Optics Communications 363 (2016) 134–137

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Can different media generate scattered field with identical spectral coherence?

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ARTICLE INFO

Article history:
Received 8 August 2015
Received in revised form
28 September 2015
Accepted 7 November 2015

Keywords:
Light wave scattering
Random medium
Spectral coherence

ABSTRACT

The possibility for different media to generate scattered field with identical spectral coherence is discussed. It is shown that two random media, with different characters of correlation function, may generate scattered field with identical spectral coherence property. An example of light waves on scattering from Gaussian-Schell model media is discussed, and a condition for identical spectral coherence of the far-zone scattered field is obtained.

1. Introduction

The potential scattering theory, which discusses the relationship between the property of the scattered field and the character of the scattering medium, is a topic of considerable importance because of its potential applications in area such as remote sensing, detecting, and medical diagnosis. In 1989, Wolf and his collaborators found that the spectrum of a polychromatic light wave will change as it is scattered from a random medium [1]. After that, numerous papers were published to discussing the effect of the character of scattering medium on the property of far-zone scattered field (see, for examples [2–7]). It is shown that when a light wave is scattered from a random medium, both the density function and the correlation coefficient play roles in the property of the far-zone scattered field [8]. Recently, some progress was made on the potential scattering theory. For examples, the scattering theory was generalized from scalar light wave to vector light wave [9,10], the scattering medium was generalized from isotropic medium to anisotropic medium [11], ellipsoidal medium [12], and semisoft boundary medium [13], and the effect of the property of the incident light wave on the scattered field is also discussed [14–16] (for a review of these work, please see [17]).

On the other hand, it is well known that two sources, which have different distributions of intensity and of degree of spatial coherence, may generate far-field with identical intensity, which is known as the "equivalence theorem" [18]. Most recently, the same phenomenon was also found in the scattering of light waves, i.e. two random medium with different correlation function may produce far-zone field with identical intensity [19]. Then one may wonder that whether two different random medium may also produce scattered field with identical distribution of the spectral coherence? It is an interesting problem especially in the inverse scattering problem due to the fact that some structure information about the scattering medium can be found from the measurements of the spectral coherence of the scattered field. However, to the best of our knowledge, there is no detail report on it. In this manuscript, we will show that two random medium with different correlation function, may produce far-zone field with the identical distribution of the spectral coherence.

2. Theory

Let us consider that a monochromatic spatially coherent plane light wave, with a propagation direction specified by a real unit vector $\mathbf{s}_0$, is incident on a random medium occupying a finite domain D (see Fig. 1). The property of incident field at a pair of points $\mathbf{r}'_1$ and $\mathbf{r}'_2$ is governed by the cross-spectral density function of the incident field, with a form of [20]

$$W^{(i)}(\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{s}_0, \omega) = S^{(i)}(\omega)\exp\left[ik\mathbf{s}_0\cdot(\mathbf{r}'_2 - \mathbf{r}'_1)\right], \tag{1}$$

where $S^{(i)}(\omega)$ is independent on the position, and $k = \omega/c$ with c being the speed of light in vacuum.

For a random medium, the scattering potential $F(\mathbf{r}', \omega)$ then is not a well-defined function of the position vector $\mathbf{r}'$. In this case, the properties of the scattering medium should be described by a

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http://dx.doi.org/10.1016/j.optcom.2015.11.017

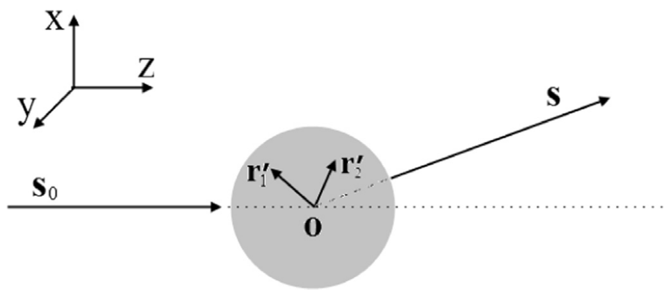

Fig. 1. Illustration of the notations.

correlation function of scattering potentials at a pair of points specified by position vectors $\mathbf{r}'_1$, $\mathbf{r}'_2$, which is defined as

$$C_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = \langle F^*(\mathbf{r}'_1, \omega)F(\mathbf{r}'_2, \omega)\rangle_m, \tag{2}$$

where the asterisk denotes the complex conjugate, and the angular brackets denotes the average, taken over the ensemble of the random medium. Assume that the scattering medium is weak so that the scattering can be analyzed within the accuracy of the first-order Born approximation [21]. Then the far-zone cross-spectral density function of the scattered field can be expressed as [20]

$$W^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2}\tilde{C}_F[-k(\mathbf{s}_1 - \mathbf{s}_0), k(\mathbf{s}_2 - \mathbf{s}_0), \omega], \tag{3}$$

where

$$\tilde{C}_F[\mathbf{K}_1, \mathbf{K}_2, \omega] = \iint_D C_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega)\exp\left[-i(\mathbf{K}_1\cdot\mathbf{r}'_1 + \mathbf{K}_2\cdot\mathbf{r}'_2)\right]d^3r'_1d^3r'_2 \tag{4}$$

is the six-dimensional Fourier transform of the correlation function $C_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega)$, and

$$\mathbf{K}_1 = -k(\mathbf{s}_1 - \mathbf{s}_0), \quad \mathbf{K}_2 = k(\mathbf{s}_2 - \mathbf{s}_0). \tag{5}$$

Now let us consider the spectral coherence of the scattered field. The spectral degree of coherence of the scattered field, which can be obtained from the cross-spectral density function and its spectral density, is defined by [20]

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{W^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega)}{\sqrt{S^{(s)}(r\mathbf{s}_1, \mathbf{s}_0, \omega)}\sqrt{S^{(s)}(r\mathbf{s}_2, \mathbf{s}_0, \omega)}}, \tag{6}$$

where

$$S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) = W^{(s)}(r\mathbf{s}, r\mathbf{s}, \mathbf{s}_0, \omega) \tag{7}$$

is the far-zone spectral density of the scattered field. On substituting from Eq. (3) together with Eq. (7) into Eq. (6) and after some calculations, one can find the spectral degree of coherence of the scattered field, with a form of

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{\tilde{C}_F[-k(\mathbf{s}_1 - \mathbf{s}_0), k(\mathbf{s}_2 - \mathbf{s}_0), \omega]}{\sqrt{\tilde{C}_F[-k(\mathbf{s}_1 - \mathbf{s}_0), k(\mathbf{s}_1 - \mathbf{s}_0), \omega]}\sqrt{\tilde{C}_F[-k(\mathbf{s}_2 - \mathbf{s}_0), k(\mathbf{s}_2 - \mathbf{s}_0), \omega]}}. \tag{8}$$

As shown in Eq. (8), when a spatially coherent plane light wave is scattered from a random medium, the spectral degree of coherence of the far-zone field is governed by the six-dimensional Fourier transform of the correlation function. For the simplicity of following discussion, let us represent the correlation function as its density function and its normalized correlation coefficients, i.e.

$$C_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = \left[I_F(\mathbf{r}'_1, \omega)I_F(\mathbf{r}'_2, \omega)\right]^{1/2}\mu_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega) \tag{9}$$

where $I_F(\mathbf{r}', \omega) \equiv C_F(\mathbf{r}', \mathbf{r}', \omega)$ is the density of the correlation function, and $\mu_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega)$ is known as the normalized correlation coefficient. Then the Fourier transform of the correlation function can be rewritten as

$$\tilde{C}_F[\mathbf{K}_1, \mathbf{K}_2, \omega] = \iint_D \left[I_F(\mathbf{r}'_1, \omega)I_F(\mathbf{r}'_2, \omega)\right]^{1/2}\mu_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega)\exp\left[-i(\mathbf{K}_1\cdot\mathbf{r}'_1 + \mathbf{K}_2\cdot\mathbf{r}'_2)\right]d^3r'_1d^3r'_2. \tag{10}$$

On substituting from Eq. (10) into Eq. (8), one readily finds that both the density function I_F and the correlation coefficient μ_F play roles in the distribution of the spectral coherence of the scattered field. Therefore, one can make a “trade-off” between the density function I_F and the normalized correlation coefficient μ_F without changing the distribution of the spectral coherence of the far-zone scattered field. It can be further concluded that two random media with different density function and different normalized correlation coefficient I_F, μ_F and I'_F, μ'_F may produce identical spectral coherence when a plane light wave is scattered from a medium.

3. An example

As an example, let us consider the spectral coherence of light waves on scattering from a *Gaussian–Schell model* medium. In this case, the density function and the normalized correlation coefficient of the correlation function are both Gaussian function [19], i.e.

$$I_F(\mathbf{r}', \omega) = A\exp\left[-\frac{\mathbf{r}'^2}{2\sigma^2}\right], \tag{11a}$$

$$\mu_F(\mathbf{r}'_2 - \mathbf{r}'_1, \omega) = \exp\left[-\frac{(\mathbf{r}'_2 - \mathbf{r}'_1)^2}{2\mu^2}\right]. \tag{11b}$$

On substituting from Eq. (11) into Eq. (9), one can obtain the correlation function of the scattering potentials of the medium, with a form of

$$C_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = A\exp\left[-\frac{\mathbf{r}'^2_1 + \mathbf{r}'^2_2}{4\sigma^2}\right]\exp\left[-\frac{(\mathbf{r}'_2 - \mathbf{r}'_1)^2}{2\mu^2}\right]. \tag{12}$$

For the simplicity of the following integration, let us rewrite the correlation function in a more compact form, i.e.

$$C_F(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = A\exp\left[-a(\mathbf{r}'^2_1 + \mathbf{r}'^2_2) + 2b\mathbf{r}'_1\cdot\mathbf{r}'_2\right] \tag{13}$$

with

$$a = \frac{1}{4\sigma^2} + \frac{1}{2\mu^2}, \quad b = \frac{1}{2\mu^2}. \tag{14}$$

On substituting from Eq. (13) into Eq. (10), and make using of the following variable transformation

$$\mathbf{r}'_1 + \mathbf{r}'_2 = 2\mathbf{r}'_S, \quad \mathbf{r}'_2 - \mathbf{r}'_1 = \mathbf{r}'_D. \tag{15}$$

One can rewrite the six-dimensional Fourier transform of the correlation function as

$$\tilde{C}_F[\mathbf{K}_1, \mathbf{K}_2, \omega] = A\int_D \exp\left[-2(a-b)\mathbf{r}'^2_S\right]\exp(-i\mathbf{K}_S\cdot\mathbf{r}'_S)d^3r'_S \times \int_D \exp\left[-\frac{a+b}{2}\mathbf{r}'^2_D\right]\exp(-i\mathbf{K}_D\cdot\mathbf{r}'_D)d^3r'_D, \tag{16}$$

where

$$\mathbf{K}_S = \mathbf{K}_1 + \mathbf{K}_2, \quad \mathbf{K}_D = \frac{\mathbf{K}_2 - \mathbf{K}_1}{2}. \tag{17}$$

After manipulating the six-dimensional Fourier transform, one finds