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Higher nonclassical properties and entanglement of photon-added two-mode squeezed coherent states

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ABSTRACT

We investigate how photon addition operations affect some higher-order nonclassical properties and the intermodal entanglement of a non-Gaussian entangled state generated by adding photons to both modes of a two-mode squeezed coherent state (TMSCS). We show that the photon addition operation can enhance the degrees of the sum squeezing and the difference squeezing for appropriate combinations of several parameters involved in the TMSCS. We consider the existing Hillery-Zubairy entanglement criterion, and find that the quantity of Hillery-Zubairy E gets more negative not only for a larger value of the squeezing parameter r but also for greater values of photon addition numbers (m, n). Our results may imply that the highest enhancement is obtained when same number of operations (i.e., $m=n$) is applied to both modes. However, for the photon-added TMSCS (PA-TMSCS), the more violation to the Hillery-Zubairy inequality does not necessarily following strong sum squeezing and difference squeezing. This because their behaviors with three phases and squeezing parameter r involved in the TMSCS are somewhat different.

1. Introduction

Non-Gaussian states generated by photon subtraction and photon addition have received increasing attention from both experimentalists and theoreticians recently [1–10]. This because that non-Gaussian states with highly nonclassical properties may constitute useful resources in quantum information with continuous variables, and overcome some limitations found in the use of Gaussian states for quantum information processing. Agarwal and Tara [2] first introduced photon-added coherent state (a non-Gaussian state) and studied its nonclassical properties. In 2004, the photon addition operation was successfully demonstrated experimentally [9] via a nondegenerate parametric amplifier with small coupling strength. The photon subtraction was implemented [10] with a beam splitter of high transmissivity and was considered in enhancing not only the entanglement but also the performance of a quantum-noise-limited amplifier [11–14]. For a review about quantum state engineering by photon subtraction and addition, we refer to Refs. [15,16] (and references therein).

Non-Gaussian two-mode squeezed thermal states were introduced and their nonclassical properties have been studied via Mandel Q parameter, anti-bunching effects and Wigner functions [17–19]. Ourjoumtsev et al. [20] demonstrated experimentally that the entanglement between Gaussian entangled states can be increased by subtracting one photon from two-mode squeezed vacuum states. Yang and Li [21] investigated the entanglement properties of those non-Gaussian squeezed vacuum states, and showed that the partial von Neumann entropy of all the resulting states is greater than that of the original two-mode squeezed vacuum state. For multiple-photon addition and subtraction, the optimal enhancement is obtained when the same number of operations is applied to both modes, where both addition and subtraction give the same entanglement enhancement [22]. Lee et al. [23] proposed a coherent superposition of photon subtraction and addition to enhance quantum entanglement of two-mode squeezed vacuum sate. For two-mode squeezed coherent state (TMSCS), Wang et al. [24] firstly introduced the non-Gaussian TMSCS which is generated by single-mode photon addition, and they studied those nonclassical properties of resulting states in terms of Mandel's Q parameter, the cross-correlation function and Wigner functions. Different from the work in Ref.[24], we consider the photon-added TMSCS (PA-TMSCS) generated by repeatedly adding different photons to both modes of the TMSCS, and showed that the photon statistical properties are sensitive to the compound phase involved in the TMSCS [25]. And the photon subtraction and addition applied to both modes can enhance the

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cross-correlation and anti-bunching effects of resulting states. Very recently, Truong et al. [26] showed that the non-Gaussian TMSCS, which is generated by single-mode photon addition, can also possess higher-order nonclassical properties such as sum squeezing, the difference squeezing and the antibunching.

Besides the entanglement, squeezing has attracted considerable attention in quantum optics and quantum information processing. In this work, different from the work in Refs. [22,26], we will further study the higher-order nonclassical properties of the PA-TMSCS, as well as its entanglement properties. This paper is organized as follows. In Section 2, we make a brief review about the PA-TMSCS generated by simultaneously adding different photons to each mode of the TMSCS and derive the expectation value of a general product of the operator $a^k a^{\dagger l} b^h b^{\dagger j}$. In Section 3, we derive the general analytic expressions for the sum squeezing and the difference squeezing, and explore numerically how photon addition operations affect the squeezing properties of the PA-TMSCS. In Section 4, we prove that the PA-TMSCS is two-mode entangled as characterized by Hillery-Zubairy entanglement criterion. Our results may imply that the entanglement can be increased with the number of photon additions, and the optical enhancement is obtained when the same number of operations is applied to both modes. Our main results are summarized in Section 5.

2. The PA-TMSCS and its normalization factor

Let us firstly make a brief review about the PA-TMSCS and its normalization factor. Theoretically, the PA-TMSCS can be obtained by repeatedly operating photon creation operators $a^\dagger$ and $b^\dagger$ on both modes of a TMSCS [25], i.e.,

$$|\xi\rangle_{pa} = C_{m,n}^{-1/2} a^{\dagger m} b^{\dagger n} D(\lambda_1, \lambda_2) S_2(\xi) |0_a, 0_b\rangle, \tag{1}$$

where m and n are the numbers of adding photons to each mode of the TMSCS, respectively. And $S_2(\xi)$ and $D(\lambda_1, \lambda_2)$ are the two-mode squeezing and displacement operators, respectively, defined by

$$S_2(\xi) = \exp\left[\xi a^\dagger b^\dagger - \xi^* ab\right],$$
$$D(\lambda_1, \lambda_2) = \exp\left[\lambda_1 a^\dagger - \lambda_1^* a + \lambda_2 b^\dagger - \lambda_2^* b\right], \tag{2}$$

with

$$\xi = r\exp[i\theta], \quad \lambda_1 = |\lambda_1| \exp[i\phi_1], \lambda_2 = |\lambda_2| \exp[i\phi_2]. \tag{3}$$

The corresponding normalization factor $C_{m,n}$ in Eq. (1) is [See Appendix in detail]

$$C_{m,n} = \sum_{p=0}^{m}\sum_{q=0}^{n} \frac{(m!n!)^2 \cosh^{2p+2q} r}{p!q!|\lambda_1|^{2p-2m}|\lambda_2|^{2q-2n}} \times \left| \sum_{f=0}^{\min(m-p,n-q)} \frac{(2|\lambda_1\lambda_2|e^{i\chi})^{-f}(\sinh 2r)^f}{f!(m-p-f)!(n-q-f)!} \right|^2 \tag{4}$$

Although there are three phases ϕ_1, ϕ_2 and θ in the TMSCS, Eq. (4) shows that $C_{m.n}$ is actually a periodic function of the compound phase χ. The compound phase χ is defined by

$$\chi = \phi_1 + \phi_2 - \theta. \tag{5}$$

For example, when $m=n=1$

$$C_{1,1} = (|\lambda_1|^2 + |\lambda_2|^2 + \cosh 2r) \cosh^2 r + |\lambda_1\lambda_2|(|\lambda_1\lambda_2| + \cos\chi \sinh 2r), \tag{6}$$

Obviously, $C_{1,1}$ is a periodic function of χ involved in the TMSCS with a period 2π.

For the PA-TMSCS state given by Eq. (1), we can obtain the expectation value of a general product of the operator $a^k a^{\dagger l} b^h b^{\dagger j}$ by the same approach as deriving Eq. (4)

$$\left\langle a^k a^{\dagger l} b^h b^{\dagger j}\right\rangle = \frac{1}{C_{m,n}}\left\langle \xi|a^m b^n a^k a^{\dagger l} b^h b^{\dagger j} a^{\dagger m} b^{\dagger n}|\xi\right\rangle = \frac{C_{m+k,m+l,n+h,n+j}}{C_{m,n}}, \tag{7}$$

where we set

$$C_{m+k,m+l,n+h,n+j} = e^{i(k-l)\phi_1 + i(h-j)\phi_2}|\lambda_1|^{k+l}|\lambda_2|^{h+j} \sum_{p=0}^{\min[m+k,m+l]} \sum_{q=0}^{\min[n+h,n+j]} \times \frac{\cosh^{2p+2q} r(m+k)!(n+h)!(m+l)!(n+j)!}{p!q!|\lambda_1|^{2p-2m}|\lambda_2|^{2q-2n}} \times \sum_{f=0}^{\min(m+k-p,n+h-q)} (f!)^{-1} \frac{(2|\lambda_1\lambda_2|e^{i\chi})^{-f}(\sinh 2r)^f}{(m+k-p-f)!(n+h-q-f)!} \times \sum_{s=0}^{\min(m+l-p,n+j-q)} (s!)^{-1} \frac{(2|\lambda_1\lambda_2|e^{-i\chi})^{-s}(\sinh 2r)^s}{(m+l-p-s)!(n+j-q-s)!}. \tag{8}$$

Eqs. (4) and (7) are important for further studying higher-order nonclassical properties of the PA-TMSCS. When $k=l$ and $h=j$ are satisfied, Eq. (8) actually reduces to Eq. (4), i.e., $C_{m+k,m+l,n+h,n+j} \equiv C_{m+k,n+h}$.

3. Sum and difference squeezing

In this section, with the help of the normalization factor we shall discuss how the photon addition affects the higher-order nonclassical statistical properties of the PA-TMSCS in terms of the sum squeezing and the difference squeezing. Especially, we shall present how the compound phase χ and photon addition operations affect those nonclassical properties.

3.1. Sum squeezing

Sum and difference squeezing are both higher-order, two-mode squeezing effects [27,28]. In quantum optics, squeezing is an earliest studied nonclassical phenomenon. For two arbitrary modes a and b, the sum squeezing is associated with a so-called two-mode quadrature operator V_φ of the form [27]

$$V_\varphi = \frac{1}{2}\left(e^{i\varphi} a^\dagger b^\dagger + e^{-i\varphi} ab\right) \tag{9}$$

where φ is an angle made by V_φ with the real axis in the complex plane. A state is said to be sum squeezed for a φ if

$$\left\langle (\Delta V_\varphi)^2 \right\rangle < \frac{1}{4}\left\langle a^\dagger a + b^\dagger b + 1\right\rangle. \tag{10}$$

where $\left\langle (\Delta V_\varphi)^2 \right\rangle = \left\langle V_\varphi^2\right\rangle - \langle V_\varphi\rangle^2$. From Eq. (10), one can define the degree of the sum squeezing S in the form of antinormally ordered operators as following

$$S = \frac{4\left\langle (\Delta V_\varphi)^2\right\rangle - \left\langle aa^\dagger + bb^\dagger - 1\right\rangle}{\left\langle aa^\dagger + bb^\dagger - 1\right\rangle}. \tag{11}$$

Substituting Eq. (9) into Eq. (11), we obtain S as

$$S = \frac{B - 2A}{A}, \tag{12}$$

where

$$A = \left\langle aa^\dagger + bb^\dagger - 1\right\rangle,$$
$$B = 2\left\langle aa^\dagger bb^\dagger\right\rangle + 2\,\mathrm{Re}\left(e^{-2i\varphi}\left\langle a^2 b^2\right\rangle\right) - 4\,\mathrm{Re}^2\left(e^{-i\varphi}\langle ab\rangle\right). \tag{13}$$

Then its negative value in the range $[-1, 0]$ indicates sum squeezing (a higher-order nonclassicality). It is clear that S has a lower