Optics Communications 350 (2015) 301–308

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Far field diffraction of an optical vortex beam by a fork-shaped grating

Lyubomir Stoyanov [a], Suzana Topuzoski [b], Ivan Stefanov [a], Ljiljana Janicijevic [b], Alexander Dreischuh [a,*]

[a] Department of Quantum Electronics, Faculty of Physics, Sofia University "St. Kliment Ohridski," 5, J. Bourchier Blvd., Sofia 1164, Bulgaria
[b] Faculty of Natural Sciences and Mathematics, Institute of Physics, University "Ss. Cyril and Methodius," Skopje 1000, Republic of Macedonia

ARTICLE INFO

Article history:
Received 15 January 2015
Received in revised form
2 April 2015
Accepted 6 April 2015
Available online 9 April 2015

Keywords:
Diffraction
Computer-generated holograms
Singular optics
Optical vortices
Topological charge
Optical angular momentum

ABSTRACT

In this work we report experimental data confirming the analytically predicted transformation of the topological charge (TC) of an input optical vortex (OV) beam, generated by means of fork-shaped binary computer-generated hologram (CGH), after a second fork-shaped binary CGH. The final TC of the vortex is confirmed to be equal to the TC of the incident beam plus the diffraction order (with its sign) times the TC encoded in the binary grating. The radii of the transformed OVs in the far field also are found to agree fairly well with these predicted by the analytical theory.

1. Introduction

Recent years have seen an increased interest and research about the optical fields possessing screw phase singularities carried by optical vortex beams. The transverse cross-sections of the optical vortices (OVs) are associated with isolated point singularities with helical phase wavefront around them. The central singular point of the helix possesses undefined phase and therefore the intensity must vanish, leading to a characteristic toroidal intensity profile (vortex ring). The study of the phase singularities is important from the viewpoint of both fundamental and applied physics. Light beams possessing phase singularities are used in the experiments for particle trapping and manipulation [1], atom trapping and guiding [2], as information carriers [3] for multiplexing in free-space communications [4], for interferometry [5], and for realizing of electron vortex beams [6], just to mention a few.

Only 20 years ago, the Laguerre–Gaussian (LG) laser modes having an azimuthal mode number different from zero (eigenmodes of a laser cavity described in cylindrical coordinates) and other vortex beams having helical wavefront structure with phase dependence $\exp(il\varphi)$ (where l is an integer and φ is the azimuthal coordinate) were recognized to have orbital angular momentum (OAM) $l\hbar$ per photon in their propagation direction [7]. The number l showing the total phase change $2\pi l$ over the azimuthal coordinate φ is referred as the topological charge of the OV beam. It was shown that this OAM can be transferred to a captured microparticle causing its rotation in a direction determined by the sign of the topological charge (TC) [8].

The intensive research in the field of singular optics has shown that optical vortex beams can be generated from incident chargeless optical beams by means of diffractive optical elements (DOEs) with embedded phase dislocations, such as spiral phase plate (SPP) [9], helical axicon [10], computer-generated holograms [11], spiral zone plates [12] as well as fork-shaped gratings [13–15]. Some of them, when build in dispersionless optical systems, provide a useful means to create phase singularities in the beams of ultrafast femtosecond lasers with broad bandwidth [16–18]. OV creation succeeded even in near-single-cycle regime of ultrashort laser pulse generation [19] by using thermally tuning reflective spiral micro-electro-mechanical elements [20]. The transformation of the incident vortex beam through some types of DOEs with encoded phase singularities was studied in detail in [21,22]. In [21] the authors have shown theoretically that in the process of diffraction of LG beam with zeroth radial mode number and arbitrary azimuthal mode number l, by a fork-shaped grating with integer forked dislocations p, in the positive and negative m-th diffraction order, the diffracted beam carries topological charge s which is determined as an algebraic sum $s = l + mp$ or $s = l - mp$, respectively. In the paper of Mair and co-authors [23] a fork-shaped grating is used as a filter for estimating the TC of optical vortices. A

* Corresponding author. Fax: +359 2 868 88 13.
E-mail address: ald@phys.uni-sofia.bg (A. Dreischuh).

http://dx.doi.org/10.1016/j.optcom.2015.04.020
0030-4018/

photon with angular momentum $p\hbar$ before the fork-shaped grating which possesses phase singularity of order p can be detected by a mono-mode fiber detector placed in the negative first diffraction order. In the same setup a photon with zero angular momentum can be detected by diffracting the beam far away from the forked section of the grating where the grating is nearly rectilinear. In this experiment [23] the authors confirmed the conservation of the OAM of (entangled) photons in the process of spontaneous parametric down-conversion. Soskin and co-authors [24] have studied theoretically and experimentally the behavior of vortices in a beam composed of singular and"background" Gaussian waves, in order to check the principle of TC conservation. The topological charge in nonlinear optics has been studied by Soskin and Vasnetsov [25] who suggested its conservation in a stimulated down-conversion process. The experiment of Maleev et al. [26] has not shown, however, any evidence of such conservation. It was recently shown that the TC conversion in cascaded four-wave frequency mixing process obeys the transformation law analogous to the one for the frequency [27]. The conservation of the TC was demonstrated for the surface plasmons: when the LG beam having azimuthal mode number l is transferred through plasmonic vortex lens with TC equal to m, then surface plasmon vortices with orbital angular momentum are generated, and inherit the optical angular momentum of light beam and plasmonic vortex lens, since their topological charge n was equal to $n = l + m$ [28].

The aim of this work is to confirm experimentally the transformation of the topological charge of the incident vortex beam during the transfer through a fork-shaped grating with embedded topological defects of integer number predicted for the first time in [21,29]. For this purpose we generate OVs with TCs equal to 2, 3 and 4 by fork-shaped gratings and let them, subsequently, diffract to the far field by another single- and twofold-charged fork-shaped gratings. The reversed case is also studied. The theoretical results [21,29] for the algebraic transformation of the TCs of the OVs were checked by making interferograms of the diffracted components and a plane wave, while the vortex ring radii of the transformed beams were compared by using their radial intensity profiles.

2. Theoretical background

In the experiment described and illustrated by the setup in Fig. 1, the vortex beam passing normally through the bottom of the fork-shaped area of the grating 2 (CGH2) is previously generated by letting a chargeless Gaussian laser beam to pass normally through the fork-shaped grating 1 (CGH1). The Gaussian beam axis is passing through the centre of the forked dislocation of the CGH1, where the pole of the cylindrical system with coordinates r', φ' and z' is situated. The gratings possess TCs p_1 and p_2, respectively, which are encoded in the fork-shaped singularities. Recalling the theoretical results in [29], in the process of Fresnel diffraction the grating 1 (situated in a plane

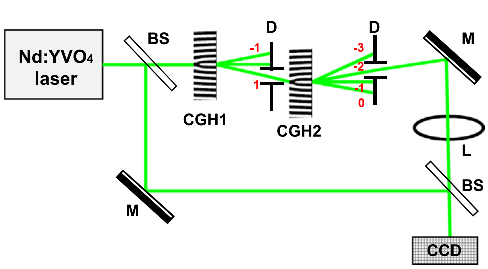

Fig. 1. Experimental setup: Nd: YVO_4 laser – continuous-wave frequency-doubled laser emitting at a wavelength of 532 nm. BS – beam splitters. CGH1, CGH2 – binary computer-generated holograms. D – iris diaphragms. L – focusing lens (f=100 cm). M – flat mirrors. CCD – charge-coupled device camera located at the beam waist.

$z' = 0$) splits the incident Gaussian beam $U'(r', \varphi', z' = 0)$= $\exp(-r'^2/w_0^2)$ (where w_0 is its waist radius) into a fan of beams. The direct zeroth-diffraction-order beam is a chargeless Gaussian beam. The higher positive and negative m'-th diffraction-order beams appear as optical vortex beams carrying phase singularities with TCs $m'p_1$ and $-m'p_1$, respectively. This can be seen from expression (17) in the same article. In our experiment we will use expression (17) adapted to the so-called far-field approximation (where the distance between the two gratings is such that $1/R(z') \to 0$, $1/z' \to 0$, $Q^{-1}(z') = (1/2)[(R^{-1}(z') - z'^{-1}) - 2i/(kw^2(z'))] \to -i/(kw^2(z'))$, and $\exp[i \arctan(2z'/kw_0^2)] \to 1$). Also, we are interested in the paraxial region of the OV beam distribution $0 \le r_{\pm m'} \le w(z')$. Instead of the coordinates $\rho_{\pm m}$ and $\vartheta_{\pm m}$ in Eq. (17) from [29] we use polar coordinates $r_{\pm m'}$ and $\varphi_{\pm m'}$, and denote with $w(z')$ the vortex beam amplitude profile radius at position z'. Therefore, we will neglect all $n \ne 0$ members in the sum by which the Kummer function is represented, i.e. for the far field $M(|m'p_1|/2, |m'p_1| + 1, r^2_{\pm m'}/w^2(z')) \approx 1$. In this approximation the m'-th diffraction order beam, emerging from the CGH1, is described by the expression

$$U_{\pm m'}(r_{\pm m'}, \varphi_{\pm m'}, z') = C_{m'p1}\left[\frac{r_{\pm m'}\sqrt{2}}{w(z')}\right]^{|m'p1|} \times \exp\left(-\frac{r^2_{\pm m'}}{w^2(z')}\right)\exp[-i(kz' \mp m'p_1\varphi_{\pm m'})]. \quad (1)$$

Here $C_{m'p_1}$ is the complex amplitude given by

$$C_{m'p1} = \frac{w_0}{w(z')} t_{\pm m'} \frac{\Gamma(|m'p_1|/2 + 1)}{\Gamma(m'p_1 + 1)\sqrt{2^{|m'p1|}}} \exp(im' p_1\pi/2) \quad (2)$$

with $t_{\pm m'}$ being the transmission coefficients of the first grating [29]. Expression (1) is recognized as zero radial mode LG laser beam with azimuthal mode number $\pm m'p_1$ equal to its phase singularity order. The same approximation for the higher diffraction orders of the beams generated by fork-shaped grating is used by the authors in [14,30–32].

In our experiment z' is the distance between the two gratings. The incident beam on the second grating carrying TC $p_2 = p$ is selected by the first diaphragm (Fig. 1). It is a vortex beam of type (1) with TC $l = m'p_1$ having entrance radius $w(z') = w_0$ of the transverse amplitude profile. Instead of the coordinates $(r_{\pm m'}, \varphi_{\pm m'}, z')$ now, for simplicity, we will use (r, φ, z), and take l to be a positive integer. Thus, the incident beam on the second fork-shaped grating is approximated as $LG_0^{(l)}$ mode

$$U_0^{(l)}(r, \varphi, z = 0) = A_{l,0}\left(r\sqrt{2}/w_0\right)^l \exp(-r^2/w_0^2)\exp(il\varphi), \quad (3)$$

where $A_{l,0}$ is an amplitude coefficient.

Further, we treat the Fraunhofer (far-field) diffraction of this singular field by the second fork-shaped grating with an encoded topological charge p, having transmission function in polar coordinates as follows:

$$T(r, \varphi) = \sum_{m=-\infty}^{\infty} t_m \exp\{-im[(2\pi/D)r\cos(\varphi) - p\varphi]\}. \quad (4)$$

The transmission coefficients of this binary amplitude grating are $t_0 = 1/2$, $t_{\pm m=\pm(2n'-1)} = \mp i/[\pi(2n' - 1)]$, $t_{\pm m=\pm 2n'} = 0$ $(n' = 1, 2, 3\ldots)$.