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Optimization of polarizer angles for measurements of the degree and angle of linear polarization for linear polarizer-based polarimeters

Run-Qiu Xia, Xia Wang*, Wei-Qi Jin, Jian-An Liang

MoE Key Laboratory of Photo-electronic Imaging Technology and System, Beijing Institute of Technology, Beijing 100081, China

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ABSTRACT

Linear polarizer-based polarimeters that measure the degree of linear polarization (DoLP) and the angle of polarization (AoP) were considered in this study. Variances of DoLP and AoP of the region of interest (ROI) to be measured were analyzed using a statistical method. To simplify the calculation, only additive noise and shot noise were considered. Optimized combinations of the polarizers that can minimize the variances of DoLP and AoP were determined by investigating the variances of different polarizer combinations. Several regularities were found when analyzing the data obtained from the optimized combinations. Some variables in the combinations are inversely proportional to the cube or square root of the signal-to-noise ratio of the output signals from sensors without polarizer filtering, and these variables are functions of the DoLP of the ROI to be measured.

1. Introduction

Polarization measurements have been used for many applications, including material classification, shape extraction and target detection [1–5]. In many cases, particularly for most passive measurements, the fourth element of the Stokes vector, which corresponds to elliptical polarization, can be considered negligible when measuring the radiation and reflection from outdoor objects [6,7]. The degree of linear polarization (DoLP) and angle of polarization (AoP) can be estimated by the first three elements [7].

The first three components of the Stokes vector can be calculated using the results of three or more intensity measurements filtered by linear polarizers that are oriented to transmit at different angles [6,7]. Based on principal-component analysis, Tyo has described an optimum linear combination method for an N-channel polarization-sensitive imaging system [8]. Other issues such as optimized strategies based on condition number and singular value decomposition for Stokes vector measurements have also been addressed [8–11].

Our work focuses on DoLP and AoP measurements for regions of interest (ROIs). Noise from sensors used in polarimeters directly impacts the precision of DoLP and AoP measurements. Using an error-propagation strategy [12], references [13–15] address the characteristics of the variances of the Stokes vector, DoLP and AoP acquired from a polarimeter. This strategy is based on the first-order linear estimation. The study by Goudail et al. is an example of using the error-propagation strategy to analyze the variances of the DoLP and AoP dependent on N (the number of the channels filtered by polarizers, which is mentioned in the article by Tyo [16]) when measuring the DoLP and AoP [14]. These authors pointed out that there are some limitations to this strategy. To simplify the calculation, they carried out the solution by restricting the range of the calculation to make sure that the estimate is valid. In our study, we will use a statistical method instead of a linear estimation method to calculate the variances of the DoLP and AoP measurements. The discrete probability density will be applied instead of continuous ones. Two types of noise will be considered: additive noise which is independent of the intensity of the signal to be measured and Poisson shot noise. We will show optimized combinations of the linear polarizers for minimizing the variances of the measurements of the DoLP and AoP of the ROI when specific numbers of polarizers are used.

2. Estimations of the DoLP and the AoP

The input linearly polarized light can be defined with first three elements of the Stokes vector $\mathbf{S} = (S_0, S_1, S_2)^T$ (the superscript T denotes transposition). We assume that the responses of the detectors are linear. Theoretically, the expected values of the intensities of the output signal from the detectors filtered by a set of N ideal linear polarizers can be described as $\langle \mathbf{I} \rangle = \mathbf{WS}$ [17], where $\mathbf{I} = [I_1 \ \cdots \ I_N]$, $\mathbf{W}$ is an $N \times 3$ matrix:

* Corresponding author.
E-mail address: angelniuniu@bit.edu.cn (X. Wang).

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$$\mathbf{W} = \frac{1}{2}\begin{bmatrix} 1 & \cos 2\theta_1 & \sin 2\theta_1 \\ \vdots & \cdots & \vdots \\ 1 & \cos 2\theta_N & \sin 2\theta_N \end{bmatrix} \tag{1}$$

where θ_n is the angle at which the nth ($1 \le n \le N$) linear polarizer is oriented to transmit. This relationship between the $\langle \mathbf{I} \rangle$ and $\mathbf{S}$ mentioned above is valid only when the intensity of the input light stays within the linear response range of the detector. When noises from the detectors are $\boldsymbol{\sigma}$, the output signals acquired from detectors filtered by the polarizers can be written as

$$\mathbf{I} = \langle \mathbf{I} \rangle + \boldsymbol{\sigma} \tag{2}$$

$\boldsymbol{\sigma}$ is dependent on the noise characteristics of the detectors that are employed. Ideally, the linear Stokes vector of the input light can be calculated by $\mathbf{S} = \mathbf{W}^{\dagger}\langle \mathbf{I} \rangle$ with $\mathbf{W}^{\dagger} = (\mathbf{W}^{\mathrm{T}}\mathbf{W})^{-1}\mathbf{W}^{\mathrm{T}}$. This relationship does not contain any variance caused by the detectors. When the perturbations as Eq. (2) are taken into account, the estimate of S can be written as

$$\hat{\mathbf{S}} = \mathbf{W}^{\dagger}\mathbf{I} \tag{3}$$

Once the estimate of the linear Stokes vector of the input light has been calculated, the estimates of DoLP and AoP can be written as [14]

$$\hat{P}(\mathbf{I}) = \frac{\sqrt{\hat{S}_1^{\,2} + \hat{S}_2^{\,2}}}{\hat{S}_0} = \frac{\sqrt{(\mathbf{W}_2^{\dagger}\mathbf{I})^2 + (\mathbf{W}_3^{\dagger}\mathbf{I})^2}}{\mathbf{W}_1^{\dagger}\mathbf{I}} \tag{4}$$

$$\hat{\alpha}(\mathbf{I}) = \frac{1}{2}\arctan\ 2(\hat{S}_2, \hat{S}_1) = \frac{1}{2}\arctan\ 2(\mathbf{W}_3^{\dagger}\mathbf{I}_1, \mathbf{W}_2^{\dagger}\mathbf{I}_2) \tag{5}$$

where arctan 2 is the four-quadrant inverse tangent function, whose return value lies in the closed interval $[-\pi, \pi]$, $\mathbf{W}_n^{\dagger}$ is the nth row of the matrix $\mathbf{W}^{\dagger}$. P denotes the true DoLP of scenes, α denotes the true AoP of scenes, and sign^ denotes the estimates of variables. Obviously, the variances of the measured $\mathbf{I}$ will be propagated to $\hat{P}$ and $\hat{\alpha}$ by Eqs. (4) and (5).

3. Variances of the DoLP and the AoP

3.1. Noise propagation method

As mentioned above, $\hat{\mathbf{S}}$ used for calculating $\hat{P}$ and $\hat{\alpha}$ contains types of noise from $\mathbf{I}$ generated in the process of photoelectric conversion and analog-to-digital conversion. Eqs. (4) and (5) are nonlinear functions of Stokes-vector components, and it is difficult to express the variances of $\hat{P}$ and $\hat{\alpha}$ intuitively.

To simplify the computation, Goudail et al. used a linear approximation to estimate variances of $\hat{P}$ and $\hat{\alpha}$ [14,18]. In this method, let $\mathbf{X}$ be an N-dimensional random vector with mean $\langle \mathbf{X} \rangle$ and covariance matrix $\boldsymbol{\Gamma}^{\mathbf{X}}$. The variable $y = f(\mathbf{X})$ is a function of the vector $\mathbf{X}$. Then, the estimation of variance of y can be written as $\mathrm{VAR}(y) = [\nabla f(\langle \mathbf{X} \rangle)]^{\mathrm{T}}\boldsymbol{\Gamma}^{\mathbf{X}}\nabla f(\langle \mathbf{X} \rangle)$, where $\nabla f(\mathbf{X}) = [\partial f/\partial X_1 \cdots \partial f/\partial X_N]^{\mathrm{T}}$ is the gradient of the function $f(\mathbf{X})$, and X_n is the nth component of $\mathbf{X}$.

The method is based on the linear estimation. This means that the variations of $\mathbf{X}$ around $\langle \mathbf{X} \rangle$ are sufficiently small and $f(\mathbf{X})$ around $\langle \mathbf{X} \rangle$ is sufficiently smooth. Goudail et al. have pointed out that this method is fulfilled only if components S_1, S_2 of the Stokes vector $\mathbf{S}$ are not closed to zero and P reaches a sufficiently high value [14].

For a more comprehensive analysis of the variances of polarization parameters, a statistical method is applied to calculate the variances of $\hat{P}$ and $\hat{\alpha}$ in this work. The outputs of detectors are most often discrete digital signals. If the discrete joint probability

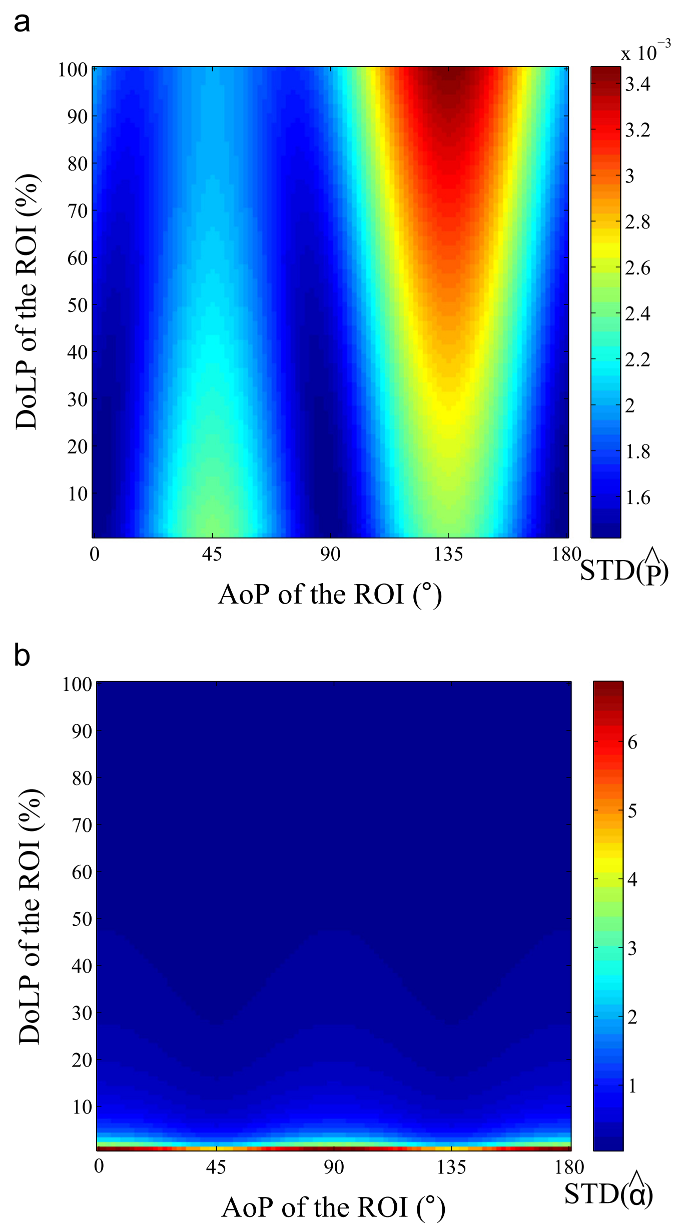

Fig. 1. STD of $\hat{P}$ (a) and $\hat{\alpha}$ (b) when the expected measurement result of S_0 of the ROI is 4000.

density function of the output signals $\mathbf{I}$ is $p(\mathbf{I})$, the expected values $\langle P \rangle$ and $\langle \alpha \rangle$ and their variances can be written as

$$\langle P \rangle = \sum_{\mathbf{I}} \hat{P}(\mathbf{I})p(\mathbf{I}) \tag{6}$$

$$\langle \alpha \rangle = \sum_{\mathbf{I}} \hat{\alpha}(\mathbf{I})p(\mathbf{I}) \tag{7}$$

$$\mathrm{VAR}(\hat{P}) = \sum_{\mathbf{I}} [\hat{P}(\mathbf{I}) - \langle P \rangle]^2 p(\mathbf{I}) \tag{8}$$

$$\mathrm{VAR}(\hat{\alpha}) = \sum_{\mathbf{I}} [\hat{\alpha}(\mathbf{I}) - \langle \alpha \rangle]^2 p(\mathbf{I}) \tag{9}$$

If all the components of $\mathbf{I}$ are independent, the joint probability density function can be written as