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Mixed Rabi Jaynes–Cummings model of a three-level atom interacting with two quantized fields

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ABSTRACT

The quantum Rabi model describes the ultrastrong interaction of a two-level atom coupled to a single quantized bosonic mode. As compared to the Jaynes–Cummings model, in the Rabi model the absorption and emission processes do not need to satisfy energy conservation and the usual rotating wave approximation (RWA) breaks down. As a result, the atom-field dynamics in the Hilbert space splits into two independent parity chains, exhibiting a collapse-revival pattern and exact periodic dynamics in the limit of degenerate atomic levels. Here we introduce a mixed Rabi Jaynes–Cummings model by considering a three-level atom interacting with two quantized bosonic fields, in which the RWA is made for one transition (with a weak atom-field coupling) but not for the other one (with an ultrastrong atom-field coupling). As a result, we show that the field in the weak coupled atomic transition can be used as a tool to control the atom-field dynamics of the other (strong coupled) transition, thus realizing an effective two-level quantum Rabi model with a controllable field. In particular, a periodic temporal dynamics of the atom-field state can be realized by appropriate tuning of the weak control field, even for non-degenerate atomic levels. A photonic simulator of the mixed Rabi Jaynes–Cummings model, based on light transport in evanescently coupled optical waveguide lattices, is also briefly discussed.

1. Introduction

The well-known quantum Rabi model [1–4], describing a two-level atom coupled to a quantum harmonic oscillator, continues to produce rich and surprising physics, with plenty of applications in a variety of physical systems. The quantum Rabi model has been applied to numerous experimental systems in quantum optics or condensed matter, such as cavity quantum electrodynamics (QED) [5–7], quantum dots [8], superconducting qubits [9,10] and trapped ions [11,12]. In most cases, when the external field is weak enough [13], the rotating wave approximation (RWA) is applied and in such a way the famous Jaynes–Cummings model is obtained [14]. However, in recent years, new regimes have been explored [15–23], in which the effect of counter-rotating terms cannot be neglected. Such regimes are the ultrastrong coupling of light-matter interactions [17–21] and the deep strong coupling (DSC) [22,23]. In the DSC regime, the absorption and emission processes do not need to satisfy energy conservation and the atom-field dynamics is more involved and splits into two parity chains in Hilbert space. As a result, the atom-field state undergoes revival and collapse dynamics in Hilbert space [22]. Remarkably, in the limit of degenerate atomic levels the dynamics becomes exactly periodic [22]. Recent works have shown that the quantum Rabi model can be simulated by using light transport in engineered waveguide superlattices [24–26]. This could allow the DSC regime, which is hard to access experimentally in cavity QED, to be successfully simulated in other physical contexts. In spite of the vast research in this area and the relative simplicity of the Rabi model, its integrability has been proven just recently [27–29].

In the past few decades, several theoretical and experimental works have shown that many interesting coherent phenomena can be observed when more than two atomic levels are involved in the dynamics. In particular, the three level system exhibits a plethora of coherent phenomena such as two-photon coherence [30], resonance Raman scattering [31], double resonance process [32], three-level super radiance [33] and quantum jumps [34], to mention a few. The quantum dynamics of a three-level atom interacting with two resonant or near resonant modes of a steady field has been generally studied by assuming either quantized fields, using a dressed-state formalism within the RWA (extended Jaynes–Cummings models [35–37]), or the Floquet approach for

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classical fields [38–41]. In such previous works, the strong coupling regime was considered solely in the semiclassical limit, i.e. for classical fields, where a Floquet analysis of the underlying time-periodic equations for the atomic population amplitudes can be employed for sinusoidal external fields [42]. However, for quantized fields the ultrastrong coupling regime in the three-level atomic system, which breaks the RWA, was not investigated.

In this paper, we study in detail a three-level atomic system interacting with two quantized fields, where one field is near-resonant and weakly coupled with one atomic transition whereas the other field is strongly coupled to the other atomic transition. Such a quantum model can be refereed to as mixed Rabi Jaynes–Cummings model, because the RWA can be applied to one transition, but not to the other one. We derive the coupled differential equations, describing the temporal evolution of the quantum system in Hilbert space, and show that the weak-coupling filed can be used as a tool to control the dynamics in Hilbert space of the atom-field state for the other transition. In particular, the weak control field can be tuned to realize exact periodic of the atom-field state even if the strongly coupled atomic levels are not degenerate. A possible physical implementation of the mixed Rabi Jaynes–Cummings model, using arrays of coupled optical waveguides with engineered coupling constants, is also briefly discussed.

2. The model

We consider a three-level quantum system, with atomic states $|1\rangle_{at}$, $|2\rangle_{at}$, and $|3\rangle_{at}$, interacting with two bosonic (e.g. electromagnetic) fields of states $|n, k\rangle_{bos}$, where n and k are the number of bosons in the two fields (Fig. 1). Such system is described by the Hamiltonian

$$H = \sum_{i=1}^{3} E_i \sigma_{ii} + \hbar\omega_1 a_1^\dagger a_1 + \hbar\omega_2 a_2^\dagger a_2 + \hbar g_1(\sigma_{13} + \sigma_{31})(a_1 + a_1^\dagger) + \hbar g_2(\sigma_{23} + \sigma_{32})(a_2 + a_2^\dagger), \tag{1}$$

where $\omega_{1,2}$ are the frequencies of the fields that are responsible for the $|1\rangle_{at}$–$|3\rangle_{at}$ and $|2\rangle_{at}$–$|3\rangle_{at}$ atomic transitions, respectively. The coupling strengths are parameterized by g_1 and g_2, $a_{1,2}^\dagger$ and $a_{1,2}$ are the creation and annihilation operators for the two modes and $\sigma_{ij} = |i\rangle_{at}\,{}_{at}\langle j|$. We now assume $\omega_1 \ll \omega_2$ and a resonant and weak coupling for the $|2\rangle_{at}$–$|3\rangle_{at}$ transition, hence $\hbar\omega_2 = E_3 - E_2$ and $\hbar g_2\sqrt{k} \ll E_3 - E_2$. Under these conditions we can apply the RWA for this transition, discarding the counter-rotating terms $\sigma_{32}a_2^\dagger$ and $\sigma_{23}a_2$, and the Hamiltonian now reads

$$H = \sum_{i=1}^{3} E_i \sigma_{ii} + \hbar\omega_1 a_1^\dagger a_1 + \hbar\omega_2 a_2^\dagger a_2 + \hbar g_1(\sigma_{13} + \sigma_{31})(a_1 + a_1^\dagger) + \hbar g_2(\sigma_{32} a_2 + \sigma_{23} a_2^\dagger). \tag{2}$$

The model described by the Hamiltonian (2) is a mixed Rabi and Jaynes–Cummings model, because the RWA is performed for one of the two transitions (like in standard Jaynes–Cummings model) but not for the other one (like in the quantum Rabi model). To study the exact temporal evolution of the atom-field state $|\Psi(t)\rangle$ in the mixed Rabi Jaynes–Cummings model, let us expand the state vector of the system as

$$|\Psi(t)\rangle = \sum_{n,k}\left[C_{n,k}^{(1)}(t)|1\rangle_{at} + C_{n,k}^{(2)}(t)|2\rangle_{at} + C_{n,k}^{(3)}(t)|3\rangle_{at}\right]|n, k\rangle_{bos}, \tag{3}$$

where $C_{n,k}^{(l)}$ is the probability amplitude to have (n,k) bosons in the two fields and the atom in level $|l\rangle_{at}$. Substitution of the ansatz (3) into the Schrödinger equation

$$i\hbar\partial_t|\Psi(t)\rangle = H|\Psi(t)\rangle, \tag{4}$$

yields the following coupled differential equations for the probability amplitudes:

$$i\hbar\dot{C}_{n,k}^{(1)} = E_1 C_{n,k}^{(1)} + \left(n\hbar\omega_1 + k\hbar\omega_2\right)C_{n,k}^{(1)} + \hbar g_1\sqrt{n+1}\,C_{n+1,k}^{(3)} + \hbar g_1\sqrt{n}\,C_{n-1,k}^{(3)} \tag{5a}$$

$$i\hbar\dot{C}_{n,k}^{(2)} = E_2 C_{n,k}^{(2)} + \left(n\hbar\omega_1 + k\hbar\omega_2\right)C_{n,k}^{(2)} + \hbar g_2\sqrt{k}\,C_{n,k-1}^{(3)} \tag{5b}$$

$$i\hbar\dot{C}_{n,k}^{(3)} = E_3 C_{n,k}^{(3)} + \left(n\hbar\omega_1 + k\hbar\omega_2\right)C_{n,k}^{(3)} + \hbar g_1\sqrt{n+1}\,C_{n+1,k}^{(1)} + \hbar g_1\sqrt{n}\,C_{n-1,k}^{(1)} + \hbar g_2\sqrt{k+1}\,C_{n,k+1}^{(2)}. \tag{5c}$$

The coupling between the amplitudes can be visualized as two sets of uncoupled chains, where each lattice site corresponds to a different state of the atom-field system. Depending on the initial condition, one of the two sets is realized and the other one is irrelevant. The chains in each set are uncoupled in the k direction, because of the RWA, while in the n direction they are semi-infinite with a gradient in the boson number. This is depicted in Fig. 2, where we show the two coupling schemes, for a particular value of the second-mode boson number $k = \kappa$. The top part of the picture shows the coupling scheme with only even number n of bosons in the first mode when the atom is in state $|1\rangle_{at}$. On the opposite, the bottom part contains only odd number of bosons n when the atom is in $|1\rangle_{at}$. In what follows, we will assume that the initial condition

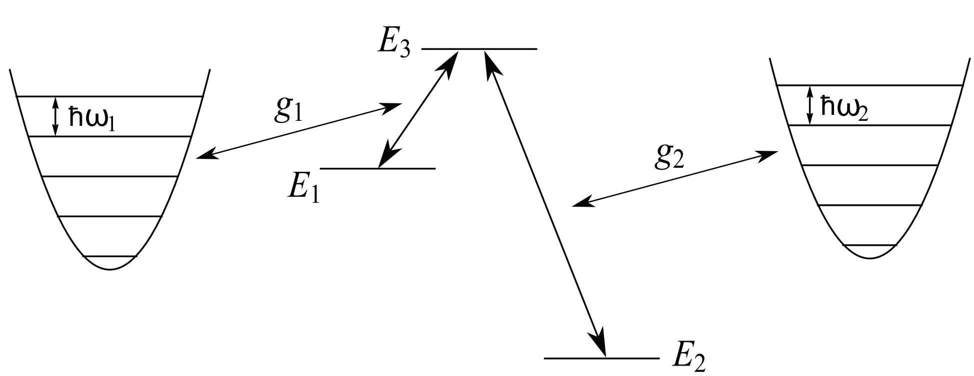

Fig. 1. Schematic of a three-level system interacting with two bosonic modes with n quanta at frequency ω_1 and k quanta at frequency ω_2.