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Azimuthally modulated vortex solitons in bulk dielectric media with a Gaussian barrier

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ABSTRACT

By means of the variational approach, three classes of azimuthally modulated vortex solitons are constructed for the $(2+1)$-dimensional nonlinear Schrödinger equation with a type of transverse nonperiodic modulation. These solutions include numerical single layer azimuthal vortex solitons in the medium with Gaussian type modulation and analytical azimuthons in medium with the anharmonic potential. We present the results for two types of nonlinearities which are common in optical materials. The properties of vortex solitons, which are characteristic of the topological charge m, propagation constant k_0, angular velocity ω and radial quantum number L, are studied. The numerical simulations are used to verify the propagation properties of these vortex solitons.

1. Introduction

In recent years, the investigation of spatial and spatiotemporal localized structures in many branches of physics has been attracted enormous attention [1–16]. In nonlinear optics, optical vortices are the spatially localized self-trapped modes carrying a nonzero angular momentum, and share many common properties with the vortices observed in other systems [14–26]. The experimentally successful demonstrations for vortex solitons in various media, including Kerr, saturable-atomic and photorefractive nonlinear media [17,18], stimulate us to investigate the vortex solitons. From a short review of the vortex concept, we know that the vortex solitons always experience the symmetry breaking azimuthal instability, and they decay into several fundamental solitons in conservative nonlinear media. Ref. [16] revealed that spatially localized vortex solitons become stable in self-focusing nonlinear media when the vortex symmetry-breaking azimuthal instability is eliminated by a nonlocal nonlinear response. In a homogeneous medium, stable vortex solitons were proposed to exist in the so-called cubic–quintic or other similar nonlinear media, based on competing self-focusing and self-defocusing nonlinearities [24–26]. However, the experimental realization of vortex solitons in such media is hard, as the requirement of very high energy flow of light usually excites other higher-order nonlinearities, which may be dominant and suppress the occurrence of competing nonlinearities [27].

Probably the most interesting feature of the vortex physics is vortex response to external factors. Recent progress in the experimental study of nonlinear optical effects in bulk dielectric media opens up many novel possibilities in the study of transverse self-trapping of light and the formation of spatial optical solitons. These studies revealed that vortex solitons also can be stabilized in some confined systems, such as graded-index optical fibers [23,29], nonlinear photonic crystals with defects, optical lattices, or optical lattices with defects, where the azimuthal instability of vortices can be suppressed by the corresponding confining potentials [27,30–33]. Thus far, analytical vortex solitons are still poorly investigated. The optical settings allowing stable higher-charged vortex solitons are rare. Main efforts in bulk or in periodic optical lattice-modulated nonlinear media were devoted to the analysis of vortex solitons with charges less than or equal to two.

The nonlinear Schrödinger equation (NLSE) has been used to describe a very large variety of physical systems [34–45]. Some interesting spatial solitons are supported by NLSE with a type of transverse nonperiodic modulation, including Gaussian term [23,45]. However, no azimuthally modulated vortex solitons to these kinds of NLSE have been obtained. In this paper, the existence, stability and propagation dynamics of azimuthally

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modulated vortex solitons in local Kerr and Saturable mediums with transverse modulation are investigated. Variational approach and numerical technique allow us to construct azimuthally modulated vortex solitons in the (2+1)-dimensional (2D) NLSE with external potential, including numerical single layer azimuthal vortex solitons in the medium with Gaussian type modulation and analytical azimuthons in medium with the anharmonic potential. We revealed the properties of vortex solitons which are characteristic of the topological charge m, propagation constant k_0, angular velocity ω and radial quantum number L. In particular, these vortex solitons with various charges exist in certain region of angular velocity ω. Specific examples and figures are given illustrating discussed features. The numerical simulations are used to verify the propagation properties of these vortex solitons. Our focus will be on nonlinear optical systems, but our ideas can be applied to BECs with external potential. Note that, some of our results coincide with those of radial counterparts in local Kerr media in the absence of external potential in Ref. [46].

2. The model and the variational approach

We consider the paraxial propagation of light in a nonlinear medium with an instantaneous response governed by the 2D NLSE [47]

$$i\frac{\partial \varepsilon}{\partial z} + \beta\Delta\varepsilon + \chi V(x, y)\varepsilon + \sigma N(I)\varepsilon = 0, \tag{1}$$

with the refractive index n, $n(x, y, z) = n_0 + n_1\chi V(x, y) + n_2\sigma N(I)$, where Δ is the 2D Laplacian, $\varepsilon(x, y, z)$ is the complex envelope of the electrical field, $I = |\varepsilon|^2$ is the intensity of the propagating light beam. x, y and z are the dimensionless variables. Moreover, the first two terms of the refractive index n describe the linear contribution to the refractive index and the last intensity-dependent term represents the nonlinearity. We assume $n_1 > 0$, but the dimensionless profile function $\chi V(x, y)$ can be negative or positive, depending on whether the graded-index medium acts as a focusing or defocusing linear lens.

Looking for a more general class of self-trapped beams, we consider solution allowing self-similar rotation [28,46] and use the cylindrical coordinates rotating with the angular velocity ω:

$$\varepsilon = U(r, \varphi - \omega z)\exp(ikz). \tag{2}$$

The complex envelope U satisfies the stationary NLSE:

$$i\omega\frac{\partial U}{\partial \theta} + kU - \beta\left(\frac{\partial^2 U}{\partial r^2} + \frac{1}{r}\frac{\partial U}{\partial r} + \frac{1}{r^2}\frac{\partial^2 U}{\partial \theta^2}\right) - \chi V(r)U - \sigma N(|U|^2)U = 0, \tag{3}$$

where $k = k_0 + m\omega$, and k and k_0 are the propagation constants in the rotating frame (r, θ) and in the laboratory frame (r, φ), respectively, with $\theta = \varphi - \omega z$.

In the nonlinear case, $\sigma \neq 0$, Eq. (3) is not integrable, and here we resort to variational analysis to find approximate stationary states. Eq. (3) is the associated Euler–Lagrange equation [48] of the following calculus of variations:

$$\delta\int_0^\infty\int_0^{2\pi}\wp\left(r, U, U_*, \frac{\partial U}{\partial \theta}, \frac{\partial U_*}{\partial \theta}, \frac{\partial U}{\partial r}, \frac{\partial U_*}{\partial r}\right)d\theta\, dr = 0. \tag{4}$$

The specific expression of $\wp$ is determined by the certain potential $V(x, y)$ and nonlinearity $N(I)$.

2.1. Numerical solution in nonlinear medium with Gaussian barrier

Below we present the results for two types of nonlinearities that are common in optical materials.

2.1.1. Nonlinear Kerr medium

The most familiar local Kerr medium corresponds to the case $N(I) = I$. Under this condition, we have

$$\wp = \frac{i}{2}\omega r\left(U\frac{\partial U_*}{\partial \theta} - U_*\frac{\partial U}{\partial \theta}\right) - \beta\frac{1}{r}\left|\frac{\partial U}{\partial \theta}\right|^2 - \beta r\left|\frac{\partial U}{\partial r}\right|^2 + r\chi V|U|^2 - kr|U|^2 + \frac{r\sigma}{2}|U|^4. \tag{5}$$

Based on the method of separation of variables and the results got in Refs. [28,46], it is reasonable for us to assume a symmetric, azimuthally modulated field in cylindrical coordinates (r, θ, z), for which U takes the form

$$U(r, \theta) = \rho G(r)\Phi(\theta), \tag{6}$$

where $G(r)$ is the real envelope and $\Phi(\theta)$ is the azimuthal complex function

$$\Phi(\theta) = \cos(m\theta) + iq\ \sin(m\theta), \tag{7}$$

and topological charge m is an integer. Then, the distributions of the intensity of vortex solitons are dependent on the azimuthal angle. The choice for modulation depth parameter $q \in [0, 1]$ illustrates the fact that azimuthally modulated vortex can be linked both with typical vortex ($q=1$) and multi-pole soliton ($q=0$). In this assumption, the beam propagates in the z direction with transverse amplitude variation dependent on the radial and axial coordinates r and z.

Substituting Eq. (6) with Eq. (7) to Eq. (5) leads to an average calculus of variations

$$\delta\int_0^\infty \Gamma(r)\, dr = 0, \tag{8}$$

where

$$\Gamma(r) = \frac{A^2\pi}{8r}\left\{8(1 + q^2)\beta r^2\left(\frac{\partial G}{\partial r}\right)^2 - \sigma A^2(2q^2 + 3q^4 + 3)r^2G^4 + [-16\omega qm + 8(k - \chi V)(1 + q^2)]r^2G^2 + 8m^2(1 + q^2)\beta G^2\right\}. \tag{9}$$

In this case, we consider a 2D transverse modulation adding a Gaussian term to the usual harmonic potential, resulting in a potential of the form [23,45]

$$V(r) = -ar^2 - b\ \exp(-cr^2), \tag{10}$$

where a, b and c are adjustable positive real parameters. According to the requirement

$$\frac{d\Gamma}{dG} - \frac{d}{dr}\left[\frac{d\Gamma}{d\left(\frac{dG}{dr}\right)}\right] = 0, \tag{11}$$

we have

$$\frac{1}{r}\frac{dG}{dr} + \frac{d^2G}{dr^2} - \mu G - \frac{m^2}{r^2}G + VG + G^3 = 0, \tag{12}$$

with

$$\mu = \frac{k}{\beta} - \frac{2\omega qm}{\beta + \beta q^2}, \quad \chi = \beta, \quad \rho = \sqrt{\frac{4\beta q^2 + 4\beta}{\sigma(3 + 2q^2 + 3q^4)}}. \tag{13}$$

It should be noted that equation parameter $\mu > 0$ is related to k, β, ω, q, m.

Eq. (12) gives the exact profiles for the vortex solitons. We start our analysis with Eq. (12) numerically and find localized solutions for any $m^2 \geq 1$. Some sample profiles for various parameters are