Optics Communications 345 (2015) 86–98

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Non-Gaussian features from excited squeezed vacuum state

Xu-bing Tang [a,b], Fang Gao [b], Yao-xiong Wang [b], Jian-guang Wu [a], Feng Shuang [b,c,*]

[a] School of Mathematics & Physics Science and Engineering, Anhui University of Technology, Ma'anshan 243032, China
[b] Institute of Intelligent Machines, Chinese Academy of Sciences, Hefei 230031, China
[c] Department of Automation, University of Science & Technology of China, Hefei 230027, China

ARTICLE INFO

Article history:
Received 30 November 2014
Received in revised form
16 January 2015
Accepted 19 January 2015
Available online 23 January 2015

ABSTRACT

In this work, we introduce a non-Gaussian quantum state named excited squeezed vacuum state (ESVS), which can be utilized to describe quantum light field emitted from the multiphoton quantum process occurred in some restricted quantum systems. We investigate its nonclassical properties such as Wigner distribution in phase space, photon number distribution, the second-order autocorrelation and the quadrature fluctuations. By virtue of the methods of Hilbert–Schmidt distance and quantum relative entropy (QRE), we quantify the non-Gaussianity of the ESVS, respectively. Due to the similar photon statistics, we examine the fidelity between the ESVS and the photon-subtraction squeezed vacuum state (PSSVS), and then find the optimal fidelity by monitoring the relevant parameters. Considering a thermal lossy channel, we examine the evolution of Wigner function for the ESVS.

1. Introduction

Current research suggests that non-Gaussian states, endowed with the qualitative role of quantum coherence or entanglement for revealing the fascinating quantum phenomena, can preserve their nonclassicality much better than Gaussian ones in quantum information process (QIP). Also non-Gaussian operations become an essential ingredient for some quantum tasks such as entanglement distillation [1,2] and noiseless amplification [3]. To say the least, non-Gaussian regime has powerfully extended to quantum information tasks, e.g. metrology [4], cloning [5], communication [6], computation [7,8] and testing of quantum theory [9].

In the frame of non-Gaussian mechanism, much attention has been focused on generation schemes [10–14], nonclassicality investigation [15–17], quantum protocols [18–20]. In general, due to the lack of high order nonlinearity, it is very difficult to deterministically generate non-Gaussian states of light via optical material media. Quantum systems with restricted dimensions have proved to be fertile ground for discovering non-Gaussian light. Fig. 1 is a sketch of a cavity QED system for heralded generating non-Gaussian states. In the presence of a magnetic field along the cavity axis, two nondegenerate spin ground states are coupled to an electronic excited state. Due to quantum noise in cavity, a small Faraday rotation occurs as the incident polarized photons passes through the cavity. By detecting photons with select polarization, one can generate a non-Gaussian states in those quantum system. Similar schemes have been proposed in Refs. [21,22]. In Ref. [23], an extra resonance has been observed as well as vacuum Rabi resonance. In a photonic crystal cavity containing a strongly coupled quantum dot, authors have discovered the photon-induced tunneling phenomena, which is a nonclassical transmitted light [24]. In circuit quantum electrodynamics (QED), the giant self-Kerr effect can be detected by measuring the second-order correlation function and quadrature squeezing spectrum [25]. In those coupled microscopic quantum systems, strong interactions can generate highly nonclassical light, which has possible uses in quantum communication [26] and metrology [27].

One important question that arises is how to describe quantum states of nonclassical light emitted from the restricted quantum system. Due to strong coupling, the composite system consisting of a multi-level atom (or quantum dot) coupled to a cavity and driven by a weak coherent field can be described as Jaynes–Cummings (JC) model. Quantum-optical effects can be demonstrated in the interaction processes of photon emission and absorbtion with atom between ground and excited states. Its energy-level structure is discrete, the so-called dressed states [28,29], which is an overall description of evaluation. Multi-photon processes originated from quantum nonlinearity can be monitored via the fluorescent resonance [30–32] and the other extra resonance [33]. Only considering an effective measurement to optical field,

* Corresponding author at: Institute of Intelligent Machines, Chinese Academy of Sciences, Hefei 230031, China.
E-mail addresses: ttxxbb@ahut.edu.cn (X.-b. Tang), fshuang@iim.ac.cn (F. Shuang).

http://dx.doi.org/10.1016/j.optcom.2015.01.053
0030-4018/

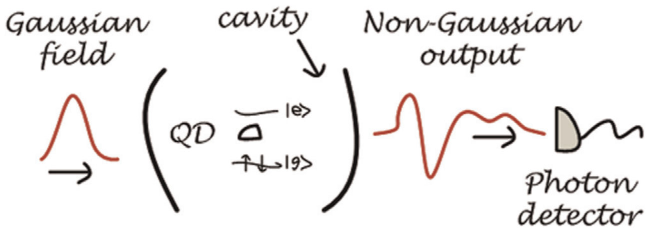

Fig. 1. Experimental script of a cavity QED system. A two-level quantum dot (QD) assembles in a optical harmonic cavity. Its dynamics processes can be described by Jaynes–Cummings (JC) model. Due to the strong coupling interaction between QD and the cavity, non-Gaussian field can be generated as the incident Gaussian field passes through the cavity.

photon-statistics methods [24], such as the second-order coherence function at time delay zero $g^{(2)}(0) = \langle a^{\dagger}a^{\dagger}aa\rangle / \langle a^{\dagger}a\rangle^2$ or high order differential correlation function $C^{(n)}(0) = \langle a^{\dagger n}a^n\rangle - \langle a^{\dagger}a\rangle^n$, are often utilized to study nonclassical characteristics of emitting light. For the case of weak coherent incident field, the quantum state of emitted light can be expressed as a series of excited coherent states $\sum_m C_m a^{\dagger m}|\alpha\rangle$ or a superposition of the different Fock states $|\psi\rangle = \sum_n C_n|n\rangle$ (due to $|\alpha\rangle = \sum_n (\alpha^n/\sqrt{n!})|n\rangle$) [24]. Its photon-statistics can exhibit similar behavior to that of an excited quantum state (e.g. $a^{\dagger m}|\varphi\rangle$).

It is interesting to consider a single-mode squeezed vacuum field to be an initial state of the incident source. Therefore, abundant nonclassicality of emitting field can be demonstrated by studying excited squeezed vacuum state (ESVS). In Ref. [34], considering the interaction of a two-level atom with a squeezed vacuum, authors have calculated the second-order intensity correlation function, the spectrum of squeezing, the coherent spectrum and discussed nonclassical behavior of light field. Similar issues have been addressed in Refs. [35–40]. A recent experimental study found a twofold reduction of the transverse radiative decay rate of a superconducting artificial atom coupled to continuum squeezed vacuum [41]. More attention has been paid to investigate Wigner function and tomogram of ESVS in Refs. [42,15–17]. In this work, we focus on a single-mode squeezed vacuum field with any number of photon addition, and investigate its nonclassical properties. In Section 2, by virtue of quantum phase space technique, we derive an analytical expression of quasi-probability distribution Wigner function, negativity of which can exhibit nonclassical behavior of the ESVS. And then we investigate its photon number statistics, calculate the Mandel's Q parameter, examine the quadrature fluctuations $\langle\Delta X\rangle$, $\langle\Delta Y\rangle$ and the correlation $\langle\Delta X\rangle\langle\Delta Y\rangle$, which can be measured outside the cavity by using a homodyne detection with a controllable phase. In Section 3, we evaluate its non-Gaussianity via methods of the Hilbert–Schmidt distance (HSD) [43] and the quantum relative entropy (QRE), respectively. As results, we shall study how the photon number modulation affects the non-Gaussianity of the ESVS and provide a guide to enhance non-Gaussianity of a desired quantum state. It is found that photon number distribution of the ESVS is similar to that of the photon-subtraction squeezed vacuum state (PSSVS). Fidelity between the ESVE and the PSSVS is obtained and the optimal fidelity has been discussed in Section 4. Considering a thermal lossy channel, we discuss the evolution of Wigner function for ESVS in Section 5. We end with the main conclusions of our work.

2. Nonclassical properties investigation of the ESVS

As is well known, a single mode squeezed field is an approximation with a superposition of all even number photon states, i.e.,

$$S(r)|0\rangle = \exp\left[\frac{r}{2}(a^{\dagger 2} - a^2)\right]|0\rangle = \frac{1}{\sqrt{\cosh r}}\sum_{n=0}(-1)^n\frac{\sqrt{(2n)!}}{2^n n!}\tanh^n r|2n\rangle, \tag{1}$$

in which r is the squeezing parameter. Adding one single photon to a weak squeezed field can be described by

$$a^{\dagger}S(r)|0\rangle \to \sum_{n=0}(-1)^n\frac{\sqrt{(2n+1)!}}{2^n n!}\tanh^n r|2n+1\rangle, \tag{2}$$

which is a superposition of all odd number photon states. Theoretically, by repeating (n times) application of the photon creation operator $a^{\dagger}$ on the squeezed vacuum field, we can obtain the excited squeezed vacuum state (ESVS) $a^{\dagger n}S(r)|0\rangle$. For the case of small value of n, the ESVS can be generated via the spontaneous parametric down-conversion (SPDC) occurred in nonlinear optical media or the multiphoton processes in the above-mentioned restricted quantum systems. Thus its density operator reads

$$\rho(r, n) = C_n^{-1}a^{\dagger n}S(r)|0\rangle\langle 0|S^{-1}(r)a^n \equiv C_n^{-1}a^{\dagger n}\rho(r)a^n, \tag{3}$$

in which $\rho(r) \equiv S(r)|0\rangle\langle 0|S^{-1}(r)$ denotes the squeezed vacuum field, $C_n = \mathrm{Tr}[\rho(r, n)] = n!\cosh^n r P_n(\cosh r)$ is a normalized constant. $P_n(\cosh r)$ is the expression of the Legendre polynomials and this result has also obtained in Ref. [42].

2.1. Quasi-probability distribution: Wigner function

In order to interview the nonclassical properties of a quantum light fields, the Wigner function, although not positive definition in general, provides a closely parallel interpretation as a probability distribution function. Based on the Weyl's mapping rule [46,47], the classical correspondence of density operator ρ is just the Wigner function, namely

$$\rho = \iint_{-\infty}^{\infty} dq\, dp\, \Delta(q, p)W(q, p) \tag{4}$$

or

$$W(q, p) = \mathrm{Tr}[\rho\Delta(q, p)], \tag{5}$$

$W(q, p)$ is the Wigner function of ρ and $\Delta(q, p)$ denotes the Wigner operator, defined in the coordinate representation $|q\rangle$ as

$$\Delta(p, q) = \int_{-\infty}^{\infty}\frac{dv}{2\pi}e^{ipv}\left|q + \frac{v}{2}\right\rangle\left\langle q - \frac{v}{2}\right|. \tag{6}$$

Noting that $\alpha = \frac{1}{\sqrt{2}}(q + ip)$, we can see $W(\alpha, \alpha^*) = \mathrm{Tr}[\rho\Delta(\alpha, \alpha^*)]$, where

$$\Delta(\alpha, \alpha^*) = \int\frac{d^2z}{\pi^2}|\alpha + z\rangle\langle\alpha - z|e^{\alpha z^* - \alpha^* z} = \frac{1}{\pi}:\exp[-2(a^{\dagger} - \alpha^*)(a - \alpha)]: = \frac{1}{2}\,\vdots\,\delta(\alpha - a)\delta(\alpha^* - a^{\dagger})\,\vdots, \tag{7}$$

$\langle z| = \langle 0|\exp\left[-\frac{1}{2}|z|^2 + z^* a\right]$ is the Glauber coherent state [48], the symbols ‘: :’ and ‘$\vdots\ \vdots$’ denote the normal ordering and Weyl ordering, respectively. In particular, a unitary operator (e.g. S with its identities $SaS^{-1} = \mu a + \nu a^{\dagger}$, $Sa^{\dagger}S^{-1} = \sigma a + \tau a^{\dagger}$) can ‘run across’ the ‘border’ of ‘$\vdots\ \vdots$’ and directly transforms bosonic operators, i.e.

$$SF(a^{\dagger}, a)S^{-1} = F(\mu a + \nu a^{\dagger}, \sigma a + \tau a^{\dagger}) = \vdots\, f(\mu a + \nu a^{\dagger}, \sigma a + \tau a^{\dagger})\,\vdots, \tag{8}$$