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Nonclassicality generated by repeatedly operating photon annihilation-then-creation and creation-then-annihilation on squeezed vacuum ☆

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ABSTRACT

We theoretically examine nonclassical properties of the field states generated by repeatedly applying photon annihilation-then-creation operation (AC) and creation-then-annihilation operation (CA) to the squeezed vacuum (SV). We firstly derive their normalization factors and then compare their statistical properties, such as photon number distribution, mean photon number, Mandel Q parameter, and the Wigner function. It is found that the ACSV can present more clear nonclassicality than the CASV.

1. Introduction

Nonclassicality of optical fields has been an interesting topic in quantum optics and quantum information processing [1,2]. The negative value in Wigner distribution, for example, becomes a sufficient, though not necessary, signature of nonclassicality [3,4]. The other is to observe individual nonclassical effects such as antibunching, sub-Poissonian statistics, and squeezing. The investigation of nonclassical states generated by photon addition and subtraction [5–7] manipulation has attracted a lot of interest during the past two decades. It is shown that these operations enable us to generate highly nonclassical quantum states [8], such as photon-addition coherent state, photon-added thermal states [9–11], photon-subtracted squeezed states [12,13], and so on. In addition, single photon operations such as photon subtraction and addition can enhance quantum linear amplifier [14], continuous-variable quantum key distribution [15], entanglement [16–19], nonlocality [21–23], multipartite quantum correlation [20], and the fidelity of continuous variable teleportation [24–26]. The entanglement distillation can also be achieved by performing photon subtraction or addition combined with coherent displacement or local squeezing [27–29].

Much attention has been paid to the sequential operations [30–32] such as photon creation-then-annihilation (CA) $aa^\dagger$ and annihilation-then-creation (AC) $a^\dagger a$ operations. In particular, the photon CA operation was considered in achieving a noiseless amplifier [33], quantifying bosonic behavior in a composite-particle system [34], and distinguishing quantum particles from classical particles [35]. Furthermore, several papers have examined how to prove the commutation relation of single photon operations [36–39]. In fact, the noncommutativity of the photon operations, $[a, a^\dagger] = 1$, breaks the symmetry between AC and CA, which has been experimentally proved the commutation relation [40]. Recently, Lee et al. demonstrated that coherent superposition of single mode operations $(ta + ra^\dagger)$ can outperform each component operation (a and $a^\dagger$) for generating single mode nonclassicality [41]. They also examined nonclassical properties of the field states generated by applying AC and CA to the thermal and coherent states and studied the effects of repeated applications of AC and of CA [31]. Due to the difference between squeezed vacuum (SV) and thermal state (coherent state), such as photon-number distribution and squeezing, thus it will lead to the different effects by operating non-Gaussian operations on the SV. Actually, AC (CA) on

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the SV can also be considered as a combination between non-Gaussian and Gaussian operations. Here, we focus on investigating nonclassical properties of the field states generated by repeatedly applying the AC and the CA on the SV and analytically deriving some distribution functions. Due to the noncommutativity, it is necessary to analyze and compare the nonclassical properties of the states that result after AC and CA.

The rest of the paper is structured as follows. In Section 2 we introduce two kinds of new quantum states (ACSV and CASV). We explicitly derive the compact expressions for their normalization factors and discuss the effect of the AC and CA operations via the fidelity with original SV. In Section 3, we discuss their quantum characters in terms of photon number distribution (PND), mean photon number (MPN), and Mandel Q parameter. In Section 4, we give the analytical expressions of their Wigner functions (WF) and discuss their nonclassicality. In the last section we summarize the conclusions.

2. The ACSV and the CASV

In quantum optics and quantum information, squeezed vacuum state is widely used because of its nonclassicality and Gaussianity. The SV is given by $|S_0(r)\rangle = S(r)|0\rangle$, where $S(r) = \exp[r/2(a^{\dagger 2} - a^2)]$ is the squeezing operator, and r is the real squeezing parameter. Here we introduce two kinds of quantum state generated from the SV. Theoretically, by repeatedly operating $a^\dagger a$ on a SV, the photon annihilation-then-creation squeezed vacuum (ACSV) can be obtained

$$|S_{AC}^m(r)\rangle = N_{AC,m}^{-1/2}\left(a^\dagger a\right)^m S(r)|0\rangle, \tag{1}$$

Similarly, by repeatedly operating $aa^\dagger$ on a SV, one can generate another non-Gaussian state, i.e., the creation-then-annihilation squeezed vacuum (CASV)

$$|S_{CA}^m(r)\rangle = N_{CA,m}^{-1/2}\left(aa^\dagger\right)^m S(r)|0\rangle, \tag{2}$$

where m can be any positive integer, and $N_{AC,m}$ $(N_{CA,m})$ is the normalization constant. When m=0, both the ACSV and the CASV are reduced to the SV.

Noticing that the SV can be expressed as

$$|S_0(r)\rangle = \cosh^{-1/2} r \exp\left(\frac{\tanh r}{2} a^{\dagger 2}\right)|0\rangle, \tag{3}$$

we have

$$\begin{aligned}&\left(a^\dagger a\right)^m S(r)|0\rangle \\ &= \cosh^{-1/2} r\partial_s^m e^{sa^\dagger a} \exp\left(\frac{\tanh r}{2} a^{\dagger 2}\right)|0\rangle|_{s=0} \\ &= \cosh^{-1/2} r\partial_s^m \exp\left(\frac{\tanh r}{2} e^{2s} a^{\dagger 2}\right)|0\rangle|_{s=0}\end{aligned} \tag{4}$$

and

$$\begin{aligned}&\langle 0|S^\dagger(r)(a^\dagger a)^m \\ &= \cosh^{-1/2} r\partial_\tau^m \langle 0|\exp\left(\frac{\tanh r}{2} e^{2\tau} a^2\right)|_{\tau=0}\end{aligned} \tag{5}$$

where $\partial_s^m \equiv \partial^m/\partial s^m$, we have used $a^\dagger a|0\rangle = 0$ and Baker–Hausdoff theorem

$$e^A B e^{-A} = B + [A,B] + \frac{1}{2!}[A,[A,B]] + \frac{1}{3!}[A,[A,[A,B]]] + \cdots. \tag{6}$$

Eqs. (4) and (5) are important in our latter calculation.

To fully describe quantum states, their normalizations are usually necessary. For this purpose, using Eq. (4) and inserting the completeness relation of coherent state $(\int d^2\alpha/\pi)|\alpha\rangle\langle\alpha| = 1$ into $\langle 0|S^\dagger(r)(a^\dagger a)^{2m}S(r)|0\rangle$, we obtain

$$\begin{aligned}&N_{AC,m} \\ &= \cosh^{-1} r\partial_s^{2m}\int\frac{d^2\alpha}{\pi}e^{-|\alpha|^2 + \tanh r/2\alpha^2 + \tanh r/2 e^{2s}\alpha^{*2}}|_{s=0} \\ &= \cosh^{-1} rU_{2m}(r),\end{aligned} \tag{7}$$

where we have set

$$U_n(r) = \partial_s^n\left[\left(1 - e^{2s}\tanh^2 r\right)^{-1/2}\right]|_{s=0}. \tag{8}$$

Similarly, we also have

$$N_{CA,m} = \cosh^{-1} rV_{2m}(r) \tag{9}$$

with

$$V_n(r) = \partial_s^n\left[\left(1 - e^{2s}\tanh^2 r\right)^{-1/2} e^s\right]|_{s=0}. \tag{10}$$

In particular, when m=0, i.e., the SV case, $N_{AC,0} = N_{CA,0} = 1$ as expected.

The normalization factors in Eqs. (8) and (10), which include the partial derivative, are important for further studying analytically the statistical properties of these quantum state. For example, in order to study the effect of the AC and the CA operations ($(a^\dagger a)^m$ and $(aa^\dagger)^m$) from the SV, we discuss that the fidelity between the ACSV and the CASV, i.e.,

$$F^{(m_1,m_2)}(r) = |\langle S_{AC}^{m_1}(r)||S_{CA}^{m_2}(r)\rangle|^2. \tag{11}$$

Thus, substituting Eqs. (1) and (2) into Eq. (11) leads to

$$F^{(m_1,m_2)}(r) = \frac{[\partial_\tau^{m_1}\partial_s^{m_2}(\Lambda_{r,s,\tau}e^s)|_{s=\tau=0}]^2}{U_{2m_1}(r)V_{2m_2}(r)}, \tag{12}$$

where we have set $\Lambda_{r,s,\tau} = \left(1 - e^{2\tau+2s}\tanh^2 r\right)^{-1/2}$.

According to the definition of Eqs. (1) and (2), when $m_2 = 0$, $F^{(m,0)}(r)$ is actually the fidelity between $|S_0(r)\rangle$ and $|S_{AC}^m(r)\rangle$. Similarly, while $m_2 = 0$, $F^{(0,m)}(r)$ is actually the fidelity between $|S_0(r)\rangle$ and $|S_{CA}^m(r)\rangle$. For the same m, $F^{(m,m)}(r)$ is the fidelity between $|S_{CA}^m(r)\rangle$ and $|S_{AC}^m(r)\rangle$. To see clearly the variation of the fidelity, we plot the graph of $F^{(m,0)}(r)$, $F^{(0,m)}(r)$ and $F^{(m,m)}(r)$ as a function of squeezing parameter r for different m (see Fig. 1(a,b,c). In addition the ACSV and the CASV approach each other as m moves toward a large number can clearly be seen in Fig. 1(d), in which $F^{(m,m)}(r)$ is plotted as a function of m for different r. Interestingly, $|S_{CA}^m(r)\rangle$ is very close to $|S_{AC}^m(r)\rangle$ in view of quantum fidelity $F^{(m,m)}(r)$ for all r, and $F^{(m,m)}(r) \to 1$ as $m \to \infty$.

3. Quantum statistical properties of the ACSV and the CASV

In this section, we investigate and compare some quantum statistical properties of the ACSV and the CASV, such as PND, mean photon number, Mandel Q parameter.

3.1. Photon number distribution

The PND is a key characteristic of every optical field. All interesting states of the field are constructed as a combination of Fock states, and different combinations have different quantum properties. The PNDs, i.e., the probability of finding n photons in ρ, is defined as $p_n = \langle n|\rho|n\rangle$.

For the ACSV, the PND is $p_n^{AC} = |\langle n||S_{AC}^m(r)\rangle|^2$. Using the un-normalized coherent state $|\alpha\rangle = \exp[\alpha a^\dagger]|0\rangle$ ($\langle\alpha.|\beta\rangle = e^{\beta\alpha^*}$), leading