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Photonic crystal with triangular stack profile

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ABSTRACT

The enhanced localized reflectivity at a plane where the refractive index derivatives are discontinuous is increased using a periodic triangular stack. This optical system is studied using three different methods: a slowly varying refractive index approximate solution, numerical solutions to the nonlinear amplitude equation and the exact analytical solution using a matrix formalism. The field intensity within a photonic crystal with a periodic triangular profile is evaluated as a function of penetration depth and wavelength. These results are compared with the standard binary layered structures.

1. Introduction

An enhanced reflectivity and specific phase change upon reflection have been predicted for dielectric planes where the refractive index derivatives are discontinuous [1]. This reflectivity enhancement calculation was performed at normal incidence and takes place even if the refractive index itself is continuous. The reflectivity, at planes where the first order derivative is discontinuous, may reach values close to 1% if a convenient refractive index profile is chosen. The phase change upon reflection is $\pm\pi/2$, the sign depends on whether the refractive index slope increases or decreases [2]. In contrast, zeroth order derivative discontinuity corresponds to the Fresnel reflection of a dielectric step function, where reflectivities are around 4% (air–glass interface) and phase changes upon reflection are 0 or π.

A periodic dielectric structure with triangular profile of the refractive index has been studied analytically using a semiclassical coupled wave theory [3]. An important advantage of wave solutions is, as the authors rightly assert, their superior physical insight. Recently, a sawtooth refractive index profile that encompasses the triangular profile has been exactly solved in terms of Bessel functions using the transfer matrix method [4]. However, the transfer matrix method does not lend itself to map the field or the intensity throughout the material. The field distribution is better represented in terms of Floquet–Bloch waves, since this procedure permits the evaluation of the field as it propagates through the periodic medium [5]. The light field distribution in photonic crystal structures has also been tackled using Fermat's principle [6,7]. 1D photonic crystals have important technological applications in fast optical switching [8]. Graded multi-layer structures open up the possibility of shaping the refractive index's profile, and hence tailor the reflection properties to specific designs.

The localized enhanced reflectivity due to the interface between two linear refractive indices with different slopes can be increased by a sequence of such derivative discontinuities. The triangular stack is the simplest scheme of this type. This triangular structure is a good experimental candidate to observe the reflectivity enhancement and phase shift upon reflection due to discontinuous first order derivatives.

In this communication, we explore the optical characteristics of the triangular stack using different theoretical methods. In Section 2, the reflection coefficient of a derivative discontinuity plane is derived in the amplitude and phase representation of waves. The nonlinear amplitude equation can be solved in power series of the slowly varying refractive index (SVRI) derivative, often called in the physics community, the JWKB approximation. At the discontinuity planes, where this SVRI scheme is not valid, the solutions at either side of the singularity are matched via field continuity conditions. In Section 3, the triangular stack is solved in the

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SVRI approximation. First, the complex reflection coefficients for the two types of vertices needed to build up a triangular stack are obtained. Section 3.1 gives a rule of thumb to evaluate the reflectivity of a triangular quarter wavelength stack. Whereas in Section 3.2, the reflectivity as a function of incident wavenumber is evaluated. The numerical solution to the nonlinear amplitude equation, considering a triangular stack profile, is presented in Section 4. Once the amplitude and phase representation of the fields are understood in terms of an Ermakov pair, the implementation of these types of solutions is straightforward. Field intensity plots as a function of layer depth and wavelength can be readily produced in this scheme. In Section 5, the procedure outlined by Morozov et al. [4] is performed. The exact solution is compared with the amplitude numerical solution of the previous section. In Section 6, the triangle stack is compared with standard multilayer devices consisting of two alternating homogeneous media. Conclusions are drawn in the last section.

2. Reflection coefficient evaluated from the amplitude equation

Consider an isotropic, non-magnetic, transparent, dielectric medium with a linear response and no free charges, stratified in the z direction. Let the electric field $\mathbf{E} = E(z)e^{-i\omega t}\,\hat{\mathbf{e}}_x$ represent monochromatic plane waves polarized in the x direction and propagation normal to the stratification planes. The non-autonomous ordinary differential equation (ODE) for the electric field is

$$\frac{d^2E}{dz^2} + k_0^2 n^2(z)E = 0. \tag{1}$$

The wave vector squared magnitude is $k_0^2 = \omega^2\mu_0\varepsilon_0$ and the refractive index is equal to the square root of the relative permittivity $n(z) = \sqrt{\varepsilon(z)/\varepsilon_0}$. The amplitude $A(z)$ and phase $\phi(z)$ representation of the complex field is $E(z) = Ae^{i\phi}$, where the amplitude and phase are real functions. Inserting this ansatz in the field equation leads to the nonlinear ordinary differential equation for the electric field amplitude [9]:

$$\frac{d^2A}{dz^2} - \frac{Q^2}{A^3} = -k_0^2 n^2 A. \tag{2}$$

The electric field solution $E(z)$ is the total field at any propagation position z. Whether there are counter-propagating waves or not and in which ratio is not discernible at this stage. Correspondingly, $A(z)$ is the amplitude of the total field. The linearity of the field ODE guarantees superposition of field solutions. For the amplitude, it is necessary to invoke a nonlinear superposition principle in order to compose two or more amplitude contributions [10]. In one dimensional problems, the amplitude and phase functions are related by the invariant:

$$Q = A^2\frac{d\phi}{dz}. \tag{3}$$

This invariant can be derived from the scalar wave equation when a conservation equation is constructed from two linearly independent solutions [11]. The complementary fields associated with these two solutions exchange energy in a dynamical equilibrium both, in the time and spatial domains. In the one dimensional degenerate case, the conserved quantity becomes a constant.

Allow for an interval along the z-axis, where the medium is homogeneous, *i.e.* constant n. Let the field solution be written as the sum of two waves with opposite phase:

$$E(z) = A_+ \exp\left[i\left(k_0 nz + \varphi_{0+}\right)\right] + A_- \exp\left[-i\left(k_0 nz + \varphi_{0-}\right)\right], \tag{4}$$

where A_+, A_- are constant real amplitudes and φ_{0+}, φ_{0-} are real phase constants. Consider the semi-space where the wave is incident. In this region, these counter-propagating waves can be associated with the incident and reflected waves $E(z) = E_{\text{incident}} + E_{\text{reflected}}$. Evaluation of the field derivative (4) gives

$$\frac{dE(z)}{dz} = ik_0 nE_{\text{incident}} - ik_0 nE_{\text{reflected}}. \tag{5}$$

From these last two expressions, the reflected to incident fields ratio $r(z)$ can be expressed in terms of the total field E and its first derivative [12]:

$$r(z) = \frac{E_{\text{reflected}}}{E_{\text{incident}}} = \frac{iEk_0 n - \frac{dE}{dz}}{iEk_0 n + \frac{dE}{dz}}, \tag{6}$$

where i stands for the imaginary unit. Alternatively, this ratio can be written in terms of the squared amplitude A^2 and its derivative

$$r(z) = \frac{nA^2k_0 - Q + iA\frac{dA}{dz}}{nA^2k_0 + Q - iA\frac{dA}{dz}}, \tag{7}$$

where the invariant relationship (3) has been invoked. The ratio $r(z)$ is interpreted as the complex reflection coefficient at a given plane of the incident semi-space. The square modulus of this quantity is the reflectivity $R = rr^*$, it is constant in the incident semi-space before reaching the reflecting surface.

The nonlinear amplitude equation (2) can be approximately solved as a power series of the inverse wavenumber k_0^{-1} for a slowly varying refractive index (SVRI) but otherwise arbitrary function $n(z)$ [13]:

$$A_{\text{SVRI}} = \sum_{m=0}^{\infty} \frac{1}{k_0^m} A_m, \tag{8}$$

where only the even order terms are non-vanishing. The A_m amplitude term retaining only the highest order derivative of the refractive index is [2]

$$A_m = \left(\frac{(-1)^{(m/2)+1}}{n^{m+(3/2)}2^{m+1}}\right)\frac{d^m n}{dz^m} + O(<m), \tag{9}$$

for even m. Solutions with higher precision can be analytically obtained by the evaluation of higher order terms in the slowly varying refractive index series expansion. Care should be taken with the convergence of the series. The field can then be evaluated from the amplitude solution together with the phase via the invariant relationship (3). In order to obtain the reflectivity at a discontinuity, the amplitude is evaluated just before and after the singularity where $n(z)$ is analytic and slowly varying. Continuity conditions are imposed on these two amplitude solutions. The complex reflection coefficient for an isolated non-vanishing discontinuity in the mth order derivative of the refractive index at z_0 is given to first order by [2]

$$r_{\text{iso}} = \frac{i^m}{(2n(z_0))^{m+1}k_0^m}\left(\left.\frac{d^m n}{dz^m}\right|_{z_0-\delta} - \left.\frac{d^m n}{dz^m}\right|_{z_0+\delta}\right), \tag{10}$$

where the light is incident from a medium where the refractive index mth derivative just before the discontinuity is $d^m n/dz^m|_{z_0-\delta}$ and just after the discontinuity is $d^m n/dz^m|_{z_0+\delta}$. The phase change upon reflection can be obtained from (10) recalling that $i^m = e^{im\pi/2}$. The phase change can be summarized in the following