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Theoretical investigation on the third harmonic generation by double resonances in waveguide directional couplers

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ABSTRACT

The cascaded third harmonic generation in waveguide directional couplers is analyzed theoretically. It is found that the phase-matching conditions for the cascaded third harmonic generation can be realized by the double resonances of the directional coupler. The core thickness and the internal separation of the two waveguides in the directional coupler can be adjusted to meet the resonance conditions. As an example, we have designed a directional coupler structure for efficient cascaded THG at 0.5167 μm, and the optimized parameters for the structure have been obtained. The results above have been well verified by numerical calculations.

1. Introduction

In recent years, much attention has been concentrated on nonlinear optical effects for frequency conversion, such as second-harmonic generation (SHG) and third-harmonic generation (THG). It is well known the phase-matching conditions are generally required for efficient nonlinear optical processes. The intensity of the harmonic wave can be enhanced only when proper phase-matching conditions are satisfied. Many different phase-matching techniques have been proposed, such as the birefringent-phase-matching (BPM) and the quasi-phase-matching (QPM) [1,2]. In addition, the study of nonlinear optical effects has been extended from bulk crystals to waveguides and coupled waveguides [3]. In 2004, Dong et al. investigated the SHG process in a waveguide directional coupler and pointed out that in this situation the required phase-matching condition can be replaced by a resonance condition of the waveguides [4]. Compared with the conventional QPM, this method has significant advantages on fabrication since no domain poling is required.

It is expected that this method can be extended to more complicated nonlinear processes such as the cascaded THG (CTHG). In a CTHG process, the third harmonic wave is achieved by the cascading of a SHG process and a sum-frequency generation (SFG) process, thus two phase-matching conditions are required to be satisfied simultaneously. In ref. [5], it shows that the phase-matching conditions for the CTHG in waveguide directional couplers can be satisfied by the combination of the waveguide resonance and the conventional QPM. This is an obvious improvement of phase-matching techniques in waveguides. However, with this approach the fabrication simplicity of the original method is lost since domain poling is still needed. In current paper, another method is proposed to realize efficient CTHG process in coupled waveguides. We found that the phase-matching conditions for the CTHG can be replaced by the double resonance conditions of the waveguides, which can be realized by simply adjusting the core thickness and the internal separation of the waveguides, thus the advantage of no domain poling is kept.

2. Theoretical model

Consider a waveguide directional coupler without any domain structure. Assuming that the nonlinear effect and the coupling between two waveguides do not significantly affect the waveguide modes, the electric fields in the coupled waveguides can be considered as the sum of the eigenmodes of each waveguide [6,7]. Supposing that $A_j(z)$ and $B_j(z)(j = \omega, 2\omega, 3\omega)$ are the slowly-varying amplitudes of the fundamental and harmonic waves in the two waveguides, the following coupled equations for the CTHG can be obtained [4,5]:

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$$\begin{cases} i\dfrac{dA_\omega}{dz} = \kappa_\omega B_\omega \exp(-i\Delta\beta_\omega z) + \eta^{2\omega}A_{2\omega}A_\omega^* \exp(-i\Delta k_1 z) \\ \qquad + \eta^{3\omega}A_{3\omega}A_{2\omega}^* \exp(-i\Delta k_2 z) \\ i\dfrac{dB_\omega}{dz} = \kappa_\omega A_\omega \exp(i\Delta\beta_\omega z) \\ \qquad + \eta^{2\omega}B_{2\omega}B_\omega^* \exp(-i\Delta k_1 z) + \eta^{3\omega}B_{3\omega}B_{2\omega}^* \\ \qquad \exp(-i\Delta k_2 z) \\ i\dfrac{dA_{2\omega}}{dz} = \kappa_{2\omega}B_{2\omega} \exp(-i\Delta\beta_{2\omega} z) + \eta^{2\omega}A_\omega^2 \exp(i\Delta k_1 z) \\ \qquad + 2\eta^{3\omega}A_{3\omega}A_\omega^* \exp(-i\Delta k_2 z) \\ i\dfrac{dB_{2\omega}}{dz} = \kappa_{2\omega}A_{2\omega} \exp(i\Delta\beta_{2\omega} z) \\ \qquad + \eta^{2\omega}B_\omega^2 \exp(i\Delta k_1 z) + 2\eta^{3\omega}B_{3\omega}B_\omega^* \\ \qquad \exp(-i\Delta k_2 z) \\ i\dfrac{dA_{3\omega}}{dz} = \kappa_{3\omega}B_{3\omega} \exp(-i\Delta\beta_{3\omega} z) + 3\eta^{3\omega}A_\omega A_{2\omega} \\ \qquad \exp(i\Delta k_2 z) \\ i\dfrac{dB_{3\omega}}{dz} = \kappa_{3\omega}A_{3\omega} \exp(i\Delta\beta_{3\omega} z) + 3\eta^{3\omega}B_\omega B_{2\omega} \exp(i\Delta k_2 z) \end{cases} \tag{1}$$

Here, κ_ω, $\kappa_{2\omega}$, $\kappa_{3\omega}$ are the mode-coupling coefficients of the directional coupler for fundamental, second- and third-harmonic waves, respectively. η^j and β_j are considered as the corresponding nonlinear optical constants and propagation constants. $\Delta\beta_j$ donates the phase mismatching between two guided modes, which can be ignored under the condition that the directional coupler consists of two identical waveguides. That is to say $\Delta\beta_\omega = \Delta\beta_{2\omega} = \Delta\beta_{3\omega} = 0$. Δk_1 and Δk_2 are the two phase mismatches in the CTHG process, which are defined as $\Delta k_1 = \beta_{2\omega} - 2\beta_\omega$ and $\Delta k_2 = \beta_{3\omega} - \beta_{2\omega} - \beta_\omega$ [8].

The phase-matching conditions required in directional couplers can be deduced from Eq. (1). Ref. [5] has discussed the situation where both of the SHG and the SFG process are phase-matched by the combination of waveguide resonance and the conventional QPM. Up to six possible phase-matching conditions for the CTHG have been obtained and are classified into three types. In current paper, we will analyze Eq. (1) in more detail. A new type of phase-matching condition is deduced where the intensity of the CTHG is also increasing on distance but at a lower speed. This result might be regarded as a complementary to the phase-matching conditions listed in ref. [5].

With the small-signal approximation, where the depletion of the fundamental wave is negligible, Eq. (1) can be simplified as

$$\begin{cases} \dfrac{dA_\omega}{dz} = -i\kappa_\omega B_\omega \\ \dfrac{dB_\omega}{dz} = -i\kappa_\omega A_\omega \\ \dfrac{dA_{2\omega}}{dz} = -i\kappa_{2\omega}B_{2\omega} - i\eta^{2\omega}A_\omega^2 \exp(i\Delta k_1 z) \\ \dfrac{dB_{2\omega}}{dz} = -i\kappa_{2\omega}A_{2\omega} - i\eta^{2\omega}B_\omega^2 \exp(i\Delta k_1 z) \\ \dfrac{dA_{3\omega}}{dz} = -i\kappa_{3\omega}B_{3\omega} - 3i\eta^{3\omega}A_\omega A_{2\omega} \exp(i\Delta k_2 z) \\ \dfrac{dB_{3\omega}}{dz} = -i\kappa_{3\omega}A_{3\omega} - 3i\eta^{3\omega}B_\omega B_{2\omega} \exp(i\Delta k_2 z) \end{cases} \tag{2}$$

To go a step further, with the initial conditions of $A_\omega(0) = A_0$, $B_\omega(0) = 0$ and $A_{2\omega}(0) = 0$, $B_{2\omega}(0) = 0$, and under the condition that the SHG process is phase-matched, Eq. (2) can be partially solved and the following equation for $A_{3\omega}$ can be obtained (the equation for $B_{3\omega}$ also has a similar form):

$$\begin{aligned} \frac{d^2A_{3\omega}}{dz^2} = {} & -\kappa_{3\omega}^2 A_{3\omega} - \frac{3i\eta^{2\omega}\eta^{3\omega}A_0^3}{4}(\Delta k_2 + \kappa_{2\omega} + \kappa_{3\omega} - \kappa_\omega) \\ & z\exp[i(\Delta k_2 + \kappa_{2\omega} + \kappa_\omega)z] - \frac{3i\eta^{2\omega}\eta^{3\omega}A_0^3}{4}(\Delta k_2 + \kappa_\omega + \kappa_{2\omega} - \kappa_{3\omega}) \\ & z\exp[i(\Delta k_2 + \kappa_{2\omega} - \kappa_\omega)z] - \frac{3\eta^{2\omega}\eta^{3\omega}A_0^3}{8} \\ & \exp[i(\Delta k_1 + \Delta k_2 + 3\kappa_\omega)z] - \frac{3\eta^{2\omega}\eta^{3\omega}A_0^3}{8} \\ & \exp[i(\Delta k_1 + \Delta k_2 - 3\kappa_\omega)z] - \frac{9\eta^{2\omega}\eta^{3\omega}A_0^3}{8} \\ & \exp[i(\Delta k_1 + \Delta k_2 + \kappa_\omega)z] - \frac{9\eta^{2\omega}\eta^{3\omega}A_0^3}{8} \\ & \exp[i(\Delta k_1 + \Delta k_2 - \kappa_\omega)z] \end{aligned} \tag{3}$$

Eq. (3) can be considered as a modified harmonic oscillator equation. It has a similar form as the corresponding equation governing $A_{2\omega}$ in the SHG process discussed intensively in ref. [4]. This equation can be further solved and the exact phase-matching conditions for the CTHG can be derived accordingly.

For example, under the resonance condition that $\Delta k_1 = \kappa_{2\omega}$, which is one of the phase-matching conditions for SHG, there are two possible resonance conditions which result in phase-matching

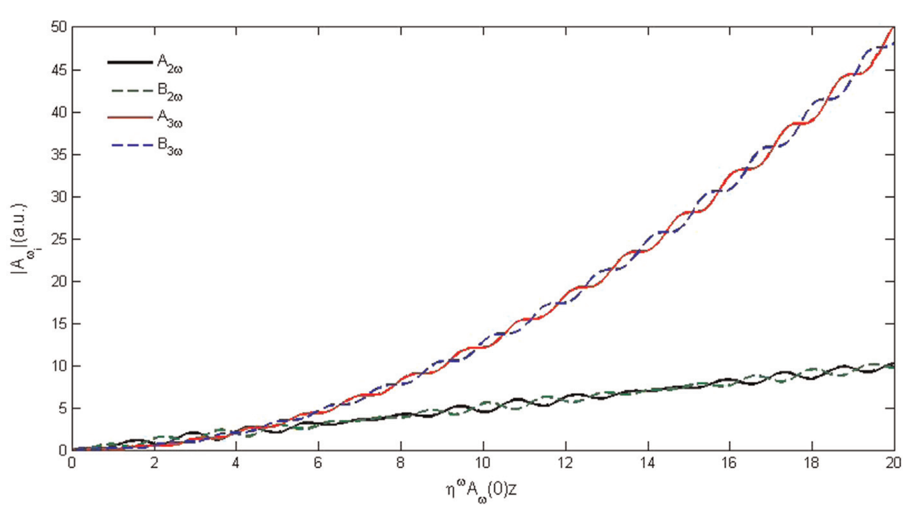

Fig. 1. The amplitudes of the harmonic waves under the double resonance phase-matching conditions $\Delta k_1 = \kappa_{2\omega}$ and $\Delta k_2 = \kappa_\omega - \kappa_{2\omega} + \kappa_{3\omega}$.