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Estimation of thermal noise for spindle optical reference cavities

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ABSTRACT

Thermal noise in optical reference cavities sets a severe limit to the frequency stability of ultra-stable lasers of the order of a few parts of 10^{-16} at 1 s. Various reference cavity designs have been developed to attempt to attain ultimate performance. Numata derived three equations based on strain energy and the fluctuation–dissipation-theorem (FDT) to estimate the noise contributions of the spacer, substrates and coatings, and these equations work well for cylindrical cavities. However, only axial strain energy has been calculated previously. Extending from that, we derive the thermal noise for a spindle spacer, including the contribution of shearing strain energy, based on FDT, and focusing on the spacer geometry and materials. We compare our new approach with finite element analysis (FEA) of the strain energy in spindle cavities, and conclude that the new analytic estimate for the spindle spacer contributions fits better with FEA.

1. Introduction

Ultra-stable lasers play an important role in many fields, such as high-precision laser spectroscopy, optical frequency standards, gravitational wave detection, tests of relativity, and transfer of optical frequencies by fiber networks [1–6]. The frequency stability of a cavity-stabilized laser is limited by the length stability of the optical cavities. Most external distortions, such as pressure, temperature, and vibration, can be well suppressed by thermal and mechanical isolation.

However, a more fundamental issue is the thermal noise inducing microscopic optical length fluctuations. Several thermal noise sources have been identified, such as Brownian thermal noise and thermo-elastic noise. The dominant effect is Brownian thermal noise, and significant effort has been expended to attempt to reduce this. Levin [7] calculated thermal noise in the cavity by applying the fluctuation–dissipation-theorem (FDT), and Numata et al. [8] derived three relationships based on strain energy and FDT to estimate the noise contributions of the spacer, substrates, and coatings, which adequately address the case of cylindrical cavities.

For state-of-the-art optical cavities with substrate materials of higher mechanical quality, fractional instability has been shown on the order of a few 10^{-16}, at this level the spacer contribution is non-negligible [9]. Based on the same cavity material, the thermal noise of the cavity is proportional to the strain energy in the cavity.

Generally the strain energy in linear elastic solids comprises three parts: the axial, bending, and shearing strain energies [10]. However, only axial strain energy has been calculated in previous estimations of the thermal noise contribution of the spacer. Extending this and our earlier work [11], we derive a new relationship for the thermal noise for a spindle spacer, including the contribution of shearing strain energy.

A spindle spacer is composed of a cylinder with two tapers at both ends, which is assumed to provide greater stiffness than that of a cylinder with the same length and mass [12]. The stiffness of elastic solid contains axial stiffness, bending stiffness and torsional stiffness [13]. Based on the same length and mass, the axial stiffness and torsional stiffness of a spindle spacer are close to that of a cylinder, however, a spindle spacer provides greater bending stiffness compared with that of a cylinder. This could reduce bending strain energy of a spindle spacer, and then decrease thermal noise of a cavity. Moreover, two tapers of a cylinder could reduce thermal expansion stress due to different coefficient of thermal expansion of Ultra Low Expansion (ULE) glass spacer and fused silica (FS) mirrors for lower thermal noise [14].

We derive the thermal noise for the spindle spacer based on FDT and strain energy. Our analysis considers two characteristics of the spindle cavity: the shearing elastic energy, and the variation of cross section along the optical axis of the spindle cavity. We compare our analysis to finite element analysis (FEA) of the same case and conclude our new estimate for the spindle spacer contribution to thermal noise which provides significantly better fit to experimental results. Considering vertex angles of two tapers of spindle spacers from Saint Venant's Principle [15], in this paper

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our new analysis formula fits with any length of spindle spacer compared with our previous publication [11], and in our new work a complete explanation of the divergence between the simulation and analytical estimation is presented in detail.

We discuss the theoretical framework of the thermal noise in the cavities in Section 2. We then apply the simulation to a typical spindle cavity design in Section 3.1 and discuss the deviations from the analytical estimate for the spindle spacer. To have a better understanding of the divergence between the simulation and analytical estimation, we investigate the thermal noise of spindle spacers under various conditions in Section 3.2. In Section 4, we focus on the thermal noise of combined-material spindle cavities and the effect of FS rings on the thermal noise. We summarize our outcomes and conclusions in Section 5.

2. Theoretical framework of the thermal noise

The effect of Brownian motion thermal noise on the length stability of ultra-stable optical resonators has been discussed by Numata et al. [8], following Levin's direct approach [7]. This concept is illustrated for clearness in the following. The sketch of the horizontal spindle cavity as an example is depicted in Fig. 1.

The distance between two mirrors in the cavity is x. From FDT, the power spectral density, $S_x(f)$, of the length fluctuations of x is

$$S_x(f) = \frac{4k_BT}{\pi f}\frac{U\phi}{F^2}, \tag{1}$$

where k_B is the Boltzmann constant; T is the temperature (assumed to be 300 K); F and f are the amplitude and frequency of an oscillatory force [8], respectively; U is the maximum elastic strain energy; and ϕ is the loss angle in the cavity. The power spectral density may be converted to fractional frequency noise

$$S_x(f)/L^2 = S_\nu(f)/\nu^2, \tag{2}$$

where L is the length of cavity, and ν is the laser frequency.

The power spectral density of spacer, substrates and coatings are expressed as $S_x(f)_{spa}$, $S_x(f)_{sub}$ and $S_x(f)_{coa}$, respectively. Provided all thermal noise sources are uncorrelated, the total power spectral density $S_x(f)$ is

$$\begin{aligned} S_x(f) &= S_x(f)_{spa} + 2(S_x(f)_{sub} + S_x(f)_{coa}) \\ &= \frac{4k_BT}{\pi f F^2}(U_{spa}\phi_{spa} + 2(U_{sub}\phi_{sub} + U_{coa}\phi_{coa})), \end{aligned} \tag{3}$$

where U_{spa}, U_{sub} and U_{coa} are the strain energy of the spacer, substrate, and coating, respectively; ϕ_{spa}, ϕ_{sub}, and ϕ_{coa} are the loss angle of spacer, substrate, and coating, respectively.

The Allan deviation [16] of the relative stability of the cavity is then

$$\sigma_y = \frac{\sqrt{2\ln(2)f}\sqrt{S_x(f)}}{L}, \tag{4}$$

The mirror spans an infinite half space, and, therefore, the strain energy of the substrate is [7,17]

$$\begin{aligned} U_{sub} &= \frac{(1-\mu^2)F^2}{2\sqrt{\pi}\omega_0 E_{sub}} \\ S_x(f)_{sub} &= \frac{4k_BT}{\pi f}\frac{(1-\mu^2)F^2}{2\sqrt{\pi}\omega_0 E_{sub}} \end{aligned} \tag{5}$$

where μ_{sub} is Poisson's ratio of the substrate, E_{sub} is Young's module of the substrate, and ω_0 is the radius of the laser mode on the mirror.

If a coating has thickness d_{coa}, its strain energy can be deduced to be

$$\begin{aligned} U_{coa} &= U_{sub}\left(\frac{2}{\sqrt{\pi}}\frac{1-2\mu_{sub}}{1-\mu_{sub}}\frac{d_{coa}}{\omega_0}\right) \\ S_x(f)_{coa} &= S_x(f)_{sub}\left(\frac{2}{\sqrt{\pi}}\frac{1-2\mu_{sub}}{1-\mu_{sub}}\frac{d_{coa}}{\omega_0}\right), \end{aligned} \tag{6}$$

Kessler et al. [9] showed that thermal noise in substrates and coatings calculated by Eq. (5) and Eq. (6) agrees well with simulations.

Assuming that the spacer has a cylindrical body, the strain energy of the spacer can be written as [9]

$$\begin{aligned} U_{spa}^{cyl} &= \frac{LF^2}{2\pi(R^2-r^2)E} \\ S_x(f)_{spa}^{cyl} &= \frac{4k_BT}{\pi f}\frac{L}{2\pi(R^2-r^2)E}\phi_{spa}, \end{aligned} \tag{7}$$

where ${U_{spa}}^{cyl}$ is the strain energy of the cylindrical spacer, and $S_x(f)_{spa}^{cyl}$ is the power spectral density of cylindrical spacer. E is Young's modulus of spacer. Eq. (7) was proven by Kessler et al. [9] in terms of the strain energy principle based on Levin's direct approach [7], and building on earlier work by Numata et al. [8].

Generally, the strain energy in linear elastic solids is composed of three parts: the axial, bending, and shearing strain energies [10]. Shearing strain would only arise from an inhomogeneous cross section of an object, and hence is not applicable in the cylindrical cavity case. However, in the case of a spindle or spherical cavity, the shearing stress term cannot be neglected.

The spindle spacer is assumed to be a linearly elastic solid. The cross section area of the spindle spacer varies along the cavity length, and as such cannot be averaged over the whole front face area, in the same manner as for the cylindrical spacer. The variation of the cross-section, reduced by the area of the central bore,

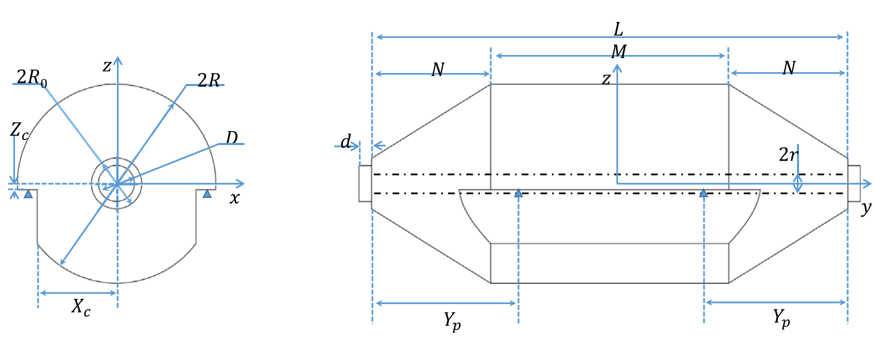

Fig. 1. The horizontal spindle cavity for estimation and simulation, where R is the spacer radius, r is the radius of the central bore, M is the cylinder length of the spindle spacer, N is the taper length, D is the diameter of the mirror (25.4 mm), d is the mirror thickness, R_0 is the radius of the front face of the spindle spacer, and L is the total length of the spacer. The optical axis lies along the y-axis. The four support points are represented by black triangles. The positions of the cutouts for support points are shown as X_c with respect to the yz plane, and Z_c with respect to the xy plane. Their distance along the y-axis from the end of the cavity is Y_p.