Optics Communications 353 (2015) 35–41

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

High harmonic generation in the undulators for free electron lasers

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ARTICLE INFO

Article history:
Received 21 February 2015
Received in revised form
22 April 2015
Accepted 28 April 2015
Available online 30 April 2015

Keywords:
Undulator radiation
Harmonics generation
Broadening
Free electron laser

ABSTRACT

We present the analysis of the undulator radiation (UR) with account for major sources of the spectral line broadening. For relativistic electrons we obtain the analytical expressions for the UR spectrum, the intensity and the emission line shape with account for the finite size of the beam, the emittance and the energy spread. Partial compensation of the divergency by properly imposed weak constant magnetic component is demonstrated in the analytical form. Considering the examples of radiation from single and double frequency undulators, we study high harmonic generation with account for all major sources of homogeneous and inhomogeneous broadening with account for the characteristics of the electrons beam. We apply our analysis to free electron laser (FEL) calculations and we compare the obtained results with the radiation of a FEL on the supposition of the ideal undulator.

1. Introduction

Undulator radiation (UR) was discovered by Motz [2] following the theoretical prediction [1] in the middle of the 20th century. It represents a form of a synchrotron radiation (SR) and is due to the photon emission by accelerated relativistic electrons, executing oscillatory trajectories in a periodic magnetic field [3]. During more than 70 years, the UR theory was brought to perfect (see, for example, [4–7]). Recent advances in technique and the appearance of free electron lasers (FEL) require UR sources with special properties, calibrated for the user's needs. In particular, high harmonics are more and more frequently exploited in the UR. Modern UR designs include precision made many period undulators with sometimes double period magnetic field [8–12] It helps to underline or reduce specific harmonic emission [13,14], for example, to reduce the hard radiation component or to enhance the frequency, needed for FEL [15,16]. High quality of the UR harmonic is of paramount importance in this context. However, an UR line is unavoidably subjected to broadening, originated from various sources, such as the electron energy spread, the beam divergency and the transport losses, not to mention non-periodic magnetic components. They may have internal or external origin [17–19], but their presence is eminent also due to the fact that the ideal periodic magnetic field simply does not satisfy Maxwell equations. The focusing magnetic components influence the beam transport; however, they are usually quite weak [17,20]. Recently the double frequency undulator was studied with account for an additional constant magnetic field and for the beam energy spread [21]. The authors concluded that the constant magnetic field shifted the resonance frequencies and caused loss of intensity. This is in perfect agreement with our recent results [20,22–24] for the on-axis UR of a planar undulator. However, some researchers conclude that undulators should not limit the spectral properties of higher UR harmonics, which should be limited only by the electron beam properties [25]. In view of that we extend to include the real size and the emittance of the electron beam in addition to the energy spread and the constant field component. We perform precise analytical treatment of the interplay of the respective homogeneous and inhomogeneous broadening contributions, giving a transparent view of their roles. We also include them to evaluate FEL performance with a double frequency undulator.

2. The UR with account for the constant magnetic components and for the electron energy spread in a beam of a finite size

The UR from an ultrarelativistic electron with $\gamma \gg 1$ and with small transverse momentum $\beta_\perp \ll 1$, $\beta_\perp H_\parallel \ll H_\perp$, $\vec{E}=0$ in a planar undulator with N periods of λ_u and the magnetic field $H_y = H_0 \sin(k_\lambda z)$, $k_\lambda = 2\pi/\lambda_u$, $\omega_0 = k_\lambda \beta_z^0 c$ where $\beta_z^0 = 1 - \frac{1}{2\gamma^2}\left(1+\frac{k^2}{2}\right)$ and $k = \frac{e}{mc^2}\frac{H_0}{k_\lambda}$ is the undulator parameter, and has the well known spectrum

$$\omega_{n0} = n\omega_{R0}, \quad \omega_{R0} = \frac{2\omega_0\gamma^2}{1+k^2/2}. \tag{1}$$

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http://dx.doi.org/10.1016/j.optcom.2015.04.079
0030-4018/

The deviation from the resonances (1) is described by the detuning parameter

$$\nu_n = 2\pi Nn\left(\frac{\omega}{\omega_n} - 1\right). \tag{2}$$

The above ideal case is characterized by the ideal width of an UR line $\frac{\Delta\omega}{\omega_{n0}} = \frac{1}{nN}$. The line has the shape, described by $\frac{\sin \nu_n/2}{\nu_n/2}$. The UR strongly depends on even minor constant magnetic constituents $H_x = \rho H_0$, $H_y = \kappa H_0$, $H_z = \delta H_0$, $\rho, \kappa =$ const in their transversal combination $H_d = H_0\kappa_1$, $\kappa_1 = \sqrt{\kappa^2 + \rho^2}$ [22–24,26]. H_d produces the effective bending angle $\theta_H = \frac{2}{\sqrt{3}}\frac{k}{\gamma}\pi N\kappa_1$. Together with the off-axis angle ψ they broaden the emission lines and shift the UR spectrum down respectively to (1) contrary to the energy spread $\sqrt{\sigma_e}$, which just broadens the line. The concept of the broadening parameters [27] qualitatively accounts for the UR line broadening [20,23,24] via broadening factors μ_i for each broadening contribution. It demonstrates that the broadening is more significant for high harmonics rather than the fundamental frequency. The total broadening can be evaluated with the help of the following formula, which accounts for the electron beam sizes $\sigma_{x,y}$ and the emittances $\varepsilon_{x,y}$, yielding the deflection angles $\Theta_{x,y} = \varepsilon_{x,y}/\sigma_{x,y}$, for H_d and for the beam energy spread σ_e:

$$\left[\frac{\Delta\omega}{\omega_n}\right]_{Tot} \cong \frac{\Delta\omega}{\omega_{n,0}}\sqrt{1 + \mu_{Tot}^2},$$

$$\mu_{Tot} = nN\sqrt{16\sigma_e + \frac{(\gamma\Theta_x)^4 + (\gamma\Theta_y)^4 + (\gamma\theta_H)^4}{(1 + k^2/2)^2}}. \tag{3}$$

In an ideal undulator $\mu_{Tot} = 0$. Note that the broadening parameters do not describe the interplay between various broadening contributions.

The UR intensity is calculated by means of classical electrodynamics [28,29]

$$\frac{d^2I}{d\omega d\Omega} = \frac{e^2}{4\pi^2 c}\left|\omega\int_{-\infty}^{\infty}[\vec{n}\times[\vec{n}\times\vec{\beta}]]\exp[i\omega(t - \vec{n}\vec{r}/c)]dt\right|^2, \tag{4}$$

where $\vec{n}$ is the observation vector, approximated by $\vec{n} \cong (\psi\cos\varphi, \psi\sin\varphi, 1 - \psi^2/2)$ for $\gamma \gg 1$. Our analytical approach, accounting for the off-axis effects and for the exact shape of the magnetic field, yields accurate expressions for the UR intensity and preserves information about its polarization. We calculate the trajectories of the electrons in the magnetic field similarly to [22] and we expand the exponential and the double vector product in the radiation integral (4) in series of $1/\gamma$, keeping the leading terms. This approach is fully justified for ultrarelativistic electrons with $\gamma > 10^2$ and, of course, it is valid for modern storage rings, where γ reaches 35,000 [30]. Eventually, we obtain integral forms, which can be transformed into the generalized Bessel functions $J_n^{(m)}(x_0, x_1, x_2, x_3)$ [31], describing the spectrum, and into the Airy-type special functions $S(\alpha, \beta, \eta) \equiv \int_0^1 d\tau e^{i(\alpha\tau + \eta\tau^2 + \beta\tau^3)}$ [11], describing the shape of the UR lines. The ideal UR line is given by $\mathrm{sinc}\,\nu_n/2$.

Consider a double frequency undulator with the following magnetic field:

$$\vec{H} = H_0(\rho, \kappa + \sin(k_\lambda z) + d\sin(hk_\lambda z), 0), \quad h \in \text{integer}. \tag{5}$$

The UR intensity, computed with account for the additional weak magnetic component H_d, for the energy spread $\sqrt{\sigma_e}$, for the beam sizes $\sigma_{x,y}$ and the emittances $\varepsilon_{x,y}$, becomes

$$\left.\frac{d^2I}{d\omega d\Omega}\right|_{\left(\frac{H_d}{H_0}\right)^2 \ll \frac{1}{(4\pi N)^2}} \approx \frac{e^2N^2\gamma^2}{c}\frac{k^2}{\left(1 + \frac{k^2}{2}\left(1 + \left(\frac{d}{h}\right)^2\right)\right)^2}\sum_{n=-\infty}^{\infty} n^2[S(\alpha, \beta, \eta)(\vec{T})]^2, \tag{6}$$

where $\alpha = \nu_{n0} + \frac{2\pi nN(\gamma\psi)^2}{1 + (k^2/2) + (\gamma\psi)^2}$ describes the influence of the finite beam size $\sigma_{x,y}$, $\nu_{n0} = 2\pi Nn\left(\frac{\omega}{\omega_{n0}} - 1\right)$, $\omega_{n0} = n\omega_{R0}$ (see (1)). The emittances $\varepsilon_{x,y}$ of the beam result in the off-axis angle $(\gamma\psi)^2 = (\gamma\Theta_x)^2 + (\gamma\Theta_y)^2$, expressed in terms of the angles $\Theta_{x,y} = \varepsilon_{x,y}/\sigma_{x,y}$. The parameter $\beta = \frac{(2\pi nN + \nu_n)(\gamma\theta_H)^2}{1 + (k^2/2)(1 + (d/h)^2) + (\gamma\theta_H)^2}$ is responsible for the effect of the constant field H_d, written in terms of the bending angle $\theta_H = \frac{2}{\sqrt{3}}\frac{k}{\gamma}\pi N\kappa_1$, and the parameter $\eta = \frac{(2\pi N)^2 nk\gamma}{1 + k^2/2 + (\gamma\psi)^2}\left(\kappa\Theta_x - \rho\Theta_y\right)$ describes the interplay of the off-axis divergence in the angles $\Theta_{x,y}$ and the field component H_d. Note, that α in $S(\alpha, \beta, \eta)$ is in fact the detuning parameter ν_n (see (2)) for the resonance frequency $\omega_0 = \frac{2n\omega_0\gamma^2}{1 + k^2/2 + (\gamma\psi)^2}$. In such terms we can write $S(\nu_n, \beta, \eta)$, where $\eta = 2\pi^2N^2(\kappa\cos\varphi - \rho\sin\varphi)\frac{\omega}{\omega_0}\left(\frac{k}{\gamma}\right)\psi$; $S|_{\psi=0} = S(\nu_{n0}, \beta, 0)$.

Vector $\vec{T} = \left\{T_{n,x}, T_{n,y}, T_{n,z}\right\}$ describes the polarization of the emitted harmonics of the UR. For the planar undulator (5) it contains only x-component, which reads

$$T_{n,x} = \left[T_{n-1}(n\mathrm{arg}) + T_{n+1}(n\mathrm{arg}) + \frac{d}{h}(T_{n+h}(n\mathrm{arg}) + T_{n-h}(n\mathrm{arg}))\right], \quad T_{n,y} = 0. \tag{7}$$

The generalized Bessel function $T_n(\mathrm{arg}) \equiv T_n(\xi, \xi_-, \xi_+, \xi_h)$ is specified in the integral form

$$T_n(\mathrm{arg}) = \frac{1}{2\pi}\int_0^{2\pi}\cos\begin{bmatrix} n\phi - \xi\sin(2\phi) - \xi_-\sin((h-1)\phi) - \\ \xi_+\sin((h+1)\phi) - \xi_h\sin(2h\phi)\end{bmatrix}d\phi. \tag{8}$$

It depends on the following arguments arg≡ξ, ξ_-, ξ_+, ξ_h:

$$\xi = -\frac{1}{4}\frac{k^2}{1 + \frac{k^2}{2}\left(1 + \left(\frac{d}{h}\right)^2\right)},$$

$$\xi_- = -\frac{4d}{h(h-1)}\xi,$$

$$\xi_+ = -\frac{4d}{h(h+1)}\xi,$$

$$\xi_h = -\frac{d^2}{h^3}\xi. \tag{9}$$

The UR spectrum of the two-frequency undulator contains the modified resonance length $\lambda_R = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{k^2}{2}\left(1 + \left(\frac{d}{h}\right)^2\right)\right)$. The exponential of the radiation integral (4) with account for the real size of the beam, for the emittance and for the constant field component H_d, yields the following formula for the UR spectrum:

$$\omega_n = n\omega_R = \frac{2n\omega_0\gamma^2}{1 + \frac{k^2}{2}\left(1 + \left(\frac{d}{h}\right)^2\right) + (\gamma\Theta_x)^2 + (\gamma\Theta_y)^2 + (\gamma\theta_H)^2 - \sqrt{3}(\gamma\theta_H)(\gamma\Omega)} \tag{10}$$