Optics Communications 346 (2015) 158–166

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Parametric registration of cross test error maps for optical surfaces

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ARTICLE INFO

Article history:
Received 21 September 2014
Received in revised form
14 February 2015
Accepted 18 February 2015
Available online 20 February 2015

Keywords:
Image registration
Non-rigid transformation
Cross test
Surface metrology

ABSTRACT

It is necessary to quantitatively compare two measurement results which are typically in the form of error maps of the same surface figure for the purpose of cross test. The error maps are obtained by different methods or even different instruments. Misalignment exists between them including the tip-tilt, lateral shift, clocking and scaling. A fast registration algorithm is proposed to correct the misalignment before we can calculate the pixel-to-pixel difference of the two maps. It is formulated as simply a linear least-squares problem. Sensitivity of registration error to the misalignment is simulated with low-frequency features and mid-frequency features in the surface error maps represented by Zernike polynomials and spatially correlated functions, respectively. Finally by applying it to two cases of real datasets, the algorithm is validated to be comparable in accuracy to general non-linear optimization method based on sequential quadratic programming while the computation time is superiorly incomparable.

1. Introduction

In optical surface metrology, it is now common to use alternative methods or instruments to measure the same surface error. One reason is the cross test helps to reduce the risk of making mistakes. As it gets more and more complicated, the optical test system is prone to introducing bigger error or even making mistakes. The figuring error due to misaligned test system for the primary mirror of the Hubble Space Telescope is a profound lesson to opticians [1]. That is why scientists have tried up to four alternatives including the principal test (interferometer null test), the SCOTS slope measurement, the scanning pentaprism test and the laser tracker test, to measure the surface error in production of 8.4 m segments for the Giant Magellan Telescope [2]. Another reason is to characterize the performance of one newly proposed method by referring it to a well developed one. For example, Koch et al. compared the Shack–Hartmann sensor with a phase-shifting interferometer in measuring large deformable mirrors and drew the conclusion that the former can replace the interferometer in many applications with particular advantages for large optics metrology [3]. In addition, combining multitool metrology can also reduce the measurement uncertainty by statistical methods [4].

The measurement results are typically in the form of error maps of the same surface figure. They are obtained by different methods or even different instruments. It requires careful physical alignment to quantitatively compare the two maps [3]. However, physical registration is difficult, time-consuming and sometimes the registration error is not negligible. Quite often we have to qualitatively compare the similarity of the two results. Fig.1 shows the surface difference induced by shifting the original error map (on a grid of 256×256 pixels) in X by 2 pixels, where PV stands for peak-to-valley and rms for root-mean-squares. The difference is about 1/10 and 1/3 of the original map for the pure second astigmatism and the mid-frequency feature with correlation length of 7.7 pixels, as shown in Fig. 1(a) and (b), respectively. Hence for the purpose of quantitative comparison, one map needs to be automatically registered to the other before we can calculate the pixel-to-pixel difference of the two maps. Registration of two maps is mathematically achieved by minimizing the sum of squares of the pixel-to-pixel deviations.

It is assumed that all error maps project the distribution of errors in height onto a certain plane, e.g., the plane perpendicular to the geometrical axis of the test surface. Possible image distortion in the interferometric null test [5], for example, is well corrected before generating the error map. Therefore the misalignment between two error maps we need to consider includes only the piston, tip-tilt, lateral shift in X and Y directions, clocking (rotation around the normal of the image plane) and scaling. The last one is non-rigid as it changes the length of geometries. It should be included allowing for the uncertainty of image unit

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http://dx.doi.org/10.1016/j.optcom.2015.02.036
0030-4018/

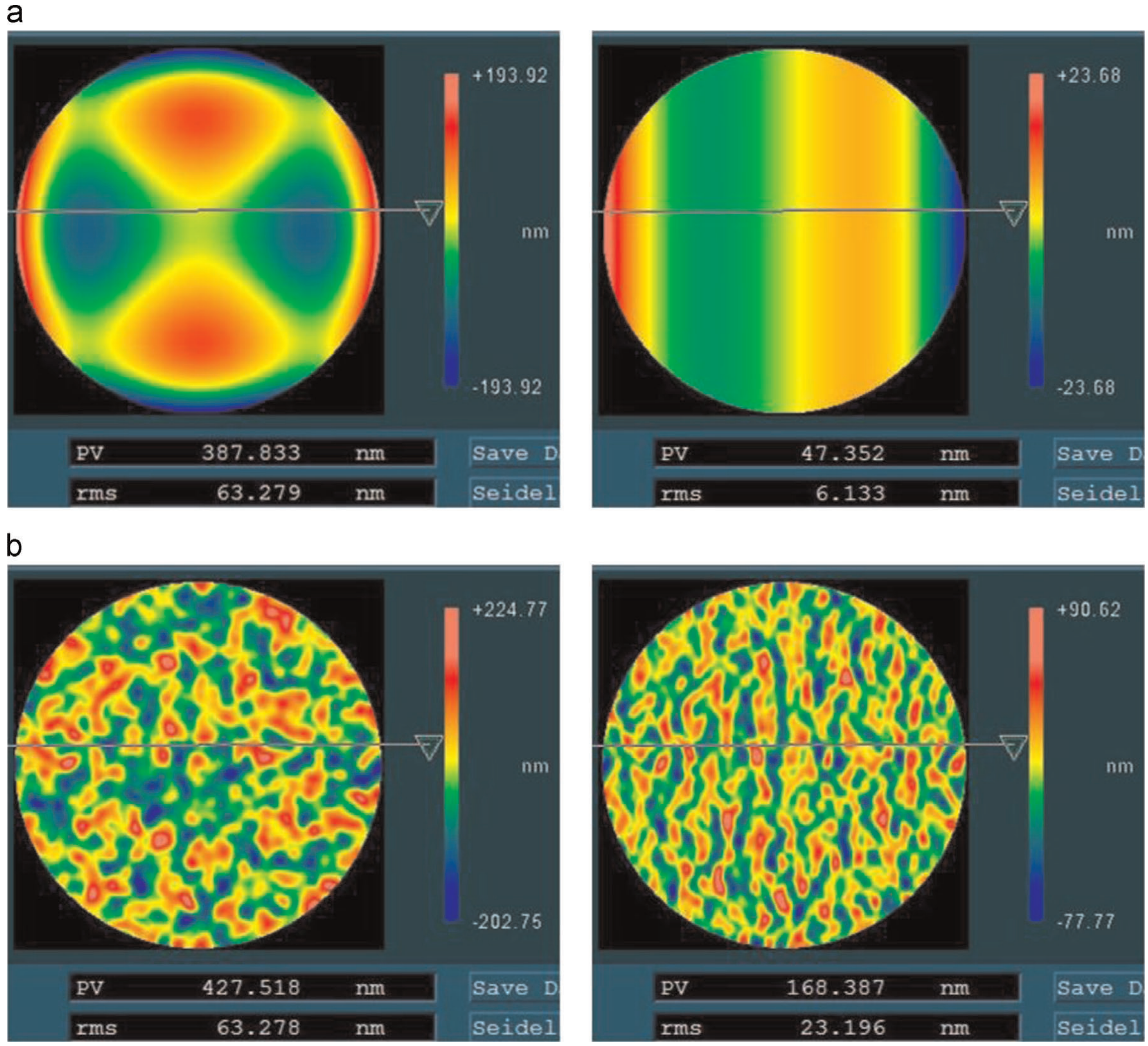

Fig. 1. Original error map and surface difference induced by shifting in *X* by 2 pixels. (a) Pure second astigmatism. (b) Mid-frequency feature with correlation length of 7.7 pixels.

calibration, i.e., calibration of the size correspondence between the sensor pixels and the test surface. In this sense, the registration algorithm based on rigid transformation for quasi-planar freeform wavefronts [6] is not applicable to cross test error maps.

Because scaling is global in nature, registration of optical error maps is generally not categorized into non-rigid registration which is applied typically to multimodal medical images with local shape variations [7]. And the misalignment between optical error maps is usually sufficiently small. Hence we do not need to employ the complex algorithms for general image registration [7,8]. On the other hand, registration of optical error maps is also different from subaperture stitching with null or near-null optics used. Because the misalignment introduces field dependent aberrations by changing the null or near-null condition [9], it can not be simply modeled by rigid transformation or scaling. In contrast, the misalignment does not change the error composition in registration of two error maps.

In this paper, we first formulate the parametric registration problem as simply a linear least-squares problem (LSP). The change of errors in height is linearly related to tip-tilt. It is also linearly related to lateral shift, clocking and scaling by means of the error surface slope. Then sensitivity of registration error to the misalignment is related to Zernike polynomials and spatial correlation length, respectively. The polynomials and the correlated function simulate low-frequency features and mid-frequency features in the surface error maps, respectively. We finally validate the algorithm by applying it to two cases of real datasets.

2. Mathematical formulation

2.1. Linear dependence of error change on misalignment

Basically the misalignment is modeled by rigid transformation and lateral scaling, which is represented by a 4×4 matrix in homogeneous coordinates. It is a non-linear function of the transformation parameters. However with small misalignment assumption, the error change Δz in height is naturally related to piston and tip-tilt as follows:

$$\Delta z_1 = a + bx + cy \quad (1)$$

where a, b, c are the coefficients of piston and tip-tilt, and (x,y) are lateral coordinates of the error map.

The error change with lateral shift can be derived from the slope of the error map [10]:

$$\Delta z_2 = s\frac{\partial z}{\partial x} + t\frac{\partial z}{\partial y} \quad (2)$$

where s and t are the coefficients of lateral shift, and the partial derivatives are slopes in X and Y directions, respectively.