Optics Communications 354 (2015) 291–298

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Decoherence analysis of multiple photon annihilation-then-creation coherent state: Amplitude decay and phase damping

Heng-Mei Li [a,*], Xue-Xiang Xu [b], Hong-Chun Yuan [c], Zhen Wang [a], Zhi-Long Wan [a], Xue-Fen Xu [d]

[a] College of Science, Changzhou Institute of Technology, Changzhou 213002, China
[b] College of Physics and Communication Electronics, Jiangxi Normal University, Nanchang 330022, China
[c] College of Optoelectronic Engineering, Changzhou Institute of Technology, Changzhou 213002, China
[d] Department of Basic Courses, Wuxi Institute of Technology, Wuxi 214121, China

ARTICLE INFO

Article history:
Received 26 April 2015
Received in revised form
29 May 2015
Accepted 3 June 2015
Available online 10 June 2015

Keywords:
Bell-polynomial
The negativity of Wigner function
Amplitude decay
Phase damping

PACS:
42.50.Dv
03.65.Ud

ABSTRACT

We mainly study on the effect of decoherence of the multiple photon annihilation-then-creation coherent state (MPACCS) in two different channels, i.e., amplitude decay and phase damping, by virtue of the time-evolution of the Wigner functions (WFs) of such state. Based on deriving the analytical expression of MPACCS's normalization factor related to the Bell-polynomial, the time-evolution of its WFs in each channel is derived analytically and discussed numerically. After undergoing the amplitude decay channel, the partial negativity of WFs diminishes gradually and the WFs have no negative region when the decay time exceeds a threshold value, while the negativity of WFs in phase damping channel varies more slowly than that of the amplitude decay. Especially, at long times WF evolves into the case of vacuum state in amplitude decay channel and a sum of weight WFs of all the Fock states in phase damping channel.

1. Introduction

Currently, various methods of quantum state engineering to manipulate or generate novel quantum states are based on the operations of photon addition, photon substraction, or the combination of the two on classical or Gaussian optical fields [1,2]. These resulting states exhibit numerous nonclassical properties and are often useful for improving loophole-free test of Bell's inequality [3], enhancing nonlocality [4], increasing quantum entanglement [5], and improving the performance in quantum teleportation [6]. Historically, Agarwal and Tara [7] first introduced the photon-added coherent state (PACS) obtained by repeated application of photon creation operator ($a^\dagger$) on a coherent state (CS), exhibiting both higher-order squeezing and higher-order sub-Poissonian character [8]. Nha's group had recently presented the coherent superposition of photon subtraction and addition [9], $ta + ra^\dagger$, as well as other coherent superpositions of second-order operations [10], $ta^2 + ra^{\dagger 2}$, for quantum state engineering. Particularly, they performed the coherent superposition on two-mode squeezed vacuum for enhancing quantum entanglement or non-Gaussian entanglement distillation [11,12]. In addition, Parigi et al. simply implemented the coherent combinations of annihilation-then-creation $a^\dagger a$ (AC) and creation-then-annihilation $aa^\dagger$ (CA) on a thermal state and proved the noncommutativity of the creation and annihilation operators [13]. Fiurášek [14] later proposed a scheme for the approximate probabilistic realization of an arbitrary operation that can be expressed as a function of photon number operator $a^\dagger a$. Lee's group [15] investigated the properties of multiple photon annihilation-then-creation coherent state (MPACCS) by operating $(a^\dagger a)^m$ and $(aa^\dagger)^m$ on CS and thermal state, respectively. Very recently, our group also repeatedly applied $a^\dagger a$ and $aa^\dagger$ on the thermal state [16] and squeezed vacuum state [17] and compared their nonclassical and non-Gaussian properties.

On the other hand, when the nonclassical optical fields propagate in the medium, they inevitably interact with their surrounding environment, which causes the dissipation or dephasing [18]. As well known, the dissipation or dephasing will deteriorate the degree of nonclassicality of the optical fields [19]. Thus, the investigations on nonclassical optical fields in different models of decoherence have inspired broad interest. For example, the decoherence of the photon-subtracted squeezed vacuum was investigated theoretically in two different decoherent channels (amplitude decay and phase damping) by Biswas and Agarwal [20]

* Corresponding author.
E-mail address: lihengm@ustc.edu.cn (H.-M. Li).

http://dx.doi.org/10.1016/j.optcom.2015.06.007
0030-4018/

and Meng et al. [21]. They indicated that the Wigner function (WF) loses its non-Gaussian nature and becomes Gaussian due to amplitude decay but remains nonclassical in the phase damping case. Later, the decoherence of such state in a thermal channel had been discussed by Hu and Fan [22], and they found that when the decay time exceeded a threshold value, this state completely lost its nonclassicality since the WF was transited from partial negative to fully positive definite in phase space. However, to our knowledge, few studies in the literature focus on the effect of decoherence for MPACCS. The main reasons are that MPACCS's normalization constant cannot be analytically derived yet, which is the key to analyze its nonclassical properties and decoherence, and the explicit expressions with time evolution in the presence of decoherence have not been previously presented due to the complicated calculations involved.

The aim of the present work is to study the effect of decoherence on MPACCS in two different channels, i.e., amplitude decay and phase damping, with the help of the time-evolution of the WFs of such state. Because the non-positive WF is regarded as a clear signature of the highly nonclassical character of the optical fields [23,24]. In the following section, we first try to derive the analytical expression of MPACCS's normalization factor, which is just related to the Bell-polynomial. From its Q-parameter, MPACCS exhibits stronger sub-Poissonian behavior numerically. In Section 3, we investigate how MPACCS evolves in amplitude decay channel by evaluating its Wigner distribution function. After deriving the analytical expression of Wigner operator in amplitude decay channel, we discuss the MPACCS's WF with time evolution both analytically and numerically. The results show that the partial negativity of WFs diminishes gradually with time and the WFs have no negative region when κt exceeds a threshold value. In Section 4, we mainly give the analytical expression of the WF's evolution of MPACCS passing through the phase damping channel. It is found that the negativity of WFs in phase damping channel varies more slowly with time than that of the amplitude decay. At long times, WF evolves into the case of vacuum state in the former channel and a sum of weight WFs of all the Fock states in the latter channel. The results are summarized in the last section.

2. The MPACCS and its normalization

Theoretically, MPACCS is obtained by repeatedly operating $a^\dagger a$ on the coherent state $|\alpha\rangle = \exp\left(-|\alpha|^2/2 + \alpha a^\dagger\right)|0\rangle$, i.e.,

$$|\alpha\rangle_m = N_m (a^\dagger a)^m |\alpha\rangle, \tag{1}$$

where $N_{AC}{}^m$ denotes the normalization *constant* to be determined, m may be any non-negative integer, $a^\dagger$ and a are the creation and annihilation operation, respectively, obeying $[a, a^\dagger] = 1$. Using the completeness relation of Fock state $\sum_{n=0}^{\infty} |n\rangle\langle n| = 1$ with $|n\rangle = (a^{\dagger n}/\sqrt{n!})|0\rangle$ and the normally ordered expansion of the vacuum project [25], $|0\rangle\langle 0| = : e^{-a^\dagger a}:$ (:: denotes normal ordering), we can directly derive

$$(a^\dagger a)^m = \sum_{n,k=0}^{\infty} n^m : \frac{(-1)^k (a^\dagger a)^{n+k}}{n!k!} : = \sum_{l=0}^{\infty} \left\{ \begin{matrix} m \\ l \end{matrix} \right\} a^{\dagger l} a^l, \tag{2}$$

where we have used $\sum_{n,k=0}^{\infty} A_n B_k = \sum_{l=0}^{\infty} \sum_{k=0}^{l} A_{l-k} B_k$ and set $\left\{ \begin{smallmatrix} m \\ l \end{smallmatrix} \right\} = 1/l! \sum_{k=0}^{l} (-1)^{l-k} \binom{l}{k} k^m$, which is just the Stiring *number* of second kind [26], and for $l > m$, $\left\{ \begin{smallmatrix} m \\ l \end{smallmatrix} \right\} = 0$ and $\left\{ \begin{smallmatrix} m \\ 0 \end{smallmatrix} \right\} = \delta_{m0}$. Then MPACCS is reformed as

$$|\alpha\rangle_m = N_m \sum_{l=0}^{m} \left\{ \begin{matrix} m \\ l \end{matrix} \right\} \alpha^l a^{\dagger l} |\alpha\rangle, \tag{3}$$

which indicates that $|\alpha\rangle_m$ is actually equivalent to a superposition of PACS.

Employing the normalization condition and using Eq. (2) as well as $a|\alpha\rangle = \alpha|\alpha\rangle$, we have

$$N_m^{-2} = \sum_{l=0}^{2m} \left\{ \begin{matrix} 2m \\ l \end{matrix} \right\} |\alpha|^{2l} \equiv B_{2m}(|\alpha|^2), \tag{4}$$

where $B_m(x) = \sum_{l=0}^{m} \left\{ \begin{smallmatrix} m \\ l \end{smallmatrix} \right\} x^l$ is just the Bell-polynomial [27]. Obviously, for the case of no-photon-operation with $m=0$, $N_0^{-2} = 1$ and $|\alpha\rangle_m$ reduces to the ordinary coherent state as expected. When $m=1$, $N_1^{-2} = |\alpha|^2 + |\alpha|^4$ and when $m=2$, $N_2^{-2} = |\alpha|^2 + 7|\alpha|^4 + 6|\alpha|^6 + |\alpha|^8$ as expected. On the other hand, its density operator is

$$\rho_0 = N_m^2 \sum_{l,k=0}^{m} \Pi_{l,k}^{m,m} \alpha^l \alpha^{*k} a^{\dagger l} |\alpha\rangle\langle\alpha| a^k, \tag{5}$$

where $\Pi_{l,k}^{m,m} \equiv \left\{ \begin{smallmatrix} m \\ l \end{smallmatrix} \right\} \left\{ \begin{smallmatrix} m \\ k \end{smallmatrix} \right\}$ and by using $\mathrm{Tr}\,(\rho_0) = 1$ and the operator identity [28]

$$a^n a^{\dagger m} = (-i)^{m+n} : H_{m,n}\left(ia^\dagger, ia\right) : , \tag{6}$$

where $H_{m,n}(\zeta, \xi)$ is two-variable Hermite polynomial, the normalization constant N_m is also calculated by

$$N_m^{-2} = \sum_{l,k=0}^{m} \Pi_{l,k}^{m,m} (-i\alpha)^l (-i\alpha^*)^k H_{l,k}\left(i\alpha^*, i\alpha\right). \tag{7}$$

As a result of Eqs. (4) and (7), we have

$$B_{2m}(|\alpha|^2) = \sum_{l,k=0}^{m} \Pi_{l,k}^{m,m} (-i\alpha)^l (-i\alpha^*)^k H_{l,k}\left(i\alpha^*, i\alpha\right). \tag{8}$$

The expansion equation (3) also leads to the following result for the scalar products:

$$_n\langle\beta|\alpha\rangle_m = \frac{e^{-(|\beta|^2/2) - (|\alpha|^2/2) + \beta^*\alpha}}{\sqrt{B_{2m}(|\alpha|^2) B_{2n}(|\alpha|^2)}} \times \sum_{l=0}^{n} \sum_{k=0}^{m} \Pi_{l,k}^{n,m} (-i\alpha)^l (-i\beta^*)^k H_{l,k}\left(i\beta^*, i\alpha\right). \tag{9}$$

Similarly, it is obtained from Eqs. (1) and (2) that

$$\begin{aligned} _n\langle\beta|\alpha\rangle_m &= N_m N_n \langle\beta|(a^\dagger a)^{m+n}|\alpha\rangle \\ &= N_m N_n \sum_{l=0}^{m+n} \left\{ \begin{matrix} m+n \\ l \end{matrix} \right\} (\alpha\beta^*)^l e^{-(|\beta|^2/2) - (|\alpha|^2/2) + \beta^*\alpha} \\ &= \frac{B_{m+n}(\alpha\beta^*) e^{-(|\beta|^2/2) - (|\alpha|^2/2) + \beta^*\alpha}}{\sqrt{B_{2m}(|\alpha|^2) B_{2n}(|\alpha|^2)}}. \end{aligned} \tag{10}$$

Further, comparing Eq. (9) with Eq. (10) yields

$$B_{m+n}(\alpha\beta^*) = \sum_{l=0}^{n} \sum_{k=0}^{m} \Pi_{l,k}^{n,m} (-i\alpha)^l (-i\beta^*)^k H_{l,k}\left(i\beta^*, i\alpha\right), \tag{11}$$

which seems a new identity. Especially, when $m=n$ and $\alpha = \beta$, Eq. (11) turns into Eq. (8).

Since the normalization factor of MPACCS is related to the Bell-polynomial, it is very convenient for further analytically studying its nonclassical properties and decoherence effects below. For instance, according to $\langle n\rangle = \mathrm{Tr}\left(a^\dagger a\rho\right)$, from Eqs. (2) and (4) we easily have mean photon number of MPACCS

$$\left\langle a^\dagger a \right\rangle = N_m^2 \langle\alpha|(a^\dagger a)^{2m+1}|\alpha\rangle = \frac{B_{2m+1}(|\alpha|^2)}{B_{2m}(|\alpha|^2)}. \tag{12}$$