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Complementarity in temporal ghost interference and temporal quantum eraser

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ABSTRACT

We present a theory for the complementarity in temporal interference and quantum erasure. We consider the case of entangled biphoton where we can get the information of single photon's arrival time without making a disturbing measurement. We find a mathematical equation for the complementary relation for a temporal double slit experiment. We also propose a quantum eraser scheme that will elucidate that the complementarity is originated from the quantum entanglement.

1. Introduction

The complementarity regarding the particle nature and the wave nature of a quantum particle has been debated since the beginning of quantum mechanics. A particle going through a double slit is a typical example of this complementarity [1]. It is well known that if one can find which slit the particle has gone through in the double slit experiment (which-path information), then there will be no interference pattern at the measurement screen. It has been argued that the measurement yielding the which-path information reveals the particle nature of the quantum particle, which destroys the interference. In earlier days, unavoidable disturbance to the particle's state associated with any physical measurement was to blame for the disappearance of interference, where the unavoidable disturbance is related to the uncertainty principle [2,3]. However, it is claimed later that a mere possibility of obtaining which-path information can destroy the interference, without any actual physical measurement on the particle. It was argued that the entanglement between the particle and the measurement device is the key to the disappearance of interference [4–6].

Further studies show that a partial, not complete, information of which-path can reduce the visibility of the interference pattern. For instance, a complementary relation $D^2 + V^2 = 1$ between the distinguishability D and the visibility V was shown to hold in a Mach–Zehnder type experiment [7].

Moreover, the quantum eraser that erases the which-path information to restore the interference was theoretically predicted [4,5] and successfully tested in several experiments [6–13].

Most of th Prod. Type: FTPe former works mentioned above have used the arrangements where particle's spatial path is distinguished. However, we notice that a complementarity exists in time domain also. A classical particle traveling in one dimensional space arrives at a particular position at a particular time, while a wave arriving at a position has finite time duration. Therefore a study about how the information of particle's arrival time is related to the coherence of the wave representing the quantum particle will give us some insights about the complementarity in time domain. In former studies on time domain, coherent pump beams have been used to generate biphoton wave packets. Two biphoton wave packets created by two coherent pump pulses were used to measure the one-photon and two-photon interferences with a narrow bandpass filter [14,15]. In another scheme, where a double pass pump beam was used, temporal interference was observed when the temporal distinguishability was erased by aligning the beam paths carefully [16]. Wigner functions of the quadrature field were also measured in the two biphotons created by two pump pulses [16,17].

In contrast to these former studies, we use a CW laser to generate biphoton wave packet and then create two indistinguishable

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signal photon wave packets by using a Michelson interferometer. The purpose of using this scheme is, first, to study how the temporal coherence changes when we change the experimental set-up to reveal the temporal information. And, secondly, we want to know whether the measurement disturbs the photon state or the mere possibility of measurement changes the temporal coherence. In our scheme, only one photon in one photon pair goes through the Michelson interferometer and we can get the temporal information of the other photon in nonlocal way. This allows us to study the change of coherence without the disturbance. We are going to consider the case of entangled photons where we can get the information of the single photon's arrival time without making a disturbing measurement. We will show that the complementary relation holds in temporal double slit experiment. We will also propose the quantum eraser scheme that will show that complementarity is originated from the quantum entanglement.

2. Theory

Let's start with single photon and study temporal distinguishability and visibility. Consider a wave packet that has well defined duration time. If we send the wave packet into a dispersive medium, the temporal width of the wave packet will be broadened as the wave packet propagates. If we send two identical wave packets in a row with small time interval between the two, as shown in Fig. 1, then the two wave packets will spread in time as they propagate and then start to overlap to each other. Interference will occur for the two pulses separated less than the longitudinal coherence length. If the two pulses have been obtained from the same pulse by splitting the original pulse with a beam splitter and recombine them with certain time delay, then the temporal coherence in the original pulse will play the role of longitudinal coherence. In terms of quantum theory, the overlap of two pulses makes the arrival time of photons indistinguishable so that the interference occurs.

We notice that the spreading of a pulse in a dispersive medium is mathematically identical to the diffraction of wave front in space [18], so that the pulse spreading is sometimes called a temporal diffraction. Therefore, we can make an arrangement for a temporal Young's double slit experiment.

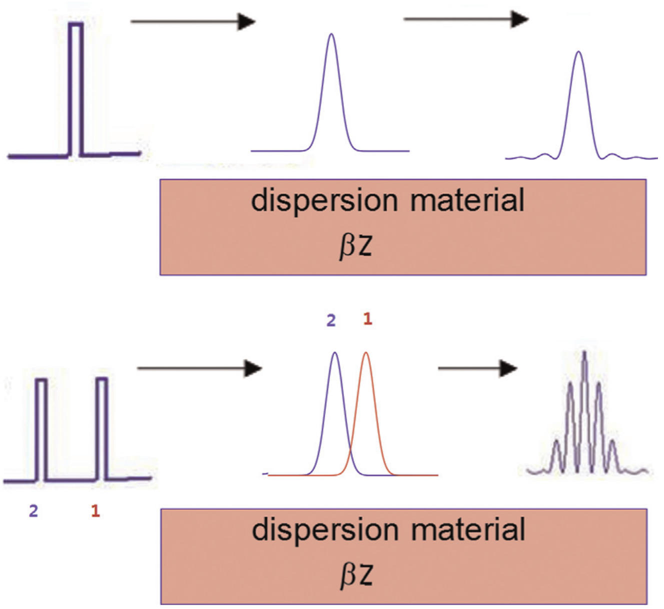

Fig. 1. (a) A single photon wave packet spreads as the wave packet propagates in a dispersive medium. (b) Two pulses overlap to each other as they spread in time and make interference. β is a dispersion coefficient, and z is the traveling distance.

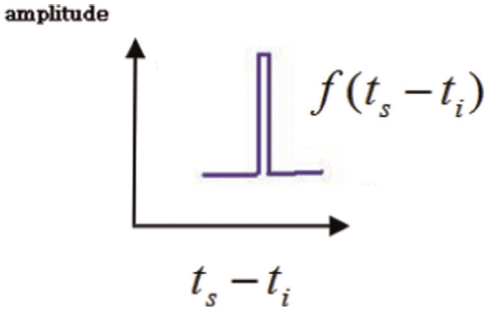

Fig. 2. Time correlation of the biphoton wave packet generated from SPDC. The signal and the idler photon have a very narrow width time correlation in the order of 10–100 fs.

In a real situation, a well defined single photon wave packet can be obtained from an entangled two photons generated from a spontaneous parametric down conversion (SPDC) process. It is well known that a time–energy entangled state, as well as the position–momentum entangled state, can be produced from SPDC. This entangled two-photon state is known as the biphoton wave packet which has temporal amplitude distribution depending on the dispersion characteristics of the nonlinear crystal used in SPDC and some geometrical conditions [19]. The signal and the idler photons generated from SPDC have very narrow time correlation function whose width is in the order of 10–100 fs, as shown in Fig. 2.

The two-photon photo detection amplitude can be written as [18,19]

$$\langle 0|\hat{E}_s^{(+)}(t_s)\hat{E}_i^{(+)}(t_i)|\Psi\rangle = e^{-i\frac{\omega p}{2}(t_s+t_i)} f(t_s - t_i), \tag{1}$$

where $|\Psi\rangle$is the entangled two-photon quantum state, t_s,t_i are the measurement times for the signal and the idler photon at the crystal, respectively. In other words, t_s,t_i are the generation times for the signal and idler photons. ω_p is the frequency of pump beam. After the signal and the idler photons travel through free space, the two-photon wave function preserves its functional form but the phase factor changes due to the time delay. For instance, let the measurement times for the signal and the idler photons after the propagation to the locations z_s, z_i to be $t_s{}'$,$t_i{}'$. Then the two-photon detection amplitude becomes

$$\begin{aligned}
&\langle 0|\hat{E}_s^{(+)}(z_s, t_s{}')\hat{E}_i^{(+)}(z_i, t_i{}')|\Psi\rangle \\
&\quad = e^{i\left(k_{s0}z_s - \frac{\omega p}{2}t_s{}'\right)} e^{i\left(k_{i0}z_i - \frac{\omega p}{2}t_i{}'\right)} \langle 0|\hat{E}_s^{(+)}(0, t_s{}' - \frac{z_s}{c})\hat{E}_i^{(+)}(0, t_i{}' \\
&\quad\quad - \frac{z_i}{c})|\Psi\rangle \\
&\quad = e^{-i\frac{\omega p}{2}\left(t_{s\prime} - \frac{z_s}{c} + t_{i\prime} - \frac{z_i}{c}\right)} f(t_s{}' - \frac{z_s}{c} - (t_i{}' - \frac{z_i}{c})) \\
&\quad = e^{-i\frac{\omega p}{2}\left(t_{s\prime} - \frac{z_s}{c} + t_{i\prime} - \frac{z_i}{c}\right)} f(t_s{}' - t_i{}' - t_{si})\Big(k_{s0} \\
&\quad = k_{i0} \\
&\quad = \frac{\omega_p}{2c}, t_{si} \\
&\quad = \frac{z_s - z_i}{c}\Big).
\end{aligned} \tag{2}$$

Let's consider the case shown in Fig. 3(a), where we put a beam splitter for the idler photon in order to make a short path and a long path for the idler photon to take. Due to the traveling time