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Vector Laguerre–Gaussian soliton in strong nonlocal nonlinear media

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ABSTRACT

In this paper, the analytical vector Laguerre–Gaussian (LG) solutions are obtained in strongly nonlocal nonlinear media by variational approach. The comparisons of analytical solutions with numerical results show that the analytical vector LG solutions are in good agreement with the numerical simulations. Furthermore, we numerically proved that the completely stationary vector LG soliton, scalar LG soliton and even (odd) LG soliton can be obtained only in strong nonlocal media. For the general and weakly nonlocal cases, the single LG beam breaks up and the single even LG beam expands during propagation, only the LG beam pairs can reduce to a quasistable soliton due to the stabilizing mutual attraction between its components.

1. Introduction

In 1997, Snyder and Mitchell presented the Snyder–Mitchell model and found the Gaussian-shaped soliton called “accessible soliton” [1]. Their work raised the upsurge in the research of the nonlocal spatial optical solitons. Such as, Krolikowski et al. obtained a sech-shaped soliton in weakly nonlocal media [2]. Guo proved that the spatial optical soliton in strongly nonlocal medium has large phase shift [3]. Wang found that there is a large phase difference between the two orthogonally polarized beams which propagate in the strong nonlocal media with anisotropy [4,5]. Furthermore, the interactions of Gaussian-shaped optical beams in strong and sub-strong nonlocal media were investigated [6,7].

In recent years, the high-order optical solitons have attracted numerous attention. For instance, Hutsebaut et al. observed stable high-order soliton in experiment in nematic liquid crystal [8]. Shen studied the instability suppression of vector-necklace-ring soliton clusters in different nonlocal media [9]. Deng discussed the propagation of Ince–Gaussian (IG) beams [10] and elegant IG beams [11] in strongly nonlocal nonlinear media. In addition, Hermite–Gaussian (HG) soliton [12], LG soliton [13], Hermite–Laguerre–Gaussian (HLG) soliton [14] and the interaction of LG solitons [15] in strong nonlocal media also have been investigated. However, to the best of our knowledge, the propagation of nonlocal LG vector soliton, which consisted of two incoherently LG beams [16–18], has not been studied. In addition, Desyatnikov found that the necklace-ring vector solitons can quasi-stably propagate in the saturable nonlinear media for the vector interactions [19]. It is worth mention that vector soliton includes temporal and spatial vector soliton. Such as Rand experimentally observe the propagation and collision of temporal vector soliton in a linearly birefringent optical fiber [20], Zhang studied Dissipative temporal vector solitons in a dispersion managed cavity fiber laser with net positive cavity dispersion [21]. This paper will study the strong nonlocal optical vector LG soliton by variational approach and numerical simulation. Furthermore, we also study the propagation of LG beam pairs, single LG beam and even (odd) LG beam in nonlocal media with different nonlocalities.

2. Physical model and variational approach

In cylindrical coordinates, the propagation of two mutually incoherent optical beams in nonlocal nonlinear media is governed as the following coupled nonlocal nonlinear Schrodinger equations (NNLSE) [3,9,16–18]:

$$i\frac{\partial\psi_j}{\partial z}+\mu\left(\frac{1}{r}\frac{\partial\psi_j}{\partial r}+\frac{\partial^2\psi_j}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\psi_j}{\partial\varphi^2}\right)$$
$$+\rho\psi_j\int_0^{+\infty}\int_0^{2\pi}R(r-r`)[\left|\psi_j(r`,\varphi`,z)\right|^2+\left|\psi_{3-j}(r`,\varphi`,z)\right|^2]r`dr`d\varphi$$
$$=0 \quad (1)$$

where ψ_j ($j=1, 2$) represent the paraxial optical beams, $\mu=1/2k$,

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$\rho k\eta$, k is the wave number in the media without nonlinearity, η is the material constant.

The Lagrange density equation, which corresponding to Eq. (1), can be written as follows:

$$L = \sum_{j=1,2} \frac{i}{2} r\left(\psi_j^* \frac{\partial \psi_j}{\partial z} - \psi_j \frac{\partial \psi_j^*}{\partial z}\right) - \mu r\left(\left|\frac{\partial \psi_j}{\partial r}\right|^2 + \frac{1}{r^2}\left|\frac{\partial \psi_j}{\partial \varphi}\right|^2\right) + \frac{1}{2}\rho r\left|\psi_j\right|^2 \int_0^{2\pi}\int_0^{+\infty} R(r - r`) \left[\left|\psi_j(r`, \varphi`, z)\right|^2 + \left|\psi_{3-j}(r`, \varphi`, z)\right|^2\right] r`\mathrm{d}r`\mathrm{d}\varphi` \quad (2)$$

For the strong nonlocal case, the characteristic length of the media is ten times larger than the beam width, therefore the response function can be expanded twice and reduced as follow [3,15]:

$$R(r - r`) \approx R_0 - \frac{1}{2}\gamma(r - r`)^2 \quad (3)$$

where $R_0 = R(0)$, $\gamma = -R^{(2)}(0)$.

Here we look for the trial solution to Eq. (1) in LG-shaped

$$\psi_j(r, \varphi, z) = A_j(z)\left[\frac{r}{a_j(z)}\right]^{m_j} L_{s_j}^{m_j}\left[\frac{r^2}{a_j^2(z)}\right] \times \exp\left[-\frac{r^2}{2a_j^2(z)} + ic_j(z)r^2 + i\theta_j(z) + im_j\varphi\right] \quad (4)$$

Inserting the trial function into Eq. (2) and integrating the Lagrange density, we obtain the average Lagrange

$$L = -\frac{\pi A_1^2(1 + m_1 + 2s_1)(m_1 + s_1)!}{(s_1!)^2}\left[\mu s_1! + 4a_1^4 s_1!\mu_1 c_1^2 + \frac{a_1^4 s_1! dc_1}{dz} + \frac{a_1^2 s_1! d\theta_1}{(1 + m_1 + 2s_1)dz}\right] + \frac{\rho\pi^2 A_1^4 a_1^4[(m_1 + s_1)!]^2}{2(s_1!)^2}[R_0 - \gamma a_1^2(1 + m_1 + 2s_1)] - \frac{\pi A_2^2(1 + m_2 + 2s_2)(m_2 + s_2)!}{(s_2!)^2}\left[\mu s_2! + 4a_2^4 s_2!\mu_2 c_2^2 + \frac{a_2^4 s_2! dc_2}{dz} + \frac{a_2^2 s_2! d\theta_2}{(1 + m_2 + 2s_2)dz}\right] + \frac{\rho\pi^2 A_2^4 a_2^4[(m_2 + s_2)!]^2}{2(s_2!)^2}[R_0 - \gamma a_2^2(1 + m_2 + 2s_2)] + \frac{\rho\pi^2 A_1^2 a_1^2(m_1 + s_1)! A_2^2 a_2^2(m_2 + s_2)!}{s_1! s_2!}\left[R_0 - \frac{1}{2}\gamma(1 + m_2 + 2s_2)a_2^2 - \frac{1}{2}\gamma(1 + m_1 + 2s_1)a_1^2\right] \quad (5)$$

Here the orthonormality of Laguerre polynomials had been used

$$\int_0^\infty z^m L_s^m(z) L_q^m(z) e^{-z} dz = \frac{\Gamma(m + s + 1)}{s!}\delta_{sq} \quad (6)$$

where

$$\delta_{sq} = \begin{cases} 0 & (s \neq q) \\ 1 & (s = q) \end{cases} \quad (7)$$

By using the variational approach, we can obtain a series of equations as follow:

$$\frac{\mathrm{d}a_j}{\mathrm{d}z} - 4\mu c_j a_j = 0 \quad (8a)$$

$$A_j^2 a_j^2 = A_{j0}^2 a_{j0}^2 = \frac{s_j! P_{j0}}{\pi(m_j! + s_j!)}, \quad (8b)$$

$$\frac{\mathrm{d}\theta_j}{\mathrm{d}z} = -\frac{2\mu(1 + m_j + 2s_j)}{a_j^2} + \rho R_0 P_{j0} - \frac{(1 + m_j + 2s_j)}{2}\rho\gamma P_{j0} a_j^2 + \rho R_0 P_{(3-j)0} - \frac{(1 + m_{3-j} + 2s_{3-j})}{2}\rho\gamma P_{(3-j)0} a_{3-j}^2 \quad (8c)$$

$$\frac{\mathrm{d}c_j}{\mathrm{d}z} = \frac{\mu}{a_j^4} - 4c_j^2\mu - \frac{1}{2}\rho\gamma P_{j0} - \frac{1}{2}\rho\gamma P_{(3-j)0} \quad (8d)$$

Where P_{j0} ($j=1, 2$) are the initial powers, a_{j0} are the initial beam widths and A_{j0} are the initial amplitudes. By combining Eqs. (8b) and (8d), the evolution equations of the beam widths can be obtained

$$\frac{\mathrm{d}^2 a_1}{\mathrm{d}z^2} = \frac{4\mu^2}{a_1^3} - 2\mu a_1 \rho\gamma P_0 \quad (9a)$$

$$\frac{\mathrm{d}^2 a_2}{\mathrm{d}z^2} = \frac{4\mu^2}{a_2^3} - 2\mu a_2 \rho\gamma P_0 \quad (9b)$$

where $P_0 = P_{10} + P_{20}$ is the total incident power. It is obvious that the evolution laws of such an LG beam pairs depend on the total initial power. Assuming $\mathrm{d}^2 a_1/\mathrm{d}{z_1}^2|_{z=0} = 0$, then the critical power of ψ_1 can be obtained by (9a)

$$P_{c1} = \frac{1}{k^2\gamma a_{10}^4 \eta} \quad (10)$$

Namely, when $P_0 = P_{10} + P_{20} = P_{c1}$, the ψ_1 will keep its initial beam width unchanged during propagation. In the same way, setting $\mathrm{d}^2 a_2/\mathrm{d}{z_2}^2|_{z=0} = 0$, we can obtain the critical power of ψ_2 by (9b)

$$P_{c2} = \frac{1}{k^2\gamma a_{20}^4 \eta} \quad (11)$$

When $P_0 = P_{10} + P_{20} = P_{c2}$, the ψ_2 will preserve its width as it travels in the straight path along the z axis. Furthermore, assuming that the total initial power is equal to the two critical powers, i.e., $P_0 = P_{10} + P_{20} = P_{c1} = P_{c2}$, the two LG beams both propagate stably in strong nonlocal media and the stable vector LG soliton can be formed. Furthermore, for $P_{c1} = P_{c2}$, we can obtain $a_{10} = a_{20}$.

Assuming

$$w_j = \frac{a_j}{a_{10}}, \quad Z = \frac{z}{k a_{10}^2}, \quad (12)$$

where w_j stand for the normalized beam widths, $Z = z/k{a_{10}}^2$ is the normalized propagation distance. Then the Eq. (9) can be normalized as follow:

$$\frac{\mathrm{d}^2 w_j}{\mathrm{d}Z^2} = \frac{1}{w_j^3} - w_j \frac{P_0}{P_{cj}} \quad (13)$$

By settting $w_j(0) = 1$, $\mathrm{d}w_j/\mathrm{d}Z|_{Z=0} = 0$, we can depict the analytical solution in Fig. 4, and compare the analytical solution with numerical simulation.