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Scattering parameter analysis of cascaded bi-anisotropic metamaterials

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ABSTRACT

Forward and backward reflection and transmission scattering (S-) parameters of a metamaterial (MM) structure of bi-anisotropic slabs in cascade are analyzed. Signal flow graph technique is applied in this analysis for a cascaded connection of two and three bi-anisotropic MM slabs with different orientations. We also investigated effects of geometrical parameters (fabrication tolerances) of individual MM cells on forward and backward reflection and transmission S-parameters of cascade connection of MM cells. We note from our analysis the following important points. First, forward and backward reflection S-parameters become equal to one another for a MM structure composed of an even number of bi-anisotropic MM slabs (individually characterized by non-identical forward and backward reflection S-parameters) with proper orientations only if there exists symmetry of the structure in the propagation direction. Second, it is noted that it is not possible to obtain identical forward and backward reflection S-parameters of a MM structure composed of an odd number of bi-anisotropic MM slabs. Finally, reflection asymmetry (bi-anisotropy) of cascade connection of MM cells generally is enhanced for a change in geometrical parameters of individual MM cells.

1. Introduction

It is known that metals such as silver behave like a plasma of free electrons (free electric charges), providing negative values of permittivity (ε_r) below plasma frequency [1]. Due to the lack of free magnetic charges in nature, benefits of plasma-like behavior cannot be used to realize negative values of permeability (μ_r). Fortunately, Pendry and his colleagues proposed in 1999 the use of Swiss rolls or split-ring-resonator (SRR) structures to achieve negative μ_r [2]. Artificial electromagnetic materials, known as metamaterials (MMs) with negative refractive index n, have attracted an increasing interest of researchers because they allow conventionally unattainable applications such as perfect lenses [3], invisibility cloaks [4], sensors [5], and tunable devices [6]. To suitably incorporate MMs for those applications, in addition to their fabrication and design, their characterization should be accurately performed.

Depending on direction and polarization of wave and on the structural inhomogeneity (substrate and/or side wall angle), and due to coupling between electric and magnetic fields, MMs can attain bi-anisotropic property [7–19]. Bi-anisotropic MMs possess different forward and backward reflection S-parameters [8,17–19], a broader stop-band in transmission [8,14], magneto-electric coupling coefficient (ξ_0) [7,8,13], and different reflected and absorbed powers in forward and backward directions [15]. It is generally thought that bi-anisotropy of a MM structure is an undesired property and should be avoided by eliminating the ξ_0 coefficient especially for optical MMs [7,12,16–19]. However, bi-anisotropic MMs have an interesting feature that they allow backward-wave propagation although the real part of the complex refractive index (n) is not negative [13]. Therefore, an analysis of their wave propagation gains importance.

In recent studies, reflection and transmission properties of symmetric and asymmetric chiral (transmission asymmetric) composite cascade materials have been analyzed [20,21]. In our present study, our main concern is to investigate reflection and transmission properties of symmetric and asymmetric bi-anisotropic (reflection symmetric) cascade MMs. In the works of Dr. Marqués and Dr. Simovski [7,12], it is thoroughly discussed how the bi-anisotropy of a MM arises and how it can be eliminated using the concept of polarizabilities. In this paper, we will revisit bi-anisotropy of a MM structure (cascade connection of even/odd number of bi-anisotropic MM slabs with proper orientations) considering directly S-parameters. From our analysis, we note that for a MM structure composed of a cascade connection of even number of bi-anisotropic MM cells, forward and backward reflection S-parameters of the whole structure can be equal to each

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other if there is a symmetry (proper orientation) of cascaded cells in the propagation direction. This arises from the fact that w.r.t. symmetry axis, the forward (backward) impedance of a given bi-anisotropic MM slab is identical to the backward (forward) impedance of the next consecutive bi-anisotropic MM slab. We think that outcomes of our investigation is helpful for analysis of cascade connection of MM slabs with various properties, which can be utilized to achieve desired goals in propagation-related applications.

2. Theoretical analysis

In this section, we will consider MM structures with square edge-coupled SRR [7,8,10,15] as shown in Fig. 1(a). The analysis, however, can also be extended to other MM structures with single-ring SRR [7,22], Ω-shaped [16,10,12], π-shaped [13], or fishnet [19,17,18] inclusions. When the incident magnetic field is in $+z$ direction (normal to the plane of metallic inclusions–magnetic excitation), circulating (surface) currents with different amplitudes ($I_o{}^m$ and $I_i{}^m$) will flow within each ring as shown in Fig. 1(b). Due to the (surface) charges accumulated at opposite sides (w.r.t. x axis) of each ring in Fig. 1(b), there will be a non-zero net electric dipole moment in $+y$ direction. On the other hand, for an external electric field in $+y$ direction (normal to the slit axis–electric excitation), different charges with opposite polarities will accumulate at opposite sides of both rings (w.r.t. x axis), yielding asymmetrical circulating currents with different amplitudes ($I_o{}^e$ and $I_i{}^e$) as shown in Fig. 1(c) [22]. Circulation of non-equal currents will eventually induce a net magnetic dipole in $\pm y$ direction. As a consequence of coupling between electric and magnetic fields, a non-zero magneto-electric coupling quantity (ξ_0) will be present in constitutive relations, resulting in bi-anisotropy in MM slabs [7–10,15]. Therefore, for a plane wave propagating in x direction (with a linear polarization in y direction) to the MM slab in Fig. 1(a), forward and backward reflection and transmission S-parameters of that slab are [10,15,23]

$$S_{11} = \frac{\Gamma_1\left(1 - T^2\right)}{1 - \Gamma_1\Gamma_2 T^2},$$
$$S_{22} = \frac{\Gamma_2\left(1 - T^2\right)}{1 - \Gamma_1\Gamma_2 T^2},$$
$$S_{21} = S_{12} = \frac{T(1 - \Gamma_1\Gamma_2)}{1 - \Gamma_1\Gamma_2 T^2}, \tag{1a}$$

$$\Gamma_{1(2)} = \frac{z_w^{+(-)} - 1}{z_w^{+(-)} + 1},$$
$$z_w^{\mp} = \frac{\mu_{zz}}{n \mp i\xi_0},$$
$$T = e^{ik_0 nL},$$
$$n = \mp\sqrt{\varepsilon_{yy}\mu_{zz} - \xi_0^2}. \tag{1b}$$

Here, Γ_1 and Γ_2 are the semi-infinite reflection coefficients (for a wave incident upon a slab infinitely long in the propagation direction); T is the propagation factor; z_w^+, z_w^-, n, and k_0 are, respectively, normalized impedances for the forward ($+x$) and backward ($-x$) waves, the refractive index of the bi-anisotropic MM slab, and the free-space wave number; L is the slab length; and ε_{yy} and μ_{zz} are the relative complex permittivity and complex permeability of the bi-anisotropic MM slab in the electric and magnetic field directions (y and z directions), respectively.

Besides, if the gap in each ring of the SRR in Fig. 1(a) is reversed 180° w.r.t. y-axis as shown in Fig. 1(d) or (e), then circulating currents and deposited charges will be as shown in Fig. 1(d) and (e) for magnetic and electric excitations, respectively. Considering the current directions (or deposited charges) and induced dipole moments in Fig. 1(b)–(e), we note two key results. First, direction of net electric dipole moment created by distributed charges in Fig. 1(d) is identical but opposite to the direction of net electric dipole moment induced by the circulating currents in Fig. 1(b). Second, direction of net magnetic dipole moment formed by the circulating charges in Fig. 1(c) is identical but opposite to the direction of net magnetic dipole moment resulted from the circulating charges in Fig. 1(e). Therefore, ξ_0 for the configuration in Fig. 1(d) (slit locations are reversed) will be negative of ξ_0 for the configuration in Fig. 1(b) (original slit position) while there is no change in ε_{yy} and μ_{zz} [8,24]. Incorporating these results into Eqs. (1a) and (1b), we found that while n does not change, z_w^+ (z_w^-) becomes equal to z_w^- (z_w^+), yielding an exchange of Γ_1 and Γ_2 and an exchange of S_{11} and S_{22} for the SRR with the directions of slits reversed.

3. Signal flow graph technique

In this section, we will analyze the overall reflection and transmission S-parameters of a cascade connection of square SRRs as shown in Fig. 1(b) and (d). For a structure composed of a cascade connection of two bi-anisotropic MM slabs (see Fig. 2(a)), the overall forward and backward reflection and transmission S-parameters (S_{11}^{2T}, S_{21}^{2T}, S_{12}^{2T}, and S_{22}^{2T}) are derived as [25]

$$S_{11}^{2T} = S_{11}^{(1)} + \frac{S_{21}^{(1)}S_{12}^{(1)}S_{11}^{(2)}}{1 - S_{11}^{(2)}S_{22}^{(1)}}, \quad S_{21}^{2T} = \frac{S_{21}^{(1)}S_{21}^{(2)}}{1 - S_{11}^{(2)}S_{22}^{(1)}}, \tag{2a}$$

$$S_{22}^{2T} = S_{22}^{(2)} + \frac{S_{21}^{(2)}S_{12}^{(2)}S_{22}^{(1)}}{1 - S_{11}^{(2)}S_{22}^{(1)}}, \quad S_{12}^{2T} = \frac{S_{12}^{(1)}S_{12}^{(2)}}{1 - S_{11}^{(2)}S_{22}^{(1)}}. \tag{2b}$$

Considering two laterally paired SRRs and assuming that the SRR configuration of the second MM slab is the mirror image of the SRR configuration of the first MM slab (w.r.t. yz plane) [$S_{22}^{(2)} = S_{11}^{(1)}$ and $S_{11}^{(2)} = S_{22}^{(1)}$] and assuming that SRR is made with reciprocal media [$S_{12}^{(1)} = S_{21}^{(1)} = S_{12}^{(2)} = S_{21}^{(2)}$] [7,15], S_{11}^{2T} becomes equal to S_{22}^{2T} (in addition to $S_{21}^{2T} = S_{12}^{2T}$ due to the reciprocity of SRR structure) resulting in degeneracy of reflection asymmetry of the whole structure. We note that although we discuss this

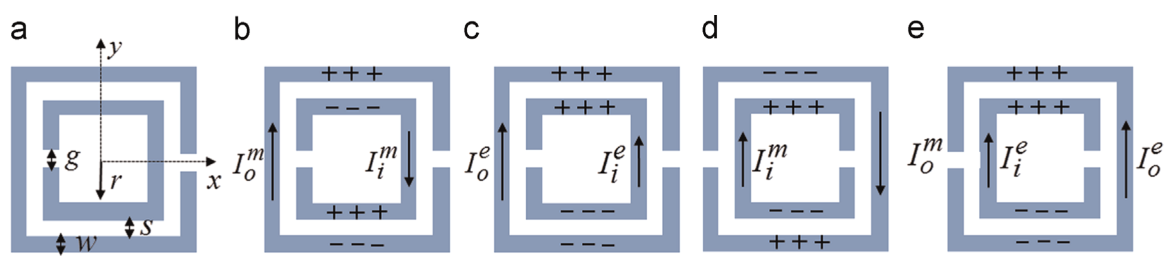

Fig. 1. (a) Configuration of square SRR resonator and schematic view of accumulated surface charges and circulating surface currents for (b) [(d)] magnetic excitation (c) [(e)] electric excitation for the SRR configuration in Fig. 1(a) [when slits in Fig. 1(a) are reversed].