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# Statistic estimation and validation of in-orbit modulation transfer function based on fractal characteristics of remote sensing images

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ABSTRACT

This paper deals with the estimation of an in-orbit modulation transfer function (MTF) by a remote sensing image sequence, which is often difficult to measure because of a lack of suitable target images. A model is constructed which combines a fractal Brownian motion model that describes natural images stochastic fractal characteristics, with an inverse Fourier transform of an ideal remote sensing image amplitude spectrum. The model is used to decouple the blurring effect and an ideal natural image. Then, a model of MTF statistical estimation is built by standard deviation of the image sequence amplitude spectrum. Furthermore, model parameters are estimated by the ergodicity assumption of a remote sensing image sequence. Finally, the results of the statistical MTF estimation method are given and verified. The experimental results demonstrate that the method is practical and effective, and the relative deviation at the Nyquist frequency between the edge method and the method in this paper is less than 5.74%. The MTF estimation method is applicable for remote sensing image sequences and is not restricted by the characteristic target of images.



## 1. Introduction

The modulation transfer function (MTF) describes a system's ability to reproduce faithfully the spatial frequency content of a scene [1]. It reflects the comprehensive effect of an imaging system and imaging conditions. Therefore, it is an important quantitative description of the imaging blurring effect. Accurate evaluation of in-orbit MTF for a remote sensing system is both beneficial to image restoration and meaningful to determine system quantitative radiometric accuracy [2,3].

There are two general image-based methods to measure the MTF of imaging systems. One method is based on fixed characteristic targets including the sine-wave method, spots, slits and edge method [4–6]. The other method is based on random targets [7]. The edge method is convenient than other methods for remote sensing imaging systems. However, some spatial remote sensing images are low-resolution geometrically, as the characteristic target is limited in these images [8]. Therefore, the MTF estimation method based on imaging a characteristic target is often invalid for some remote sensing imaging systems.

Existing research shows that natural scene and contours possess non-stationary, self-similarity and multiple scales characteristics. These characteristics are described by the stochastic fractal [9–12]. Corresponding to natural scenes, remote sensing images have stochastic fractal characteristics. In addition, its power spectrum manifests consistency [13–15]. Fractal Brownian motion (FBM) [16–18] as a generalization of the usual Brownian motion can be used as a model to describe a large number of natural phenomena and shapes. Therefore, it can be used to decouple the MTF and the ideal image of a remote sensing imaging system. However, different types of degradation affect the image power spectrum to different degrees. Therefore, it is inconvenient to apply the FBM model to MTF estimation directly.

To solve the above problem based on the analysis of remote sensing image sequence power spectrum characteristics, the relation between an ideal natural scene image power spectrum and a degradation image is deduced theoretically. Considering natural scene image power spectrum consistency, a MTF estimation model is built based on natural image power spectrum statistical characteristics. The method is independent of the characteristic target, and can be used for MTF estimation of remote sensing images that do not contain the characteristic target.

## 2. Characteristic analysis of the natural image spectrum

Natural images possess fractal characteristics of self-similarity, scale-invariability and self-affinity statistically. This stochastic

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fractal of natural phenomena is often described by fractal Brownian motion, and its power spectrum satisfies the following law [16,18]:

$$\left|\boldsymbol{F}[f]_{u,v}\right|^2 = \frac{C_i}{r^{2q}}, \tag{1}$$

where $f$ represents a natural image, $r = \sqrt{u^2 + v^2}$ is the distance between a frequency domain coordinate $(u, v)$ and the origin of coordinates, $q$ is the fractal exponent, $C_i$ is a constant. According to fractal principles, natural images are satisfied with Eq. (1) statistically. However, the analysis reveals that the differences of the fractal exponent of remote sensing images are obvious, which leads to difficulties for model application. To solve the problem of model parameter estimation, according to the natural image power spectrum fractal model the natural image amplitude spectrum is expressed as

$$f(r) = \begin{cases} \dfrac{c_i}{r^q} & r > 0 \\ V_m & r = 0. \\ \dfrac{c_i}{(-r)^q} & r < 0 \end{cases} \tag{2}$$

This is a symmetrical even function. When $r > 0$, the inverse Fourier transform of the function can be expressed as

$$\boldsymbol{F}^{-1}\left(\frac{c_i}{r^q}\right) = \frac{1}{2\pi}\int_{\upsilon_1}^{\upsilon_2} \frac{c_i}{r^q} e^{jrx} d\ r = \frac{1}{2\pi}\left(\frac{1}{jx}\frac{c_i}{r^q}e^{jrx} - \frac{q}{jx}\int_{\upsilon_1}^{\upsilon_2}\frac{c_i}{r^{q+1}}e^{jrx} d\ r\right)\Bigg|_{\upsilon_1}^{\upsilon_2}, \tag{3}$$

where $\upsilon_1$ and $\upsilon_2$ are the scopes of frequency of the inverse Fourier transform, and $x$ is a spatial domain coordinate.

A part of the inverse Fourier transform of the spectrum function is still similar to the original function; Eq. (4) can be given by an expansion of Eq. (3):

$$\boldsymbol{F}^{-1}\left(\frac{c_i}{r^q}\right) = f_1(r) f_2(r)\Big|_{\upsilon_1}^{\upsilon_2}, \tag{4}$$

where $f_1(r)$ and $f_2(r)$ can be described as Eqs. (5) and (6):

$$f_1(r) = \frac{1}{2\pi}\frac{c_i}{r^q}\left[1 + \lim_{n\to\infty}\left(\frac{q}{r^1} + \dots + \frac{q+n-1}{r^n}\right)\right] \tag{5}$$

$$f_2(r) = \frac{1}{jx}e^{jrx}. \tag{6}$$

Eq. (7) is obtained by integration of Eq. (4):

$$\boldsymbol{F}^{-1}\left(\frac{c_i}{r^q}\right) = f_1(\upsilon_2) f_2(\upsilon_2) - f_1(\upsilon_1) f_2(\upsilon_1). \tag{7}$$

In the same way, when $r < 0$,

$$\boldsymbol{F}^{-1}\left(\frac{c_i}{(-r)^q}\right) = f_1(\upsilon_1) f_2(-\upsilon_1) - f_1(\upsilon_2) f_2(-\upsilon_2). \tag{8}$$

When $r = 0$:

$$\boldsymbol{F}^{-1}[f(0)] = V_m, \tag{9}$$

where $V_m$ is an assuming value.

Adding the three parts of $r > 0$, $r < 0$ and $r = 0$:

$$\boldsymbol{F}^{-1}[f(r)] = f_1(\upsilon_2)\left[f_2(\upsilon_2) - f_2(-\upsilon_2)\right] + f_1(\upsilon_1)\left[f_2(-\upsilon_1) - f_2(\upsilon_1)\right] + V_m. \tag{10}$$

Considering that $\upsilon_1 \to 0$, $\upsilon_2 \to \infty$, while:

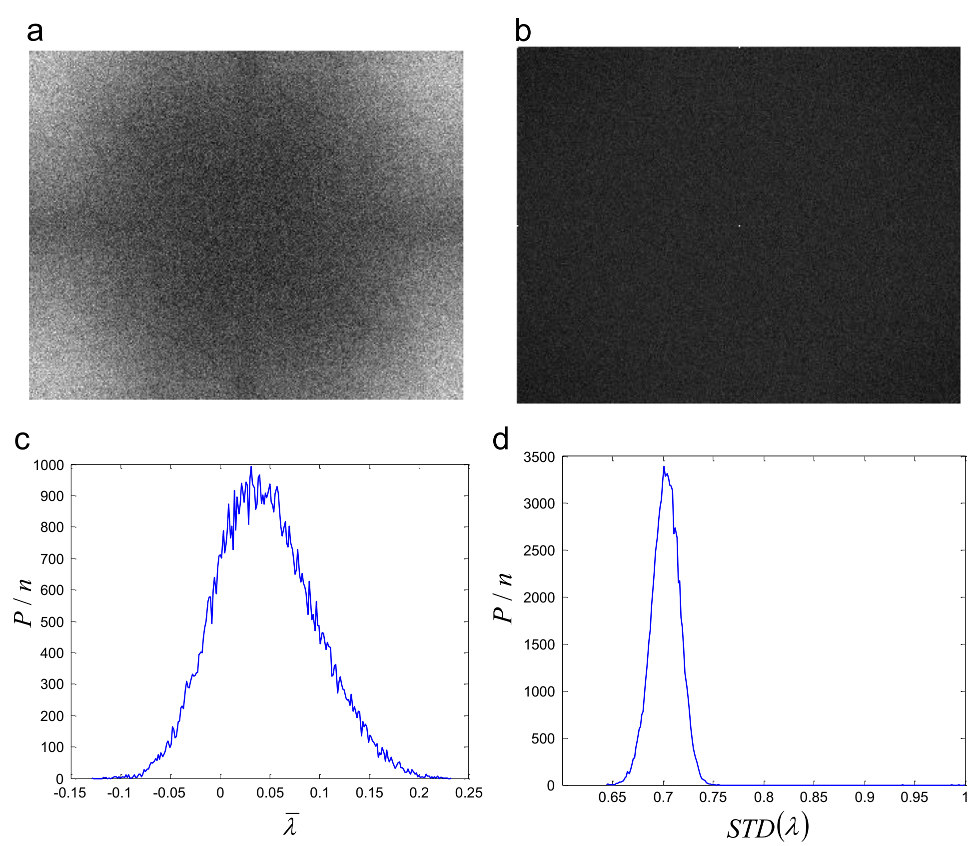


**Fig. 1.** Analysis of the noise amplitude spectrum factor.