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Modal analysis of the impact of the boundaries on transverse Anderson localization in a one-dimensional disordered optical lattice

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ABSTRACT

Impact of the boundaries on transversely localized modes of a truncated one-dimensional disordered optical lattice is numerically studied. The results show lower modal number density near the boundaries compared with the bulk, while the average decay rate of the tail of localized modes is the same near the boundaries as in the bulk. It is suggested that the perceived suppressed localization near the boundaries is due to a lower mode density: on average, it is less probable to excite a localized mode near the boundaries; however, once it is excited, its localization is with the same exponential decay rate as any other localized mode.

1. Introduction

Anderson localization was originally described as the absence of diffusion for electrons in random electronic lattices due to strong scattering and interference [1,2]. However, it was later realized that the concept is inherently a wave phenomenon and was observed in highly scattering classical wave systems including optics, acoustics, elastics, and electromagnetics [5,4,6,3,7–9], and various quantum optical systems, such as atomic lattices [10]. Transverse Anderson localization was introduced by Abdullaev et al. [11] and De Raedt et al. [12] where a beam of light is localized in transverse dimension(s) of a transversely disordered waveguide but propagates freely in the longitudinally invariant direction. Several observations of transverse Anderson localization have been reported over the past few years [14,15,13,16–18].

Recently, Anderson localization of optical waves near the boundaries has been discussed theoretically [20,19] and experimentally [21,22]. A delocalizing effect near the boundaries of one dimensional (1D) and two dimensional (2D) random lattice waveguides has been reported, so that a higher level of disorder near the boundaries is claimed to be needed to obtain the same level of localization as in the bulk [20,21]. These reports seemed to be in contrast with our experimental observation of transverse Anderson localization in a glass optical fiber, where a strong localization happens near the outer boundary of the fiber and no trace of localization is observed in the central regions [23]. The disagreements were explained by considering the non-uniform distribution of disorder in the fiber. The disorder was observed to be much stronger near the outer boundary of the fiber which resulted in stronger localization in that region.

In Ref. [21], lower mode density near the boundary of lattice is considered as the reason behind the less localized average intensity profile, which is obtained both experimentally and numerically using the beam propagation method (BPM) [24,25]. For the BPM, an initial excitation profile is propagated through the lattice waveguide and the final output pattern reveals the extent of localization. To uncover the impact of the boundary on localization, the initial excitation profile is adjusted once to cover the central regions of the waveguide and then its edges, and the output beam profiles are compared with each other. However, this method is not independent of the excitation profile and it is desirable to use a method that can quantify the effect of the boundary on localization independent of the profile of the input beam [26,27].

Recently, Karbasi et al. [28] used a modal analysis to explore localization behavior of a disordered lattice waveguide. The modal analysis offers a clear intuitive description of the localization phenomenon independent of the physical properties of the external excitation. Here, we carry out a detailed numerical investigation for the effect of boundaries on the formation of localized modes of a 1D disordered optical lattice waveguide using the modal perspective. Our results show that the average decay rate of the tail of Anderson localized modes is the same near the

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boundaries as in the bulk. Of course, the boundary-side tail of any mode which is localized near the boundary decays according to the boundary index step; however, the side that faces the lattice decays with the same average exponent as any other mode which is located anywhere else in the lattice.

Another observation presented here is that the modal number density near the boundaries is lower compared with the bulk, confirming similar results obtained by Ref. [21]. Therefore, we suggest that the suppressed localization near the boundaries is due to a lower mode number density rather than a weaker exponential decay of the near-boundary localized modes. In other words, it is less probable to excite a localized mode near the boundaries; however, once it is excited, its localization is with the same exponential decay rate as any other mode (on the disordered lattice side).

2. Modal analysis

Modal analysis as presented in Refs. [14,29,30] has two main advantages over other techniques such as the beam propagation method for the investigation of the localization-related phenomena in disordered optical waveguides: (1) A standard technique to analyze optical waveguides is the modal method. The modal method provides an intuitive framework to understand the results related to the transverse Anderson localization of light in a language that is quite familiar in the domain of optical waveguides. (2) The modal analysis allows us to study the localization phenomenon independent of the excitation profile while containing all the information provided by the beam propagation method [28]. In this work, we have chosen to calculate the transverse electric (TE) guided modes of the disordered waveguide using the finite element method (FEM) presented in Ref. [31].

Refractive index profile of the disordered lattice in this paper is the same as the one in Ref. [28]. Briefly, the wave-guiding profile is a disordered lattice, which is surrounded by a cladding with refractive index n_c on both sides. The disordered lattice is constructed by stacking a collection of dielectric slabs with refractive index of n_0 and n_1 ($n_0 < n_1$). In our analysis, we have chosen the refractive index of cladding region as $n_c = n_0$, which resembles the practical disordered waveguides written on the silica glass by femtosecond pulses [32,21,33]. Fig. 1(a) shows the refractive index profile of an ordered waveguide versus a disordered one, where for the ordered waveguide the thickness of each slab is chosen to be $\bar{\Lambda} = 2\lambda$ while for the disordered waveguide the thickness of each slab is randomly chosen with uniform probability to be in the interval of $[\bar{\Lambda} - \delta\Lambda, \bar{\Lambda} + \delta\Lambda]$ with $\delta\Lambda = \lambda$. The strength of spatial disorder is defined as $\delta\Lambda/\bar{\Lambda}$, which is kept fixed at 50% level in this manuscript for simplicity. However, the reported observations are general and apply to other levels of spatial disorder. In order to observe transverse Anderson localization, the disorder can be introduced in many different ways into the waveguide, such as diagonally [1] or off-diagonally [15,34,35]; our method is intended to be closely related to the recent implementations of disordered optical fibers of [12,16], and relies on a combination of diagonal and off-diagonal disorder. Fig. 1(b) shows the refractive index profile of the disordered structure.

For each guided mode, using Eqs. (1)–(4), position (x), width (σ_2), asymmetry (σ_3), and skewness (S) of the mode across the waveguide [36] are calculated, respectively, according to:

$$\bar{x} = \int_{-\infty}^{+\infty} xI(x)\,dx, \tag{1}$$

$$\sigma_2^2 = \int_{-\infty}^{+\infty} (x - \bar{x})^2 I(x)\,dx, \tag{2}$$

$$\sigma_3^3 = \int_{-\infty}^{+\infty} (x - \bar{x})^3 I(x)\,dx, \tag{3}$$

$$S = \frac{2\sigma_3}{\sigma_2^3}. \tag{4}$$

$I(x)$ is the intensity of the mode in the waveguide and it is normalized such that $\int_{-\infty}^{+\infty} I(x)\,dx = 1$. In these equations, $\bar{x}$ is a measure of the position of the modes across the lattice, σ_2 is a measure of width of the modes meaning that a larger σ_2 is proportional to a wider mode intensity profile distribution, σ_3 is a measure of mode asymmetry in a sense that a zero value of σ_3 is equivalent to a totally symmetric mode, and S is a measure of mode asymmetry, like the third moment (σ_3), except that it is normalized by the mode width.

We investigate these characteristic features for 2000 realizations of the random lattice for several values of refractive index difference (Δn). To study the localization behavior as a function of location in the waveguide, the lattice is divided into 24 equal bins of width 10λ each. The nth bin on the horizontal line is identified by the position belonging to the interval, $x \in [(10n + 5)\lambda, (10n + 15)\lambda]$, where $n = 0, 1, 2, \ldots, 23$. $x = 25\lambda (x = 225\lambda)$ signifies the leftmost (rightmost) side of the disordered lattice, and $x = 0 (x = 250\lambda)$ signifies the leftmost (rightmost) corner of the cladding in the waveguide.

All the calculated modes are categorized based on their position, where each mode is placed in one of the bins. Each bin is associated with certain number of modes and the statistics associated with that bin can be studied independently. Therefore, we can judge with sufficient statistics whether the position in the waveguide and the distance from the boundaries affect the characteristics (width, spatial density, skewness and decay exponent) of the modes. The simulations are carried out for a wavelength of

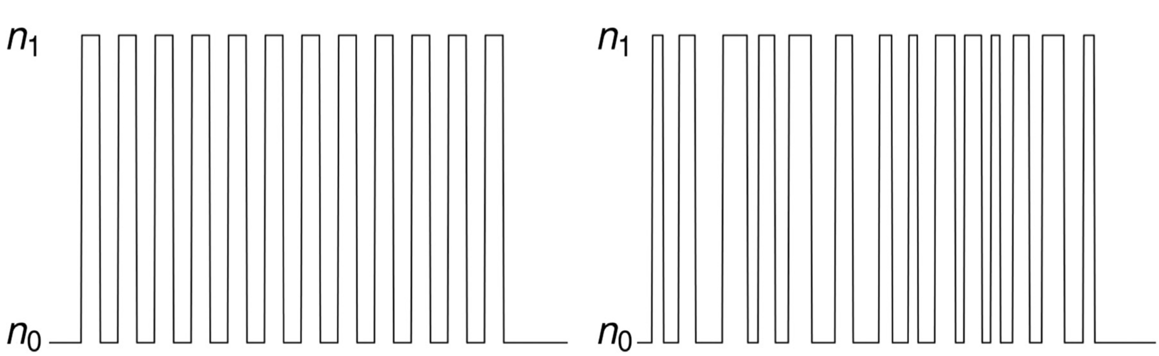

Fig. 1. Sample refractive index profiles of ordered (left) and disordered (right) slab waveguides are shown.