\sigma}_- - g\hat{\sigma}_z\hat{a}, \end{aligned} \tag{10}$$