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Dynamics of self-similar waves in asymmetric twin-core fibers with Airy–Bessel modulated nonlinearity

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ABSTRACT

We explore the exact optical similaritons of a generalized nonlinear Schrödinger equation (GNLSE) with space–time modulated dispersion, nonlinearity, external potential and inhomogeneous source. It is shown here that this equation appertains to the description of wave propagation through asymmetric twin-core fibers in which we control the dynamics of the pulse propagating through passive fiber by controlling the dynamics of the self-similar wave propagating through the active fiber, due to the linear coupling between them. By utilizing multivariate similarity transformation, we map the nonautonomous GNLSE to standard NLSE with a homogeneous external source. Furthermore, by using Möbius transformation, we find periodic waves, solitary waves, and pure cnoidal and pure snoidal solutions as exact solutions. As an application, we explicate the mechanism to control the dynamical behaviors of these similaritons for a spatial Airy and Bessel modulated nonlinearity.

1. Introduction

By now, it has been well established that the nonlinear Schrödinger equation (NLSE) and its variants describe wave propagation through nonlinear optical fibers. Solitons which emerge as the solutions of NLSE, due to a delicate balance between nonlinearity and dispersion or diffraction, are considered to be the natural carriers of high-bit-rate information via long-haul telecommunication networks [1]. Recently, there is a tremendous interest in obtaining self-similar waves or similaritons for NLSE with distributive parameters [2–6]. Self-similar waves are similar to solitons with an added advantage of their modulating behaviors both in the width and amplitude as a function of the length of the fibers. More recently, there is a resurgence of interest in obtaining space–time modulated self-similar waves that propagate self-similarly subject to simple scaling rules in a nonlinear fiber [7–13].

Despite the fact that, it is easier to fabricate twin-core fibers (TCF) with some built-in asymmetry, for example, in the geometry and the material with which the cores of the two fibers (the core of the passive fiber may be made of a material with normal dispersion and the active core may be made of a material with anomalous dispersion) are made of, the study of nonlinear wave propagation in these types of couplers has not received wide attention in the literature. The author of the present paper has studied dynamics of self-similar waves in TCF, where the relevant equation is the NLSE interacting with an external source [14–18]. Additionally, nonautonomous matter waves in Bose–Einstein condensates interacting with a spatially modulated external source has been studied [19]. In this work, we intend to study the propagation of space–time modulated self-similar waves in an asymmetric twin core fiber, under Airy and Bessel modulated nonlinearity. The model equation is NLSE with space–time modulated dispersion, nonlinearity, external potential, and an external source. Utilizing a multivariate similarity transformation, we map the nonautonomous dynamical system to an autonomous one [20–24]. Very recently, many authors have mapped different variants of nonautonomous dynamical systems to autonomous ones, and found interesting localized waves [25–33]. Then by using one more transformation, we obtain an elliptic equation. With the aid of Möbius transformation, we find periodic, solitary wave, and pure cnoidal solutions as exact solutions. Although the method described here has wide-spread applicability for studying arbitrary modulations of the coefficients, in this work we confine our study to delineate in more detail the dynamics of the self-similar waves for which the nonlinearity is spatially modulated by Bessel functions or Airy functions. The interest in NLSEs involving Bessel functions either as a nonlinearity or as an external potential, or both, has picked up after the appearance of nondiffracting Bessel beams. The imperviousness of these beams to the diffraction or dispersion makes them as the much wanted wave packets in nonlinear optics, involving long-haul telecommunication networks, where nondiffracting localized beams—solitons—

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commonly exist.

2. Model equation

The generalized nonlinear Schrödinger equation with space–time modulated parameters may be written as

$$i\frac{\partial}{\partial z}\psi + \frac{1}{2}\beta(z,x)\frac{\partial^2}{\partial x^2}\psi + \gamma(z,x)|\psi|^2\psi + R(z,x)\psi + \eta(z,x)e^{i\Phi + iB(z,x)} = 0. \tag{1}$$

Here, $\beta(z,x)$ corresponds to the diffraction coefficient, $\gamma(z,x)$ the nonlinearity coefficient, $R(z,x)$ is the external potential, $\psi(z,x)$ corresponds to the complex envelope of the electric field, and $\eta(z,x)$ is the inhomogeneous source. Here, the phase Φ in the source term contains the phase part of the wave that is propagating through the passive fiber whose amplitude is contained in η. Thus, we can control the signal that is propagating through the passive fiber, by controlling the dynamical behavior of the signal propagating via active fiber. First, we shall analyze our results using the spatial modulation of the nonlinearity in the form

$$\gamma(x) = a_0 + b_0(Ai[-(x+c_0)])^2 + b_0(Ai[+(x+c_0)])^2, \tag{2}$$

where a_0, b_0 and c_0 are constants, and $Ai(x)$ is the Airy function. Such a choice is realistic, because it avoids the appearance of singularities. Next we shall consider the spatial modulation of the nonlinearity in the form

$$\gamma(x) = d_0 + J_n^2(x), \tag{3}$$

where d_0 is a constant, $J_n(x)$ is the n-th order Bessel function of first kind and n is an integer or half-integer. The parameters a_0, b_0, c_0 and d_0 facilitate control over nonlinearity.

3. Similariton wave packets in asymmetric twin-core fibers

In order to obtain the exact analytical solutions of Eq. (1), use shall be made of the following multivariate similarity transformation [3,10,13]:

$$\psi(z,x) = A(z,x)Q(z,X(z,x))e^{i[\Phi + B(z,x)]}, \tag{4}$$

where the multivariate self-similar variable $X(z,x)$, the amplitude AQ, and the combined phase $\Phi + B$ are all functions of z and x. Writing the amplitude of ψ as a product of two auxiliary functions allows for more freedom in the treatment of Eq. (1). The ultimate aim is to transform nonautonomous Eq. (1) into the standard NLS equation with constant coefficients. Substituting Eq. (4) into Eq. (1), we may extract the following equation which is the NLSE with a homogeneous external source:

$$i\frac{\partial}{\partial z}Q + \frac{1}{2}\frac{\partial^2}{\partial X^2}Q + \sigma|Q|^2Q + Ke^{i(X-vz)} = 0, \tag{5}$$

requiring that the following relations are satisfied:

$$\beta(z,x)X_x^2 = 1, \tag{6}$$

$$\chi(z,x)A^2 = \sigma, \tag{7}$$

and

$$\eta(z,x) = KA(z,x). \tag{8}$$

Eq. (8) indicates that the space–time modulated external source is proportional to the amplitude of the self-similar wave. Fig. 1 depicts this source for (a) Bessel nonlinearity for $n = 1, 2, 3$ and (b) Airy modulated nonlinearity. Now we separately assemble terms containing Q and Q_X from the remaining terms of the equation. Equating the coefficients of Q and Q_X to zero, we get following system of

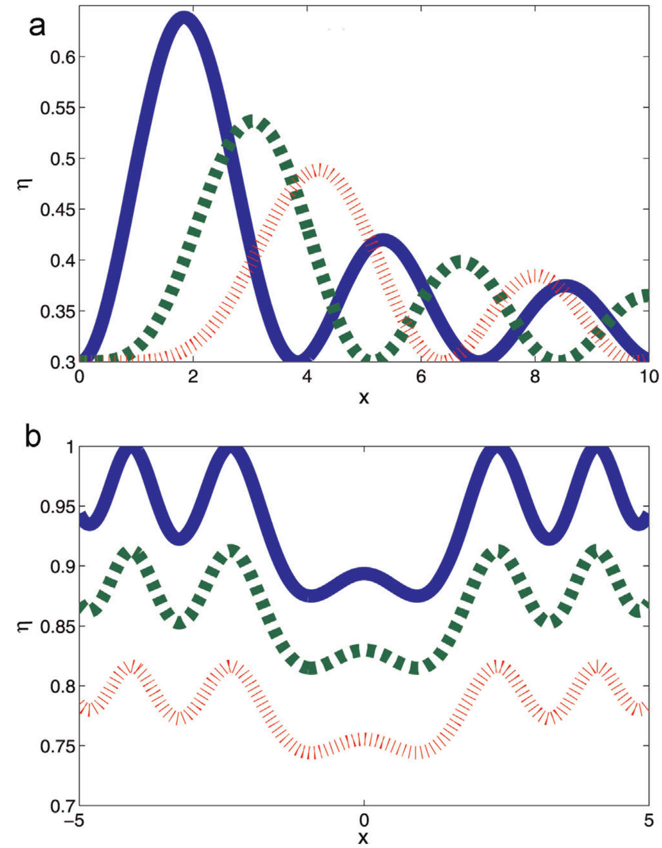

Fig. 1. Plot depicting the inhomogeneous source for (a) Bessel nonlinearity for $n = 1, 2, 3$ and (b) Airy nonlinearity.

$$\frac{\partial}{\partial x}\left(A^2\frac{\partial X}{\partial x}\right) = 0, \tag{9}$$

$$-\frac{\partial B}{\partial z} + \frac{\beta}{2A}\frac{\partial^2 A}{\partial x^2} - \frac{\beta}{2}\left(\frac{\partial B}{\partial x}\right)^2 + R = 0, \tag{10}$$

$$\frac{\partial B}{\partial x} = -\frac{1}{\beta}\frac{X_z}{X_x}, \tag{11}$$

$$\frac{\partial}{\partial z}A^2 + \beta\frac{\partial}{\partial x}\left(A^2\frac{\partial B}{\partial x}\right) = 0, \tag{12}$$

From Eq. (9) we find

$$A^2 = \frac{\lambda^2(z)}{X_x}, \tag{13}$$

where $\lambda(z)$ is an integration constant. Substituting Eq. (13) into Eq. (12) and also using Eq. (11) we obtain

$$\frac{\lambda_z}{\lambda} - \frac{X_x X_{xz}}{X_x^2} + \frac{\beta_x}{2\beta}\frac{X_z}{X_x} + \frac{X_z X_{xx}}{X_x^2} = 0. \tag{14}$$

Eq. (6) leads to

$$\frac{\beta_x}{2\beta} = -\frac{X_{xx}}{X_x}. \tag{15}$$

Substituting Eq. (15) into Eq. (14) we get