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Scintillation of electromagnetic beams generated by quasi-homogeneous sources

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ABSTRACT

We derive an expression for the far-zone scintillation index of electromagnetic beams that are generated by quasi-homogeneous sources. By examining different types of sources, we find conditions under which this index reaches its minimum or its maximum value. It is demonstrated that under certain circumstances two sources with different spectral densities can produce beams with identical scintillation indices.

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1. Introduction

When a beam-like electromagnetic field propagates in space its coherence and polarization properties and its intensity typically vary owing to the randomness of the source or the randomness of the transmitting medium. In detection, the fluctuations of intensity at the detector site are of particular interest. The contrast of intensity fluctuations, or scintillation index, has been extensively studied for beams propagating through random media such as the turbulent atmosphere [1–4]. This is motivated by the fact that efficient control and tailoring of the fluctuating intensity leads to an improved performance (with a reduced noise level) of detection systems. However, much less attention has been devoted to intensity fluctuations that occur on free-space beam propagation. We are only aware of two studies in which the evolution of the scintillation index on free-space propagation was investigated [5,6]. In those papers the analysis was restricted to so-called Gaussian Schell-model beams [7, Chapter 9]. Because of applications such as coherence tomography and laser communication in space, it is desirable to investigate the scintillation of beams that arise from other types of sources.

A broad class of partially coherent beams are those that are generated by quasi-homogeneous sources [7, Chapter 5]. These sources are characterized by a spectral degree of coherence that is homogeneous, meaning that it depends only on the separation of the two spatial points at which it is evaluated, and by an intensity profile that is a slowly varying function compared to the degree of coherence. Scalar and electromagnetic quasi-homogeneous sources and the fields they produce have been studied extensively, see, for example [8–19].

In this work we study stochastic electromagnetic beams that are generated by planar, secondary quasi-homogeneous sources with Gaussian statistics. The assumption of Gaussian statistics is applicable to many types of practical sources and it allows the intensity fluctuations to be represented in terms of second-order coherence quantities. An expression for the scintillation index along the beam axis in the far zone is derived, and its consequences are discussed by examining different examples. We demonstrate, for instance, under which circumstances the scintillation index takes on its minimum or its maximum value. In addition, a connection is made between the scintillation index of beams with Gaussian statistics and an electromagnetic degree of coherence.

2. Quasi-homogeneous, planar electromagnetic sources

Consider a stochastic, statistically stationary, planar, secondary source which produces an electromagnetic beam that propagates

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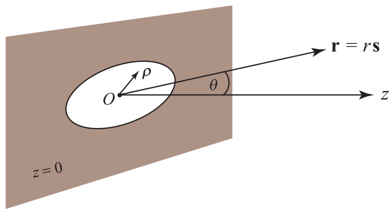

Fig. 1. Illustrating the notation. The origin O of a right-handed Cartesian coordinate system is taken in the plane of a quasi-homogeneous source that generates an electromagnetic beam that propagates along the z-axis. Source points are indicated by the vector $\rho=(x,y)$. The position of a point in the far zone is denoted by the vector $\mathbf{r}=r\mathbf{s}$. The unit vector $\mathbf{s}$ makes an angle θ with the positive z-axis.

closely along the z-axis (see Fig. 1). The state of coherence and polarization of the source field can be characterized, in the space–frequency domain, by a 2×2 *electric cross-spectral density matrix* [7, Section 9.1], namely

$$\mathbf{W}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\begin{pmatrix} W_{xx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) & W_{xy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) \\ W_{yx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega) & W_{yy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)\end{pmatrix}, \tag{1}$$

where

$$W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\langle E_i^*(\boldsymbol{\rho}_1,\omega)E_j(\boldsymbol{\rho}_2,\omega)\rangle \quad (i,j=x,y). \tag{2}$$

Here $E_i(\boldsymbol{\rho},\omega)$ is a Cartesian component of the electric field vector, at a point $\boldsymbol{\rho}$ and at frequency ω, of a typical realization of the statistical ensemble representing the source, and the angled brackets denote the ensemble average. The superscript (0) indicates quantities in the source plane $z=0$. The *spectral density* of the source field, $S^{(0)}(\boldsymbol{\rho},\omega)$, is given by the expression

$$S^{(0)}(\boldsymbol{\rho},\omega)=\mathrm{tr}\,\mathbf{W}^{(0)}(\boldsymbol{\rho},\boldsymbol{\rho},\omega), \tag{3}$$

where tr denotes the trace. The four *correlation coefficients* of the source field are defined as

$$\mu_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\frac{W_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)}{\sqrt{W_{ii}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_1,\omega)W_{jj}^{(0)}(\boldsymbol{\rho}_2,\boldsymbol{\rho}_2,\omega)}} \quad (i,j=x,y). \tag{4}$$

These coefficients obey the relations $0\le|\mu_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)|\le 1$ for all $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$. Moreover, the equal-point values satisfy $\mu_{ij}^{(0)}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)=1$ when $i=j$, since $\mu_{xx}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)$ and $\mu_{yy}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)$ are *auto-correlation functions*, whereas for the *cross-correlation functions* $\mu_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)$, with $i\neq j$, the relation $\mu_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\mu_{ji}^{(0)*}(\boldsymbol{\rho}_2,\boldsymbol{\rho}_1,\omega)$ holds and the quantities $|\mu_{ij}^{(0)}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)|$ may take on any value between 0 and 1.

In order to introduce a quasi-homogeneous, planar electromagnetic source we first assume that, at each frequency ω, the source behaves as a Schell-model source (the concept of Schell's model for scalar sources is discussed in [7, Section 5.3.1]). This means that the correlation coefficients depend only on the positions $\boldsymbol{\rho}_1$ and $\boldsymbol{\rho}_2$ through the difference $\boldsymbol{\rho}_2-\boldsymbol{\rho}_1$, i.e.,

$$\mu_{ij}^{(0)}(\boldsymbol{\rho}_1,\boldsymbol{\rho}_2,\omega)=\mu_{ij}^{(0)}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1,\omega) \quad (i,j=x,y). \tag{5}$$

In addition, the two spectral densities $S_i^{(0)}(\boldsymbol{\rho},\omega)=W_{ii}^{(0)}(\boldsymbol{\rho},\boldsymbol{\rho},\omega)$, associated with each Cartesian component of the electric field, are assumed to change much more slowly with $\boldsymbol{\rho}$ than the moduli (absolute values) of the four correlation coefficients $\mu_{ij}^{(0)}(\boldsymbol{\rho}_2-\boldsymbol{\rho}_1,\omega)$ vary with $\boldsymbol{\rho}_2-\boldsymbol{\rho}_1$. If these two conditions are met, the electromagnetic source is said to be *quasi-homogeneous*.

It was recently shown that for such sources the elements of the cross-spectral density matrix in the far zone are related to those in the source plane by four so-called *reciprocity relations* [20]. Omitting the frequency-dependence for brevity, these relations read

$$W_{xx}^{(\infty)}(r_1\mathbf{s}_1,r_2\mathbf{s}_2)=(2\pi k)^2\cos\theta_1\cos\theta_2\frac{e^{\mathrm{i}k(r_2-r_1)}}{r_1r_2}\times\tilde{S}_x^{(0)}[k(\mathbf{s}_{2\perp}-\mathbf{s}_{1\perp})]\tilde{\mu}_{xx}^{(0)}[k(\mathbf{s}_{1\perp}+\mathbf{s}_{2\perp})/2], \tag{6}$$

$$W_{xy}^{(\infty)}(r_1\mathbf{s}_1,r_2\mathbf{s}_2)=(2\pi k)^2\cos\theta_1\cos\theta_2\frac{e^{\mathrm{i}k(r_2-r_1)}}{r_1r_2}\times\tilde{S}_{xy}^{(0)}[k(\mathbf{s}_{2\perp}-\mathbf{s}_{1\perp})]\tilde{\mu}_{xy}^{(0)}[k(\mathbf{s}_{1\perp}+\mathbf{s}_{2\perp})/2], \tag{7}$$

$$W_{yx}^{(\infty)}(r_1\mathbf{s}_1,r_2\mathbf{s}_2)=(2\pi k)^2\cos\theta_1\cos\theta_2\frac{e^{\mathrm{i}k(r_2-r_1)}}{r_1r_2}\times\tilde{S}_{xy}^{(0)}[k(\mathbf{s}_{2\perp}-\mathbf{s}_{1\perp})]\tilde{\mu}_{xy}^{(0)*}[k(\mathbf{s}_{1\perp}+\mathbf{s}_{2\perp})/2], \tag{8}$$

$$W_{yy}^{(\infty)}(r_1\mathbf{s}_1,r_2\mathbf{s}_2)=(2\pi k)^2\cos\theta_1\cos\theta_2\frac{e^{\mathrm{i}k(r_2-r_1)}}{r_1r_2}\times\tilde{S}_y^{(0)}[k(\mathbf{s}_{2\perp}-\mathbf{s}_{1\perp})]\tilde{\mu}_{yy}^{(0)}[k(\mathbf{s}_{1\perp}+\mathbf{s}_{2\perp})/2]. \tag{9}$$

Here $k=\omega/c$ is the wavenumber associated with frequency ω, with c being the speed of light, and $\mathbf{s}_{p\perp}$, with $p=1,2$, is the two-dimensional projection of the directional unit vector $\mathbf{s}_p$ onto the xy plane. In these expressions the superscript (∞) indicates quantities in the far zone of the source, and we have introduced the "off-diagonal" spectral density

$$S_{xy}^{(0)}(\boldsymbol{\rho})\equiv\sqrt{S_x^{(0)}(\boldsymbol{\rho})}\sqrt{S_y^{(0)}(\boldsymbol{\rho})}. \tag{10}$$

The two-dimensional spatial Fourier transform $\tilde{S}_i^{(0)}(\mathbf{f})$ of the spectral density is defined as

$$\tilde{S}_i^{(0)}(\mathbf{f})=\frac{1}{(2\pi)^2}\int_{z=0}S_i^{(0)}(\boldsymbol{\rho})e^{-\mathrm{i}\mathbf{f}\cdot\boldsymbol{\rho}}\,\mathrm{d}^2\rho, \tag{11}$$

with strictly analogous definitions for $\tilde{S}_{xy}^{(0)}(\mathbf{f})$ and $\tilde{\mu}_{ij}^{(0)}(\mathbf{f})$. It is to be noted that these reciprocity relations are generally valid for electromagnetic beams generated by secondary, planar, quasi-homogeneous sources. We will make use of these expressions in the next sections.

3. Intensity fluctuations of stochastic electromagnetic beams

The fluctuation of the intensity of the field at an arbitrary point $\mathbf{r}$ in the beam (at frequency ω) is defined as

$$\Delta I(\mathbf{r})=I(\mathbf{r})-S(\mathbf{r}), \tag{12}$$

where $I(\mathbf{r})$ stands for the (random) intensity due to a single realization of the field, and $S(\mathbf{r})=\langle I(\mathbf{r})\rangle$ denotes the expectation value, or ensemble average, of the intensity, as defined by Eq. (3). On making use of Eq. (12) it follows at once that the *correlation of the intensity fluctuations* at two points $\mathbf{r}_1$ and $\mathbf{r}_2$ is given by the expression

$$\langle\Delta I(\mathbf{r}_1)\Delta I(\mathbf{r}_2)\rangle=\langle I(\mathbf{r}_1)I(\mathbf{r}_2)\rangle-S(\mathbf{r}_1)S(\mathbf{r}_2). \tag{13}$$

The first term on the right-hand side of Eq. (13) contains a fourth-order correlation function of the field. Under the assumption that the fluctuations of the source are governed by Gaussian statistics, one can use the *Gaussian moments theorem* [21, Section 1.6.1] to derive that for random beams [21, Section 8.4]:

$$\langle\Delta I(\mathbf{r}_1)\Delta I(\mathbf{r}_2)\rangle=\sum_{i,j}|W_{ij}(\mathbf{r}_1,\mathbf{r}_1)|^2 \quad (i,j=x,y). \tag{14}$$

Correlation of intensity fluctuations of this kind in beams generated by quasi-homogeneous sources has recently been examined [22]. It is further of interest to note that if the quantity on the right-hand side of Eq. (14) is normalized by the mean values of the intensities at points $\mathbf{r}_1$ and $\mathbf{r}_2$, one obtains the square of the

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