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Propagation of anomalous vortex beams in strongly nonlocal nonlinear media

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ABSTRACT

The propagation properties of anomalous vortex beams in strongly nonlocal nonlinear media are investigated. Two equivalent analytical expressions for the evolution of anomalous vortex beams are obtained. It is found that the input power plays a key role in the beam evolutions. Selecting a proper input power, the beam width can be broadened or be compressed periodically, even it can keep invariant during propagation. The beam order and the topological charge mainly influence the intensity evolution and the phase evolution, respectively. The evolution period, the beam width, the phase distribution and the intensity distribution are discussed in detail. The results can also be generalized to other equivalent physical systems, such as an optical fractional Fourier transform system or a medium with a quadratic graded refractive index distribution.

1. Introduction

Spatial laser beams propagating in various optical systems have attracted a lot of attention since the laser was invented. In particular, in nonlinear media, optical beams can form spatial solitons. Although the theory of spatial optical soliton was proposed as early as 1960s [1], the (1+1)-dimensional optical solitons were first observed until 1985 [2]. However, the observation of (2+1)-dimensional optical solitons in experiments was not realized for a long time. Until 1993, the (2+1)-dimensional spatial optical solitons were observed in photorefractive media, which is an epoch-making event [3] because it is found that the saturation nonlinearity can suppress the catastrophic collapse. Nowadays, various solitons have been found in different media, such as surface solitons [4–9], PT-symmetric potential solitons [10–14], optical lattice solitons [15–20], and even spatiotemporal optical solitons (namely the light bullets) [21–27], and so on. Recently, it is found that except the saturation nonlinearity, the nonlocal nonlinearity can also suppress the catastrophic collapse in bulk media. Optical beams propagating in nonlocal nonlinear media (NNM) have attracted much attention since the Snyder–Mitchell model (SMM) is proposed [28]. Nonlocality can be encountered in many physical systems, such as nematic liquid crystals, lead glasses, atomic vapors, Bose–Einstein condensates, and photorefractive crystals. The nonlocality allows the refractive index of a material at a particular point to be related to the beam intensity at other points, which is distinctly different from the conventional local nonlinearity. As a result, the optical beams propagating in NNM exhibit many novel properties, including the large phase shift, attraction between two beams with out-phases, and suppression of the collapse instability. If the characteristic response length of media is much larger than the beam width, they are usually called strongly nonlocal nonlinear media (SNNM). In SNNM, the well known nonlocal nonlinear Schrödinger equation (NNSE) is simplified to SMM [28,29]. Some optical beams have been investigated based on SMM, for instance Gaussian beams [28,29], elliptic-Gaussian beams [30], Hermite– and Laguerre–Gaussian beams [31–33], rotating parabolic cylindrical beams [34], complex-variable-function-Gaussian beams [35], and Ince–Gaussian beams [36]. It is also found that the spatiotemporal optical solitons can exist in SNNM [25,37–41].

On the other hand, the vortex beams have received an increasing attention due to their applications in free space information transfer and communications, optical trapping of particles, quantum information processing and quantum cryptography and so on. Optical vortex is always associated with the phase singularity, and the wavefront of the vortex beams carries orbital angular momentum. Some mathematical models have been established to describe vortex beams in the past decade. In experimental work, the vortex beams can be generated using a spiral phase plate or a spatial light modulator, etc. The Laguerre–Gaussian beam is a typical vortex beam, and many investigations are carried out based on it previously. However, most recently, a new type of vortex beam, i.e. the anomalous vortex beam (AVB) is proposed and generated by Cai et al. [42]. They exploited the

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propagation of the AVB in free space and found that the AVB can be considered as a virtual source to generate an elegant Laguerre–Gaussian beam, which is the first time to generate such a beam experimentally. In this paper, the AVB propagating in SNNM is investigated based on SMM. Two equivalent analytical expressions for the evolution of AVBs in SNNM are obtained. It is found that the evolution of AVBs in SNNM is periodical. Unlike the spatial soliton which keeps its patterns invariant, the AVB always transforms its transverse patterns during propagation, meanwhile its beam width can be compressed or broadened, even keep invariant, which is similar to the higher-order optical solitons in time domain. Thus the AVB propagating in SNNM can be regarded as a generalized higher-order spatial soliton or breather. In addition, the SNNM is equivalent to several other physical systems, such as optical fractional Fourier transform systems [43,44] and quadratic-index media [45,46], therefore the results in this paper can also be expanded to study the propagation of AVBs in these equivalent physical systems.

2. Analytical expressions

The propagation of paraxial optical beams in NNM is governed phenomenologically by the nonlocal nonlinear Schrödinger equation (NNLSE) [47,48],

$$2ik\frac{\partial\Phi}{\partial z}+\Delta_\perp\Phi+2k^2\frac{\Delta n}{n_0}\Phi=0, \tag{1}$$

where Φ is the complex amplitude of paraxial optical beams, $k=n_0\omega/c$ is the wave number in the media without nonlinearity, n_0 is the linear refractive index of media, $\Delta_\perp=\partial^2/\partial x^2+\partial^2/\partial y^2$, $\Delta n=n_2\iint R(\mathbf{r}-\mathbf{r}')|\Phi(\mathbf{r}',z)|^2\,d^2\mathbf{r}'$ is the nonlinear perturbation of refraction index, $\mathbf{r}=(x,y)$ represents the two dimensional transverse coordinates, n_2 is the nonlinear index coefficient, and $R(\mathbf{r})$ is the nonlocal response function of the media. If $R(\mathbf{r})=0$, i.e. the last term on the left of Eq. (1) is ignored, Eq. (1) reduces to the paraxial wave equation in free space, which is an important partial differential equation. Although it has been extensively investigated in the past decades, people are still making effort to search new solutions to describe the potential new laser beams. If $R(\mathbf{r})$ is a delta function, Eq. (1) reduces to the well-known nonlinear Schrödinger equation in local nonlinear media, which also attracts a lot of attention in the past. In this paper, we take $R(\mathbf{r})$ as a normalized symmetrical real nonlocal nonlinear response function, for instance, the Gaussian function [31,49,50]. For the case of strongly nonlocality, which means that the optical beam width is much narrower than the width of the response function, the response function can be expanded in Taylor's series. If the response function is twice differentiable at the original point, Eq. (1) is degenerated to SMM [28,29] which takes the form in cylindrical coordinates

$$2ik\frac{\partial\Phi}{\partial z}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\theta^2}-k^2\gamma^2P_0r^2\Phi=0, \tag{2}$$

where $\gamma^2>0$ is a material constant associated with the response function R, and $P_0=\int_0^\infty\int_0^{2\pi}|\Phi|^2r\,dr\,d\theta$ is the input power of optical beams. Eq. (2) is a partial differential equation similar to the paraxial wave equation in form. Considering the importance of the paraxial wave equation, Eq. (2) also has significant research value.

The electric field of AVBs at the initial plane is defined as [42]

$$\Phi(r_0,\theta_0,0)=C_0\left(\frac{r_0}{w_0}\right)^{2n+|m|}\exp\left(-\frac{r_0^2}{w_0^2}\right)\exp(-im\theta_0), \tag{3}$$

where $C_0=\sqrt{2^{2n+m+1}P_0/\pi\Gamma(2n+m+1)w_0^2}$ is a normalized constant which ensures the input power equals P_0, $\Gamma(\cdot)$ is the Euler gamma function, w_0 is the beam waist width of the Gaussian beam, n is the beam order of AVBs, m is the topological charge, r_0 and θ_0 are the radial and azimuthal coordinates, respectively. If $n=0$ and $m\neq0$, Eq. (3) represents the $(0,m)$ order Laguerre–Gaussian beams (also called the ordinary Gaussian vortex beams); if $m=0$ and $n\neq0$, it represents the hollow Gaussian beams [51]; if $m=0$ and $n=0$, it reduces to the fundamental Gaussian beams.

In theory, one can investigate the propagation of AVBs in SNNM based on Eqs. (2) and (3). However, it is almost impossible to analytically obtain the propagation expressions of AVBs in SNNM by reason of the complication in mathematical calculations. Fortunately, for the strongly nonlocal case Eq. (2) can be reduced to the SMM. As shown in our previous work [43,44,52], the beam propagation in SMM can be solved based on the matrix optics [53] or the fractional Fourier transformer [54–56]. The beam propagation expression for SMM can be obtained by the well-known Collins formula and the ABCD matrix. Therefore the analytical expression of paraxial optical beams propagating in SNNM can be obtained by the following integral formula:

$$\begin{aligned}\Phi(x,y,z)=&-\frac{ik\sqrt{\gamma^2P_0}}{2\pi\sin(\sqrt{\gamma^2P_0}z)}\exp\left[\frac{ik\sqrt{\gamma^2P_0}(x^2+y^2)}{2\tan(\sqrt{\gamma^2P_0}z)}\right]\\&\times\iint\Phi(x_0,y_0,0)\\&\exp\left[\frac{ik\sqrt{\gamma^2P_0}(x_0^2+y_0^2)}{2\tan(\sqrt{\gamma^2P_0}z)}-\frac{ik\sqrt{\gamma^2P_0}(xx_0+yy_0)}{\sin(\sqrt{\gamma^2P_0}z)}\right]dx_0\,dy_0.\end{aligned} \tag{4}$$

In cylindrical coordinates, Eq. (4) turns into

$$\begin{aligned}\Phi(r,\theta,z)=&-\frac{ik\sqrt{\gamma^2P_0}}{2\pi\sin(\sqrt{\gamma^2P_0}z)}\exp\left[\frac{ik\sqrt{\gamma^2P_0}r^2}{2\tan(\sqrt{\gamma^2P_0}z)}\right]\\&\times\int_0^{2\pi}\int_0^\infty\Phi(r_0,\theta_0)\\&\exp\left[\frac{ik\sqrt{\gamma^2P_0}r^2}{2\tan(\sqrt{\gamma^2P_0}z)}-\frac{2ik\sqrt{\gamma^2P_0}rr_0\cos(\theta-\theta_0)}{\sin(\sqrt{\gamma^2P_0}z)}\right]r_0\,dr_0\,d\theta_0.\end{aligned} \tag{5}$$

In Eqs. (4) and (5), all denominators in all constituents of the wavefunction simultaneously become zero at some particular propagation distances. We should note that at these particular propagation distances, one must use the limit theory to solve this problem, which has been pointed out in Ref. [54]. In fact, these positions are the end planes of each evolution period, i.e., at these planes, the beam recovers its initial shape.

Substituting Eq. (3) into Eq. (5) and recalling the integral formula [57]

$$J_0(x)=\frac{1}{2\pi}\int_0^{2\pi}\exp(ix\cos\theta)\,d\theta \tag{6}$$

and

$$\begin{aligned}&\int_0^\infty x^\mu e^{-\alpha x^2}J_\nu(\beta x)\,dx\\&=\frac{\beta^\nu\Gamma\left(\frac{1}{2}\nu+\frac{1}{2}\mu+\frac{1}{2}\right)}{2^{\nu+1}\alpha^{(\mu+\nu+1)/2}\Gamma(\nu+1)}{}_1F_1\left(\frac{\nu+\mu+1}{2},\nu+1;-\frac{\beta^2}{4\alpha}\right),\end{aligned} \tag{7}$$

where ${}_1F_1(a,b;x)$ is a confluent hypergeometric function, after somewhat complicated calculations, one can obtain the following analytical expression describing the propagation of AVBs in SNNM,