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Electromagnetic scattering by spheres of topological insulators

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ABSTRACT

The electromagnetic scattering properties of topological insulator (TI) spheres are systematically studied in this paper. Unconventional backward scattering resulting from the topological magnetoelectric (TME) effect are found in both Rayleigh and Mie scattering regimes. This enhanced backward scattering can be realized by introducing an impedance-matched background. In addition to conventional resonances, interesting antiresonances in scattering coefficients are found in the Mie scattering regime. At the antiresonances, electric or magnetic fields induced by the TME effect can be completely trapped inside TI spheres. In the Rayleigh limit, a method to determine the quantized TME effect of TIs is proposed simply based on the measured electric field components of scattered waves in the far field.

1. Introduction

Topological insulators (TIs) are an emerging quantum phase in condensed matter physics [1–3] with gapless edge or surface states within the bulk energy gap which are protected by time-reversal symmetry. TI materials have been theoretically predicted and experimentally observed in various systems such as HgTe/CdTe quantum well, and $Bi_{1-x}Sb_x$, Bi_2Te_3, Bi_2Se_3 [1,2]. A novel quantized topological magnetoelectric effect (TME) is predicted in TIs: an applied electric field could induce parallel magnetization while an applied magnetic field could induce parallel electric polarization [3]. As a result, an additional term in the Lagrangian $\Delta\mathcal{L} = (\Theta\alpha/4\pi^2)\mathbf{E}\cdot\mathbf{B}$ should be incorporated [3,4], where $\alpha = e^2/\hbar c$ is the fine structure constant, $\Theta = (2p+1)\pi$ is the axion angle with p being an integer, $\mathbf{E}$ is the electric field and $\mathbf{B}$ is the magnetic field. Together with the conventional term in the Lagrangian, it can give a complete description of the electromagnetic (EM) responses of TIs. The corresponding constitutive relations for TIs should be modified as $\mathbf{D} = \varepsilon\mathbf{E} - \bar{\alpha}\mathbf{B}$, $\mathbf{H} = \mathbf{B}/\mu + \bar{\alpha}\mathbf{E}$, where $\mathbf{D}$ and $\mathbf{H}$ are respectively the electric displacement and magnetic field strength, ε and μ are respectively the permittivity and permeability of TIs, $\bar{\alpha} = \Theta\alpha/\pi$ is proportional to the fine structure constant. Indeed, many unusual EM phenomena due to the TME effect have been revealed [5–7].

Scattering of EM waves by small particles is fundamentally interesting with many important applications [8]. Compared with conventional scatterers, scattering of EM waves by scatterers of TIs shows many unusual features due to the presence of the TME effect. For instance, parity-violating scattering under oblique incidence and a strong perturbation of dipole radiations in TI spheres were predicted [9]. Broadband strong scattering in the backward direction and interesting antiresonances were found for TI cylinders [10]. Moreover, the quantization of the TME effect of TI cylinders can even be determined by measuring the electric field components of scattered waves in the far field at one or two scattering angles in the Rayleigh scattering limit [11].

In this paper, we study theoretically scattering of EM waves by spheres of TIs. Based on the standard Mie theory, we derive the scattering coefficients and scattering matrix for TI spheres analytically. Exotic backward scattering due to the TME effect is found in both Rayleigh and Mie scattering regimes, similar to the case for TI cylinders [10]. At certain frequencies, antiresonances of cross-polarized scattering coefficients of TI spheres are revealed, wherein the cross-polarized fields induced by the TME effect are trapped inside TI spheres. In the Rayleigh limit, we propose a simple way to determine the quantized TME effect of TIs by measuring the electric field components of scattered waves in the far field.

2. Mie theory for TI spheres

The system under study is shown in Fig. 1. We consider a TI sphere which is illuminated by a time-harmonic EM wave with an angular frequency ω. The radius of the sphere is a. The dielectric

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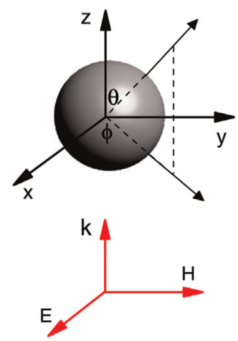

Fig. 1. Schematic view of the system under study. A TI sphere is placed at the origin. An EM plane wave polarized in the x direction is incident along the z direction.

permittivity and magnetic permeability for the TI sphere are (ε_1, μ_1), and those for the background medium are (ε_b, μ_b). The scattering problem can be solved analytically by the standard Mie theory. In the spherical coordinate system, (r, θ, ϕ), the EM fields can be expanded by the vector spherical harmonics [8]: $\mathbf{M}_{e1n}^{(I)}(kr)$, $\mathbf{M}_{o1n}^{(I)}(kr)$, $\mathbf{N}_{e1n}^{(I)}(kr)$ and $\mathbf{N}_{o1n}^{(I)}(kr)$, where subscripts e and o denote respectively the even and odd modes with respect to the x axis, k is the corresponding wavevector ($k_1 = \sqrt{\varepsilon_1\mu_1}\,\omega/c$ and $k_b = \sqrt{\varepsilon_b\mu_b}\,\omega/c$ are respectively the wavevectors in the TI sphere and background), n is an integer, and I stands for the kind of the spherical Bessel (Hankel) functions. The incident EM wave can be written as $\mathbf{E}_{inc} = E_0 e^{ik_b z}\hat{\mathbf{e}}_x$, where E_0 is the amplitude. The scattered and internal electric fields can be expanded as follows:

$$\mathbf{E}_{sca} = \sum_{n=1}^{n=\infty} E_n\left(-b_n\mathbf{M}_{o1n}^{(3)} + ia_n\mathbf{N}_{e1n}^{(3)} - b_n^{\mathrm{TI}}\mathbf{M}_{e1n}^{(3)} + ia_n^{\mathrm{TI}}\mathbf{N}_{o1n}^{(3)}\right), \tag{1}$$

$$\mathbf{E}_{int} = \sum_{n=1}^{n=\infty} E_n\left(c_n\mathbf{M}_{o1n}^{(1)} - id_n\mathbf{N}_{e1n}^{(1)} + c_n^{\mathrm{TI}}\mathbf{M}_{e1n}^{(1)} - id_n^{\mathrm{TI}}\mathbf{N}_{o1n}^{(1)}\right), \tag{2}$$

where $E_n = i^n E_0(2n+1)/n(n+1)$, $\{a_n, b_n\}$ are the conventional scattering coefficients. Different from conventional dielectric spheres, additional terms associated with two new scattering coefficients $\{a_n^{\mathrm{TI}}, b_n^{\mathrm{TI}}\}$, dubbed *cross-polarized* scattering coefficients, arise due to the TME effect. The superscripts 1 and 3 stand for the spherical Bessel function of the first kind and spherical Hankel function of the first kind, respectively. By matching the standard boundary conditions at $r=a$ and applying the modified constitutive relations $\mathbf{D} = \boldsymbol{\varepsilon}\mathbf{E} - \bar{\alpha}\mathbf{B}$, $\mathbf{H} = \mathbf{B}/\mu + \bar{\alpha}\mathbf{E}$ inside the TI sphere, the scattering coefficients a_n, b_n, and the internal coefficients c_n, d_n can be found as

$$a_n = \frac{\mu_b m^2 j_n(mx)[xj_n(x)]'\beta_2 - \mu_1 j_n(x)[mxj_n(mx)]'}{\mu_b m^2 j_n(mx)[xh_n(x)]'\beta_2 - \mu_1 h_n(x)[mxj_n(mx)]'}, \tag{3}$$

$$b_n = \frac{\mu_1 j_n(mx)[xj_n(x)]' - \mu_b j_n(x)[mxj_n(mx)]'\beta_1}{\mu_1 j_n(mx)[xh_n(x)]' - \mu_b h_n(x)[mxj_n(mx)]'\beta_1}, \tag{4}$$

$$c_n = \frac{\mu_1 j_n(x)[xh_n(x)]' - \mu_1 h_n(x)[xj_n(x)]'}{\mu_1 j_n(mx)[xh_n(x)]' - \mu_b h_n(x)[mxj_n(mx)]'\beta_1}, \tag{5}$$

$$d_n = \frac{\mu_1 mj_n(x)[xh_n(x)]' - \mu_1 mh_n(x)[xj_n(x)]'}{\mu_b m^2 j_n(mx)[xh_n(x)]'\beta_2 - \mu_1 h_n(x)[mxj_n(mx)]'}, \tag{6}$$

where j_n and h_n are respectively the spherical Bessel and Hankel functions of the first kind, $x \equiv k_b a$ is the size parameter and $m \equiv \sqrt{\varepsilon_1\mu_1}/\sqrt{\varepsilon_b\mu_b}$ is the relative refractive index. The auxiliary functions β_1 and β_2 are related to the axion angle Θ by $\beta_1 = 1 + \tilde{\alpha}^2\chi_1$ and $\beta_2 = 1 - \tilde{\alpha}^2\chi_2$, where $\tilde{\alpha} = \bar{\alpha}\sqrt{\mu_1/\varepsilon_1}$ and

$$\chi_1 = \frac{\mu_b m^2 j_n(mx)[xh_n(x)]'}{\mu_b m^2 j_n(mx)[xh_n(x)]' - \mu_1 h_n(x)[mxj_n(mx)]'}, \tag{7}$$

$$\chi_2 = \frac{\mu_b h_n(x)[mxj_n(mx)]'}{\mu_1 j_n(mx)[xh_n(x)]' - \mu_b h_n(x)[mxj_n(mx)]'}, \tag{8}$$

The cross-polarized scattering coefficients a_n^{TI} and b_n^{TI}, corresponding to n-th order of electric and magnetic multipoles but with 90° polarization-rotation compared to conventional ones, can be related to the internal coefficients by

$$a_n^{\mathrm{TI}} = -\frac{[mxj_n(mx)]'}{m[xh_n(x)]'}d_n^{\mathrm{TI}}, \quad b_n^{\mathrm{TI}} = -\frac{j_n(mx)}{h_n(x)}c_n^{\mathrm{TI}}. \tag{9}$$

The coefficients c_n^{TI} and d_n^{TI} are the cross-polarized internal coefficients of the TI sphere, related to the normal internal coefficients c_n and d_n by

$$d_n^{\mathrm{TI}} = \tilde{\alpha}\chi_1 c_n, \quad c_n^{\mathrm{TI}} = \tilde{\alpha}\chi_2 d_n. \tag{10}$$

Obviously, a_n^{TI} (electric) and b_n^{TI} (magnetic) are directly related to the internal multipolar terms c_n (magnetic) and d_n (electric), respectively. This is exactly a manifestation of the TME effect that an applied electric (magnetic) field can induce magnetic (electric) field. For a topologically trivial insulator, the axion angle $\Theta=0$, namely, $\tilde{\alpha} = 0$, leading to $\beta_1 = \beta_2 = 1$, $a_n^{\mathrm{TI}} = b_n^{\mathrm{TI}} = c_n^{\mathrm{TI}} = d_n^{\mathrm{TI}} = 0$. The scattering coefficients $\{a_n, b_n\}$ and internal coefficients $\{c_n, d_n\}$ are reduced to be the conventional ones as expected [8].

3. Scattering coefficients in the Rayleigh limit

We assume that both TI and background media under study are non-magnetic, i.e., $\mu_1 = \mu_b = 1$. In the Rayleigh scattering limit ($x \ll 1$ and $mx \ll 1$), it can be shown that only the following scattering coefficients are in the order of x^3:

$$a_1 = -i\frac{2}{3}\frac{3m^2 - 3 + 2\bar{\alpha}^2/\varepsilon_b}{3m^2 + 6 + 2\bar{\alpha}^2/\varepsilon_b}x^3 + O(x^5), \tag{11}$$

$$b_1 = i\frac{2}{3}\frac{\bar{\alpha}^2/\varepsilon_b}{3m^2 + 6 + 2\bar{\alpha}^2/\varepsilon_b}x^3 + O(x^5), \tag{12}$$

$$a_1^{\mathrm{TI}} = b_1^{\mathrm{TI}} = i\frac{2\bar{\alpha}/\sqrt{\varepsilon_b}}{3m^2 + 6 + 2\bar{\alpha}^2/\varepsilon_b}x^3 + O(x^5). \tag{13}$$

All other scattering coefficients are in the order of x^5 or higher and can be hence neglected in the Rayleigh scattering limit. The scattering coefficients a_1 and b_1 correspond to the electric and magnetic dipoles, whereas a_1^{TI} and b_1^{TI} correspond to the cross-polarized electric and magnetic dipoles, respectively.

4. Scattering efficiency

According to the Mie theory, the scattering efficiency for a TI sphere is given by

$$Q_{\mathrm{sca}} = \frac{2}{x^2}\sum_{n=1}^{\infty}(2n+1)\left(|a_n|^2 + |b_n|^2 + |a_n^{\mathrm{TI}}|^2 + |b_n^{\mathrm{TI}}|^2\right). \tag{14}$$

Different from conventional dielectric scatterers, the cross-polarized scattering coefficients also contribute to Q_{sca}. Thus, Q_{sca} is dependent on not only the materials parameters ε_1, μ_1, ε_b, μ_b, and x but also the axion angle Θ. We can further decompose Q_{sca} into two terms: the bulk scattering $Q_{\mathrm{b}} = (2/x^2)\sum_{n=1}^{\infty}(2n+1)(|a_n|^2 + |b_n|^2)$ and the scattering caused by surface Hall currents