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Hanbury Brown-Twiss effect with partially coherent electromagnetic beams scattered by a random medium

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ABSTRACT

The expressions for the correlation of intensity fluctuations in the far-zone that occurs in stochastic electromagnetic beams scattered by a random medium are derived within the validity of the first-order Born approximation. Some numerical results are presented to illustrate the influences of different source parameters and scatterer parameters on the normalized correlation of intensity fluctuations of the far-zone scattered field.

1. Introduction

Considering the potential applications in diverse fields, such as remote sensing, biological imaging, target detection and so on, light scattering is always a subject of pretty importance. The direct problem of predicting the statistics properties of the scattered field has been broadly studied for various types of incident waves and the scatterers ([1], Chap. 6), [2–8], where the incident waves may be deterministic or random nature and the scatterers may be continuous media or particle collections. Besides, the inverse scattering problem has also been massively investigated to reconstruct the media properties due to the information carried in scattered light parameters [9–11]. However, to the best of our knowledge, the researches referred above are almost restricted to the second-order scattered field.

On the other hand, it has found that the four-order correlation of the scattered field, i.e., the Hanbury Brown-Twiss effect, which was firstly introduced to determine the angular diameter of radio stars by analyzing their correlation of intensity fluctuations [12,13], could also contain information about the scattering media, and it has proved the correlation depends on spatial Fourier transforms of both the intensity and degree of spatial correlation of scattering potentials of the media when a monochromatic plane wave is scattered by a quasi-homogeneous random media [14]. Shortly afterwards, the investigations have been generalized to the case of an electromagnetic plane wave [15] and of a random field governed by Gaussian statistics [16]. Besides, Kuebel et al. [17] have derived two four-order reciprocity relations to be used to reconstruct the scattering potential of the medium, which requires relatively simple intensity correlation experiments.

In the present paper we continue to extend the ones in [14] to the case of a stochastic electromagnetic beam and use the so-called electromagnetic Gaussian Schell-model sources as an example to study the Hanbury Brown-Twiss effect occurring in stochastic electromagnetic beams scattered by a random medium. The analytical expressions for the far-zone correlation of intensity fluctuations have been derived and some analyses have been given to illustrate our results.

2. Correlation of intensity fluctuations of partially coherent electromagnetic Gaussian fields

We assume $\mathrm{E}(\mathrm{r},\omega)$ is a stochastic electromagnetic field in the space-frequency domain at a point r, which oscillates at angular frequency ω. Let us represent $E_r(\mathrm{r},\omega)$, $E_\theta(\mathrm{r},\omega)$ and $E_\varphi(\mathrm{r},\omega)$ be the three mutually orthogonal components of the electric field in the spherical coordinate system and thus the intensity of the field at a point r at frequency ω may be given as

$$I(\mathrm{r},\omega) = |E_r(\mathrm{r},\omega)|^2 + |E_\theta(\mathrm{r},\omega)|^2 + |E_\varphi(\mathrm{r},\omega)|^2. \tag{1}$$

The statistical properties of such a field at a pair of points

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specified by the position vectors r_1 and r_2 may be characterized by the cross-spectral density matrix $\overleftrightarrow{W}(\mathrm{r}_1, \mathrm{r}_2, \omega)$ with elements ([1], Section 9.1)

$$W_{\alpha\beta}(\mathrm{r}_1, \mathrm{r}_2, \omega) = \left\langle E_\alpha^*(\mathrm{r}_1, \omega) E_\beta(\mathrm{r}_2, \omega) \right\rangle, (\alpha, \beta = r, \theta, \varphi), \tag{2}$$

where the angular brackets denote the ensemble average and the asterisk denotes the complex conjugate. From this definition we can express the average intensity as

$$\left\langle I(\mathrm{r},\omega) \right\rangle = \mathrm{Tr}\, \overleftrightarrow{W}(\mathrm{r},\mathrm{r}, \omega), \tag{3}$$

where Tr denotes the trace. If we introduce the intensity fluctuations at a point r as

$$\Delta I(\mathrm{r},\omega) = I(\mathrm{r},\omega) - \left\langle I(\mathrm{r},\omega) \right\rangle, \tag{4}$$

then the correlation of intensity fluctuations at two points r_1 and r_2 can be described as

$$C(\mathrm{r}_1, \mathrm{r}_2, \omega) = \left\langle \Delta I(\mathrm{r}_1, \omega) \Delta I(\mathrm{r}_2, \omega) \right\rangle. \tag{5}$$

Next if we assume the intensity fluctuations obey Gaussian statistics, then by the use of the moment theorem for a Gaussian random process, the correlation of intensity fluctuations can be expressed with the components of the cross-spectral density matrix as ([18], Chap. 8)

$$C(\mathrm{r}_1, \mathrm{r}_2, \omega) = \sum_{\alpha,\beta} \left| W_{\alpha\beta}(\mathrm{r}_1, \mathrm{r}_2, \omega) \right|^2, (\alpha, \beta = r, \theta, \varphi), \tag{6}$$

and the normalized correlation of intensity fluctuations at two points r_1 and r_2 is then defined as

$$C_n(\mathrm{r}_1, \mathrm{r}_2, \omega) = \frac{C(\mathrm{r}_1, \mathrm{r}_2, \omega)}{\left\langle I(\mathrm{r}_1, \omega) \right\rangle \left\langle I(\mathrm{r}_2, \omega) \right\rangle}. \tag{7}$$

3. Review of weak scattering theory

Suppose now that a stochastic electromagnetic field $\mathrm{E}^i(\mathrm{r},\mathrm{s}_0, \omega)$ is incident upon a linear random medium, which occupies a finite domain V. If the field is radiated close to the z direction into the positive half-space $z > 0$, the two mutually orthogonal Cartesian components of the field $E_x^i(\mathrm{r},\mathrm{s}_0, \omega)$ and $E_y^i(\mathrm{r},\mathrm{s}_0, \omega)$, perpendicular to the beam axis, may be expressed by the angular spectrum representation of plane waves as ([18], Section 3.2)

$$E_\alpha^i(\mathrm{r}, \mathrm{s}_0, \omega) = \int_{|\mathrm{s}_{0\perp}|^2 \le 1} a_\alpha(\mathrm{s}_{0\perp}, \omega) \exp(ik\mathrm{s}_0 \cdot \mathrm{r}) d^2 \mathrm{s}_{0\perp}, (\alpha = x, y), \tag{8}$$

where $a_\alpha(\mathrm{s}_{0\perp}, \omega)$ is the amplitude of a plane wave, which propagates in a direction specified by a unit vector $\mathrm{s}_0 = \left(p, q, \sqrt{1 - p^2 - q^2}\right)$. Besides, $\mathrm{s}_{0\perp} = (p, q)$ is a real two-dimensional vector, $k = 2\pi/\lambda$ is the wave number, λ is the wavelength and the homogeneous plane waves have been considered only.

On taking the average over the ensemble of the field, the statistical properties of the incident field may be characterized by the cross-spectral density matrix $\overleftrightarrow{W}^i(\mathrm{r}_1, \mathrm{r}_2, \mathrm{s}_{01}, \mathrm{s}_{02}, \omega)$ with components ([1], Section 9.1)

$$W_{\alpha\beta}^i(\mathrm{r}_1, \mathrm{r}_2, \mathrm{s}_{01}, \mathrm{s}_{02}, \omega) = \left\langle E_\alpha^{i*}(\mathrm{r}_1, \mathrm{s}_{01}, \omega) E_\beta^i(\mathrm{r}_2, \mathrm{s}_{02}, \omega) \right\rangle, (\alpha, \beta = x, y). \tag{9}$$

On substituting Eq. (8) into Eq. (9), the elements of the cross-spectral density matrix of the incident field can be gotten as

$$W_{\alpha\beta}^i(\mathrm{r}_1, \mathrm{r}_2, \mathrm{s}_{01}, \mathrm{s}_{02}, \omega) = \int_{|\mathbf{s}_{01\perp}|^2 \le 1} \int_{|\mathbf{s}_{02\perp}|^2 \le 1} A_{\alpha\beta}(\mathbf{s}_{01\perp}, \mathbf{s}_{02\perp}, \omega) \times \exp[-ik(\mathbf{s}_{01} \cdot \mathrm{r}_1 - \mathbf{s}_{02} \cdot \mathrm{r}_2)] d^2 \mathrm{s}_{01\perp} d^2 \mathrm{s}_{02\perp}, (\alpha, \beta = x, y), \tag{10}$$

where $A_{\alpha\beta}(\mathrm{s}_{01\perp}, \mathrm{s}_{02\perp}, \omega) = \left\langle a_\alpha^*(\mathrm{s}_{01\perp}, \omega) a_\beta(\mathrm{s}_{02\perp}, \omega) \right\rangle$ is the so-called angular correlation function between components of two plane wave modes of the stochastic electromagnetic field and may be gained by ([18], Section 5.6.3)

$$A_{\alpha\beta}(\mathbf{s}_{01\perp}, \mathbf{s}_{02\perp}, \omega) = \left(\frac{k}{2\pi}\right)^4 \iint W_{\alpha\beta}^0(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega) \times \exp[-ik(\mathrm{s}_{02\perp} \cdot \boldsymbol{\rho}'_2 - \mathrm{s}_{01\perp} \cdot \boldsymbol{\rho}'_1)] d^2 \rho'_1\, d^2 \rho'_2, \ (\alpha, \beta = x, y), \tag{11}$$

where $\boldsymbol{\rho}'_1 = (x'_1, y'_1)$, $\boldsymbol{\rho}'_2 = (x'_2, y'_2)$ are two two-dimensional position vectors and $W_{\alpha\beta}^0(\boldsymbol{\rho}'_1, \boldsymbol{\rho}'_2, \omega)$ is the element of the cross-spectral density matrix of the stochastic electromagnetic field in the source plane.

Let us assume that $F(\mathrm{r}', \omega)$ is the scattering potential of the medium and suppose now that the weak scattering process is considered within the validity of the first-order Born approximation ([19], Chap. 13). In this case, the scattered field at an observation point r=rs($s^2 = 1$) can be given by [20]

$$\mathrm{E}^s(r\mathrm{s}, \omega) = -\,\mathrm{s} \times \left[\mathrm{s} \times \int_V F(\mathrm{r}', \omega) \mathrm{E}^i(\mathrm{r}', \mathrm{s}_0, \omega) G(r\mathrm{s}, \mathrm{r}', \omega) d^3 r' \right], \tag{12}$$

where $G(r\mathrm{s}, \mathrm{r}', \omega)$ is the outgoing free-space Green's function and may be approximated in the far-zone as

$$G(r\mathrm{s}, \mathrm{r}', \omega) = \frac{\exp(ikr)}{r} \exp(-ik\mathrm{s} \cdot \mathrm{r}'). \tag{13}$$

From Eq. (12) it is obvious that $\mathrm{s} \cdot \mathrm{E}^s(r\mathrm{s}, \omega) = 0$ and in this situation, the far-zone scattered field is orthogonal to s, i.e., $E_r^s(r\mathrm{s}, \omega) = 0$. That is to say, if we express such a transverse field in terms of the spherical polar coordinate system rather than in terms of the Cartesian coordinate system, it only has two non-zero components. So for the sake of simplicity, we consider the scattered field in the spherical polar coordinate system and the two non-zero components of the scattered field can be gotten by the following expressions [21]

$$E_\theta^s(r\mathrm{s}, \omega) = \int_V F(\mathrm{r}', \omega) G(r\mathrm{s}, \mathrm{r}', \omega) \times \left[\cos\theta \cos\varphi E_x^i(\mathrm{r}', \mathrm{s}_0, \omega) + \cos\theta \sin\varphi E_y^i(\mathrm{r}', \mathrm{s}_0, \omega) \right] d^3 r', \tag{14a}$$

$$E_\varphi^s(r\mathrm{s}, \omega) = \int_V F(\mathrm{r}', \omega) G(r\mathrm{s}, \mathrm{r}', \omega) \left[-\sin\varphi E_x^i(\mathrm{r}', \mathrm{s}_0, \omega) + \cos\varphi E_y^i(\mathrm{r}', \mathrm{s}_0, \omega) \right] d^3 r', \tag{14b}$$

where s=($\sin\theta \cos\varphi$, $\sin\theta\sin\varphi$, $\cos\theta$).

On substituting from Eqs. (14a) and (14b) into Eq. (2), one finds that all the four elements of the cross-spectral density matrix $\overleftrightarrow{W}^s(r\mathrm{s}_1, r\mathrm{s}_2, \omega)$ of the far-zone scattered field scattered by a random medium may be expressed as [22]

$$\begin{aligned} W_{\theta\theta}^s(r\mathrm{s}_1, r\mathrm{s}_2, \omega) = \frac{1}{r^2} [& \cos\theta_1 \cos\varphi_1 \cos\theta_2 \cos\varphi_2 F_{xx}(r\mathrm{s}_1, r\mathrm{s}_2, \omega) \\ & + \cos\theta_1 \cos\varphi_1 \cos\theta_2 \sin\varphi_2 F_{xy}(r\mathrm{s}_1, r\mathrm{s}_2, \omega) \\ & + \cos\theta_1 \sin\varphi_1 \cos\theta_2 \cos\varphi_2 F_{yx}(r\mathrm{s}_1, r\mathrm{s}_2, \omega) \\ & + \cos\theta_1 \sin\varphi_1 \cos\theta_2 \sin\varphi_2 F_{yy}(r\mathrm{s}_1, r\mathrm{s}_2, \omega)], \end{aligned} \tag{15a}$$