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Scattering analysis of Bessel beam by a multilayered sphere

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ABSTRACT

The scattering of an on-axis zero-order vector Bessel beam by a multilayered sphere is investigated within the generalized Lorenz–Mie theory. The detailed theoretical process of the scattering theory for the Bessel beam by a multilayered is presented, with special attention paid to the far-field scattering intensity. The present method is proved to be effective by the comparison of our results and the numerical results obtained from the surface integral equation method. Numerical results concerning the influences of different parameters of the incident Bessel beam and of the scattering body on the scattering intensity are displayed in detail. The study is expected to provide key support for understanding the interactions between Bessel beams and multilayered spheres.

1. Introduction

The interaction between light beam and various small particles is an important subject for many researchers, which has been widely applied in many fields, such as remote sense, communication, particle capture, and aerosol detection. During the past few decades, the scattering calculation for a multilayered sphere has attracted the attention of many investigators, due to the fact that many particles encountered in nature or produced in industrial processes can be modeled by multilayered spheres. The scattering theory of a plane electromagnetic wave by a concentric sphere was firstly derived by Aden and Kerker [1]. In this original study of multilayer sphere scattering, the electromagnetic fields were expanded by spherical vector wave functions [2], which are based on Lorenz–Mie theory. Moreover, the far zone scattering field, the total scattering cross section, and the backscattering cross section were given in detail. Since then, based on the method of Aden and Kerker, many subsequent researches including proposal or improvement of algorithms for multilayered sphere scattering and application of the corresponding theory have been reported. Toon and Ackerman [3], Bhandari [4], Wu and Wang [5], Yang [6], and Johnson [7] studied the computation methods of the multilayered sphere scattering, respectively. Moreover, Fenn and Oser [8] studied the scattering properties of a concentric sphere by visible and infrared light. Especially, according to generalized Lorenz–Mie theory (GLMT), Wu et al. [9] extended the scattering calculation of plane waves for multilayered spheres to the case of Gaussian beams for multilayered spheres. Later, Li et al. [10] researched the scattering of a multilayered sphere by Gaussian beam, using Debye series unlike the expansion of spherical vector wave functions.

As reviewed above, many studies have been carried out on the scattering of plane waves or Gaussian beams for a multilayered sphere. Recently, Bessel beam, first introduced by Durnin [11,12], has attracted the attention of many researchers as a result of its special characteristics of non-diffraction and self-reconstruction. Several researchers have been devoted to the experimental generations of Bessel beams [13–15], and the light scattering analyses of Bessel beams by homogeneous dielectric particles. Specifically, the scattering of a zero-order Bessel beam [16] and a high-order Bessel vortex beam [17] by a homogeneous water sphere were firstly studied by Mitri. In these researches, the incident beam was expanded as partial wave series involving the beam shape coefficients calculated using the method of integration. The scattering of an unpolarized Bessel beam by spheres was investigated by Ma and Li [18]. In addition, in their research, the incident Bessel beam was expanded in terms of spherical vector wave functions and the dimensionless scattering function was analyzed. Then, Li et al. [19] presented the far-field scattering result of an axicon-generated Bessel beam for a sphere by combining GLMT with Debye series. Furthermore, the scattering of Bessel beam by a uniaxial anisotropic sphere was analyzed by Qu et al. [20]. And more recently, our research group has also studied the scattering of Bessel beam by a concentric sphere [21] and a conducting spheroidal particle with dielectric coating [22]. To the best of our knowledge, the scattering of Bessel beam by a multilayered sphere has not been reported. Considering that the scattering researches on

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multilayered spheres may have some significance in guiding practice, the purpose of this paper is to study the scattering of Bessel beam by a multilayered sphere by using the GLMT.

The body of this paper is organized as follows. In Section 2 we give the detailed theoretical process of the scattering for a multilayered sphere by Bessel beam. The incident fields, the scattering fields and the internal fields in different regions are expanded in terms of spherical vector wave functions. In Section 3 we analyze the influences of different parameters of the incident Bessel beam and of the scatter on the scattering intensity. Finally, the conclusion of this paper is given in Section 4.

2. Theory background

In this section, the exact scattering theory for a multilayered dielectric sphere, located on the beam axis, by a Bessel beam is presented on the basis of GLMT. The geometry of the problem is shown in Fig. 1, in which m_l and R_l are the refractive index of the medium in the lth layer relative to the refractive index of the surrounding medium and the outer radius of the lth layer, respectively. The size parameter of lth layer is characterized by $x_l = 2\pi R_l/\lambda, l = 1, 2, \ldots, L$, where λ is the wavelength of the incident wave and the layer number is assumed to L. And the magnetic permeability of every layer for a multilayered sphere is assumed to have the free-space value $\mu = \mu_0$. Two Cartesian coordinates $O_b uvw$ and $oxyz$ are defined, the centers of which are located at the center of beam and of the multilayered sphere, respectively. The coordinates of O_b in $oxyz$ are set as $(0, 0, z_0)$. In other words, the relationship between $O_b uvw$ and $oxyz$ can expressed as

$$x = u, \quad y = v, \quad z = w - z_0. \tag{1}$$

For the analysis of the scattering theory, it is important to chance an appropriate vector expression of the incident beam. In 1991, Mishra [23] presented such vector expressions of a zero-order Bessel beam that satisfied Maxwell's equations. Apparently, it is suitable to adopt these expressions of Bessel beam for our research. The Cartesian components of the incident electric and magnetic fields of Bessel beam can read, respectively, as

$$E_u = \frac{1}{2}E_0\left[\left(1 + \frac{k_w}{k} - \frac{k_r^2u^2}{k^2r^2}\right)J_0(k_r r) - \frac{k_r(v^2 - u^2)}{k^2r^3}J_1(k_r r)\right]e^{ik_w w}, \tag{2}$$

$$E_v = \frac{1}{2}E_0 uv\left[\frac{2k_r}{k^2r^3}J_1(k_r r) - \frac{k_r^2}{k^2r^2}J_0(k_r r)\right]e^{ik_w w}, \tag{3}$$

$$E_w = \frac{1}{2i}E_0\frac{u}{kr}\left(1 + \frac{k_w}{k}\right)k_r J_1(k_r r)e^{ik_w w}, \tag{4}$$

$$H_u = \frac{\sqrt{\varepsilon}}{2}E_0 uv\left[\frac{2k_r}{k^2r^3}J_1(k_r r) - \frac{k_r^2}{k^2r^2}J_0(k_r r)\right]e^{ik_w w}, \tag{5}$$

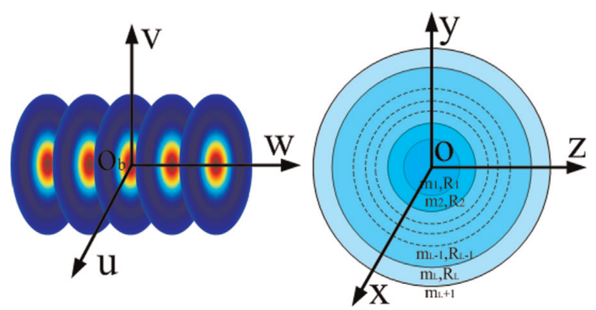

Fig. 1. Geometry for the scattering of Bessel beam by a multilayer sphere.

$$H_v = \frac{\sqrt{\varepsilon}}{2}E_0\left[\left(1 + \frac{k_w}{k} - \frac{k_r^2v^2}{k^2r^2}\right)J_0(k_r r) - \frac{k_r(u^2 - v^2)}{k^2r^3}J_1(k_r r)\right]e^{ik_w w}, \tag{6}$$

$$H_w = \frac{\sqrt{\varepsilon}}{2i}E_0\frac{v}{kr}\left(1 + \frac{k_w}{k}\right)k_r J_1(k_r r)e^{ik_w w}, \tag{7}$$

in which $J_{0,1}(\cdot)$ is the cylindrical Bessel function of the first kind of the zeroth, and first orders, respectively, E_0 and ε are the amplitude of the electric field strength and permittivity of the medium, respectively, and

$$k_r = k \sin\alpha, \tag{8}$$

$$k_w = k \cos\alpha, \tag{9}$$

$$r = \sqrt{u^2 + v^2}, \tag{10}$$

where the parameters k_r and k_w are the transversal and longitudinal components of the wave number k, and α is the half-cone angle of the incident Bessel beam. It is noted that the assuming time-harmonic factor of electromagnetic field is $\exp(-i\omega t)$, which is omitted for brevity in this paper.

In an analogous manner of the incident beam expansion described in our previous papers [23,24], the electric field of the incident on-axis Bessel beam can be expanded in terms of spherical vector wave functions $\overrightarrow{M}^{(j)}_{^o_emn}$ and $\overrightarrow{N}^{(j)}_{^o_emn}$ [2] (The index $j = 1, 2, 3, 4$ corresponds to the sphere Bessel functions of the first, second, third and fourth kind, respectively.) in $oxyz$ as

$$\vec{E}^i = \sum_{n=1}^{\infty} E_n g_n\left[\overrightarrow{M}^{(1)}_{o1n}(kR, \theta, \phi) - i\overrightarrow{N}^{(1)}_{e1n}(kR, \theta, \phi)\right], \tag{11}$$

where

$$g_n = \frac{1}{2}e^{ik_w z_0}\left[\left(1 + \frac{k_z}{k} - \frac{k_r^2}{2k^2}\right)J_0(\rho) + \frac{k_r^2}{2k^2}J_2(\rho)\right], \tag{12}$$

$$\rho = \left(n + \frac{1}{2}\right)\sin\alpha, \tag{13}$$

$$E_n = E_0 i^n\frac{2n + 1}{n(n + 1)}. \tag{14}$$

The coefficients g_n are known as the beam shape coefficients in the shaped beam scattering theory and the corresponding calculation. And these coefficients g_n are equal to one when the incident beam is a plane wave.

Also, the magnetic field of the incident on-axis Bessel beam, the electromagnetic field in the core region of the multilayered sphere, and the scattering electromagnetic field can be expressed, respectively, as

$$\vec{H}^i = -\frac{k}{\omega\mu}\sum_{n=1}^{\infty} E_n g_n\left[\vec{M}^{(1)}_{e1n}(kR, \theta, \phi) + i\vec{N}^{(1)}_{o1n}(kR, \theta, \phi)\right], \tag{15}$$

$$\vec{E}^1 = \sum_{n=1}^{\infty} E_n\left[C_n\overrightarrow{M}^{(1)}_{o1n}(m_1kR, \theta, \phi) - iD_n\overrightarrow{N}^{(1)}_{e1n}(m_1kR, \theta, \phi)\right], \tag{16}$$

$$\vec{H}^1 = -\frac{k_1}{\omega\mu}\sum_{n=1}^{\infty} E_n\left[D_n\overrightarrow{M}^{(1)}_{e1n}(m_1kR, \theta, \phi) + iC_n\overrightarrow{N}^{(1)}_{o1n}(m_1kR, \theta, \phi)\right], \tag{17}$$

$$\vec{E}^s = \sum_{n=1}^{\infty} E_n\left[iA_n\overrightarrow{N}^{(3)}_{e1n}(kR, \theta, \phi) - B_n\overrightarrow{M}^{(3)}_{o1n}(kR, \theta, \phi)\right], \tag{18}$$