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# Nonlinear optical absorption in parabolic quantum well via two-photon absorption process

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ABSTRACT

We theoretically study the nonlinear optical absorption phenomenon in a GaAs/GaAlAs parabolic quantum well via investigating the phonon-assisted cyclotron resonance (PACR) effect. We find that the two-photon absorption process (nonlinear) is comparable with the one-photon process (linear), and cannot be neglected in studying PACR effect. The additional peaks in the absorption spectrum due to transitions between Landau levels and electric subband energy accompanied by emission and absorption of LO-phonon are indicated. PACR behavior is strongly affected by the magnetic field, the temperature and the confinement frequency. The present work obtains an usefully resonant condition which is more general than and includes the previous resonant behaviors.



## 1. Introduction

The multi-photon absorption process, or a nonlinear phenomenon, has been studied in a large number of papers in recent years [1–7]. In these papers, the first- and second-orders of the nonlinear optical conductivity have been obtained using the projection-diagram method. Although these results are very clear in physical interpretation and useful in the study of optical conductivity in low-dimensional systems, their analytical calculations are quite complicated. Therefore, these results have not been applied to investigate the nonlinear phenomenon, especially, in numerical calculations. In the other works, using different methods, many researchers have succeeded in studying the linear and nonlinear optical absorptions in low-dimensional semiconductor quantum systems [8–20]. The results show that the optical absorption coefficients are strongly affected by confinement potential [11,14], structural parameters of the system [8–10,13–18], hydrostatic pressure [16,17,19], and external fields [12,20]. However, these works only investigated the optical absorption process via one-photon absorption process, the two-photon absorption process is still open for further investigation.

Phonon-assisted cyclotron resonance (PACR) has been known as one of the useful tools for investigating transport behaviour of electrons in semiconductor materials under an applied magnetic field. This effect indicates the transition of electrons between Landau levels due to the absorption of photons accompanied with the absorption or emission of phonons. Since the early theoretical predictions [21] and experimental observations [22], the cyclotron resonance and PACR have been studied both theoretically and experimentally in bulk semiconductors [23–26], in quantum wells [27–32], in quantum wires [33–35], and in quantum dots [36]. However, the study of nonlinear PACR in PQW has not been found.

It is known that in GaAs/GaAlAs systems, electron interaction with polar phonon modes is of great importance [29]. Therefore, the purpose of the present work is to investigate the effects of the magnetic field, the temperature and the confinement frequency on the linear and nonlinear optical absorption spectrum in GaAs/$Ga_{1-x}Al_xAs$ PQW in the presence of electron–LO phonon interaction. Besides, the present work also obtains an useful resonant behaviour which expresses the phonon-assisted cyclotron resonance condition, including the other types of resonances, such as the optically detected electrophonon resonance [37,38] and the optically detected magnetophonon resonance [39]. The paper is organized as follows: in Section 2, the theoretical framework used in calculations and the analytical results are presented. The discussion of the results is given in Section 3. Finally, the conclusion is given in Section 4.

## 2. Theoretical framework and analytical results

When an electromagnetic wave characterized by a time-dependence electric field of amplitude $F_0$ and angular frequency

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$\Omega$ is applied to a semiconductor system, the optical absorption power is given by [31]

$$P(\Omega) = \frac{F_0^2\sqrt{\varepsilon}}{8\pi}\sum_i W_i f_i, \tag{1}$$

where $\varepsilon$ is the dielectric constant of the medium, $f_i$ is the electron distribution function, and $W_i$ is the transition probability. The sum is taken over all the initial states $i$ of electrons. The transition probability of absorbing photon with simultaneously absorbing and/or emitting phonon $W_i^{\mp}$ can be written as [31,40]

$$W_i^{\mp} = \frac{2\pi}{\hbar}\sum_f\sum_{\mathbf{q}}|M_{fi}|^2 \sum_{\ell=-\infty}^{+\infty}\frac{1}{(\ell!)^2}\left(\frac{a_0 q_\perp}{2}\right)^{2\ell} \times\delta(E_f - E_i \mp \hbar\omega_{\mathbf{q}} - \ell\hbar\Omega), \tag{2}$$

where the upper $(-)$ and lower sign $(+)$ refer to the phonon absorption and phonon emission, respectively, $M_{fi}$ is the transition matrix element of the electron–phonon interaction, $a_0$ is the laser dressing parameter, $E_i \equiv E_{N,n}$ and $E_f \equiv E_{N',n'}$ are the energy initial and final states of the electron, respectively, and $\mathbf{q} = (q_z, q_\perp)$ is the phonon wave vector.

We consider a PQW, in which electron system is confined in $z$-direction by the potential $U(z) = m^*\omega_z^2 z^2/2$, where $\omega_z$ is the confinement frequency. When a static magnetic field $\mathbf{B} = (0,0,B)$ is applied to system, the normalized eigenfunctions in the Landau gauge for the vector potential $\mathbf{A} = (0, Bx, 0)$ and the corresponding energy are given by [41]

$$|N, n, k_y\rangle = \frac{1}{\sqrt{L_y}}\exp(ik_y y)\phi_N(x - x_0)\psi_n(z), \tag{3}$$

$$E_{N,n} = \left(N + \frac{1}{2}\right)\hbar\omega_c + \varepsilon_n, \quad N = 0,1,2,\ldots, \tag{4}$$

where $m^*$ is the effective mass of a conduction electron, $N$ and $n$ are the Landau level index and the level quantized number in $z$-direction, respectively; and $\omega_c = eB/m^*$ is the cyclotron frequency. Also, $\phi_N(x - x_0)$ represents the harmonic oscillator wave function, centered at $x_0 = -k_y/m^*\omega_c$. Here $k_y$ and $L_y$ are the wave vector and the normalization length in $y$-direction, respectively. The radius of the orbit in the $(x, y)$ plane is $a_c = (\hbar/m^*\omega_c)^{1/2}$. In Eqs. (3) and (4), the one-electron normalized eigenfunctions and the corresponding eigenvalues in the conduction band are, respectively, given by

$$\psi_n(z) = \left(\frac{1}{2^n n!\sqrt{\pi}a_z}\right)^{1/2}\exp\left(-\frac{z^2}{2a_z^2}\right)H_n\left(\frac{z}{a_z}\right), \tag{5}$$

$$\varepsilon_n = \left(n + \frac{1}{2}\right)\hbar\omega_z, \quad n = 0,1,2,\ldots, \tag{6}$$

where $H_n(x)$ is the $n$th Hermite polynomial and $a_z = (\hbar/m^*\omega_z)^{1/2}$.

The matrix elements for electron-confined LO-phonon interaction in PQW in the presence of magnetic field can be written as

$$|M_{fi}|^2 = |V_m(q)|^2|J_{nn'}(q_z)|^2|J_{NN'}(q_\perp)|^2 \times(N_{Li} + 1/2 \mp 1/2)\delta_{k_z',k_z\pm q_z}, \tag{7}$$

where $N_{Li}$ is the distribution function of confined LO-phonon for frequency $\omega_{\mathbf{q}} = \omega_{Li}$, and the coupling function is given by [42]

$$|V_m(q)|^2 = \frac{e^2\hbar\omega_{Li}}{\varepsilon_0 d_i}\left(\frac{1}{\chi_{\infty i}} - \frac{1}{\chi_{0i}}\right)\times\left(q_\perp^2 + \frac{m^2\pi^2}{d_i^2}\right)^{-1}, \quad m = 1,2,3,\ldots, \tag{8}$$

with $\varepsilon_0$ is the permittivity of free space, $\chi_{\infty i}$ ($\chi_{0i}$) and $d_i$ are the high (low) frequency dielectric constants and the thickness of region $i$ of quantum well, respectively, and

$$|J_{nn'}(q_{zm})|^2 = \frac{n!}{n'!}e^{-a_z^2 q_{zm}^2/2}(a_z^2 q_{zm}^2/2)^{n'-n} \times\left[L_n^{n'-n}(a_z^2 q_{zm}^2/2)\right]^2, \quad n \le n', \tag{9}$$

$$|J_{NN'}(q_\perp)|^2 = \frac{N!}{N'!}e^{-a_c^2 q_\perp^2/2}(a_c^2 q_\perp^2/2)^{N'-N} \times\left[L_N^{N'-N}(a_c^2 q_\perp^2/2)\right]^2, \quad N \le N', \tag{10}$$

where we have denoted $q_{zm} = m\pi/d_i$, and $L_N^M(x)$ is the associated Laguerre polynomials.

The transition probability in Eq. (2) contains contributions of absorption process of $\ell$-photons. In this paper, we restrict ourselves to considering the process of absorbing two photons ($\ell = 1, 2$). Using Eq. (1) and making a straight forward calculation of probability with the use of matrix element for confined LO-phonon scattering (Eq. (7)), we obtain the expression for the optical absorption power in PQW in the case of non-degenerate electron gas

$$\begin{aligned}P(\Omega) = A(\Omega,\omega_c)\sum_{N,N',n,n'}|I_{nn'}|e^{-E_{N,n}/k_B T}\times\{[N_{Li}\delta(p\hbar\omega_c + \Delta\varepsilon_{n'n} - \hbar\omega_{Li} - \hbar\Omega)\\ +(N_{Li}+1)\delta(p\hbar\omega_c + \Delta\varepsilon_{n'n} + \hbar\omega_{Li} - \hbar\Omega)]\\ +(N + N' + 1)\frac{a_0^2}{8a_c^2}[N_{Li}\delta(p\hbar\omega_c + \Delta\varepsilon_{n'n}\\ -\hbar\omega_{Li} - 2\hbar\Omega) + (N_{Li}+1)\delta(p\hbar\omega_c + \Delta\varepsilon_{n'n} + \hbar\omega_{Li} - 2\hbar\Omega)]\},\end{aligned} \tag{11}$$

here we denoted $p = N' - N$ (is an integer), and $\Delta\varepsilon_{n'n} = \varepsilon_{n'} - \varepsilon_n = (n' - n)\hbar\omega_z$. In Eq. (11), the overlap integral $I_{nn'}$ is defined as

$$I_{nn'} = \sum_{m=1,2,3,\ldots}\left|J_{nn'}\left(\frac{m\pi}{d_i}\right)\right|^2, \tag{12}$$

and

$$A(\Omega,\omega_c) = \frac{F_0^2\sqrt{\varepsilon}n_e V_0^2 a_0^2 e^2\omega_{Li}}{128\pi^3 L_z d_i\lambda a_c^6\varepsilon_0}\left(\frac{1}{\chi_{\infty i}} - \frac{1}{\chi_{0i}}\right), \quad \lambda = \sum_{N,n}e^{-E_{N,n}/k_B T}$$

with $n_e$ is the electron concentration, $k_B$ is the Boltzmann constant, $T$ is the temperature. In Eq. (11), the delta functions present the energy conservation law. This implies that when an electron undergoes a collision by absorbing photon energy, its energy can only change by an amount equal to the energy of a phonon involved in the transitions. The energy-conservation delta functions in Eq. (11) show resonant behaviour at the PARC condition for PQW based on the following selection rule of transition condition:

$$\ell\hbar\Omega = p\hbar\omega_c + \Delta\varepsilon_{n'n} \pm \hbar\omega_{Li}. \tag{13}$$

This condition predicates that the PACR is affected by both Landau and electric subband levels. In the case of transitions without electric subband levels, this equation reduces to $\ell\hbar\Omega = p\hbar\omega_c \pm \hbar\omega_{Li}$. This is the pure PACR condition which is only affected by the Landau levels [31]. Furthermore, we also see that the transition condition in Eq. (13) can include the other resonant behaviours, such as the optically detected electrophonon resonance (ODEPR) [37,38], and the optically detected magnetophonon resonance (ODMPR) [39]. Indeed, being only affected by the electric subband levels, ODEPR, which satisfies the condition $\hbar\Omega = \Delta\varepsilon_{n'n} \pm \hbar\omega_{Li}$, is the specific case of PACR condition in the case of $\ell = 1, p = 0$. Similarly, satisfying the condition $p\hbar\omega_c = \hbar\omega_{Li} + \Delta\varepsilon_{n'n} \pm \hbar\Omega$, ODMPR is the specific case of PACR condition with $\ell = 1$. Therefore, it can be seen that our PACR condition is more general than the previous ones.

Following the collision broadening model, the delta functions in Eq. (11) are replaced with Lorentzians [43]

$$\delta(Z_\ell^{\pm}) = \frac{1}{\pi}\frac{\hbar\Gamma_\ell^{\mp}}{(Z_\ell^{\pm})^2 + \hbar^2(\Gamma_\ell^{\mp})^2}, \tag{14}$$