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Two-dimensional solitons in triangular photonic lattices with parity-time symmetry

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A B S T R A C T

We report the existence and stability of two-dimensional (2D) fundamental, dipole-mode, vortex, and multipole solitons in parity-time (*PT*) symmetric triangular lattices with the Kerr self-focusing nonlinearity. It is demonstrated that the structure of such complex lattice potentials strongly affects the shape of the solitons, enabling the formation of stable out-of-phase dipole and multipole solitons, as well as vortices. The solitons of all these species have their stability regions in the semi-infinite gap. We also identify the point of the *PT*-symmetry-breaking phase transition in this lattice.

1. Introduction

The fact non-Hermitian Hamiltonians subjecting to the *PT*-symmetry may have entirely real spectra of eigenvalues was first demonstrated by Bender and Boettcher [1]. Then, the concept of the *PT* symmetry was introduced into optics [2]. Many works have been addressed Hamiltonians with pseudo-Hermitian (complex) potentials in diverse physical settings [3–5]. The simplest physical system with the *PT*-symmetric complex-valued potential is composed of two coupled optical waveguides, with equal amounts of linear gain and loss carried by them [4]. Increasing the strength of the gain and loss components of *PT*-symmetric potential past a critical value breaks the *PT* symmetry [6]. The corresponding transition (bifurcation) has been studied in detail, both theoretically and experimentally [2–7].

Localized nonlinear modes (solitons) exist in both one-dimensional (1D) and two-dimensional (2D) photonic lattices with the *PT*-symmetry, which has been established recently [8–21]. Photonic lattices combined with the linear [8–17], nonlinear [18,19] or mixed linear-nonlinear *PT* symmetry [20–23] support diverse families of bright solitons. *PT*-symmetric nonlinear lattices can also enable the formation of solitons, some narrow modes being stable even when the conservative nonlinear lattice potential is absent [18]. Stability of solitons in such *PT*-symmetric potentials was analyzed too [19]. The existence and stability of solitons supported by defects in *PT*-symmetric lattices in local [13] and nonlocal [14] nonlinear media were also reported. In defocusing Kerr media with embedded photonic lattices, stable in-phase multi-peak gap solitons were reported in 1D [15] and 2D [16] settings. Very recently, vector solitons in *PT*-symmetric lattices were studied, too [17].

Hexagonal and triangular lattices are viewed as basic structures in many physical systems, especially in photonic band-gap crystals [24]. Gap solitons in triangular photonic lattices originated from the first and second band of the linear transmission spectrum have been observed, see Refs. [25,26]. The structure of the triangular photonic lattices affects the formation of fundamental and dipole solitons [27]. Solitons in triangular photonic lattices with defects and saturable nonlinearity have been reported as well. It was found that, with the changes of the defect strength, solitons may exist in different bandgaps [28]. Higher-order solitons were studied too, with the conclusion that dipoles, necklaces, and necklace-vortex solitons have their stability regions in lattices with the three-fold symmetry [29].

In this paper we investigate the existence and stability of fundamental, dipole-mode, vortex, and multipole solitons in 2D triangular lattices with the *PT* symmetry based on focusing Kerr nonlinearity. In particular, we consider the effect of the distance between the two peaks of the dipole soliton on its stability. Stable 3-peak vortices and out-of-phase quadrupoles and 6-poles are presented too.

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2. *PT*-symmetric triangular lattice and the Bloch bandgap

The propagation of the probe beam in the 2D *PT*-symmetric triangular photonic lattices with the focusing Kerr nonlinearity is governed by the nonlinear Schrödinger equation [10]:

$$i\frac{\partial U}{\partial z}+\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+VU+|U|^2U=0. \quad (1)$$

here $V(x,y)=R(x,y)+iI(x,y)$ is the complex-valued potential, $U(x, y, z)$ is the slowly varying amplitude of the (extraordinarily polarized) probe beam, (x, y) are the transverse coordinates, and z is the propagation distance.

In what follows, the real part $R(x, y)$ of the *PT*-symmetric linear photonic lattices is adopted in the form of

$$R(x,y)=2V_0\left[\frac{3}{2}+\cos(\mathbf{k}_{12}\cdot\mathbf{r})+\cos(\mathbf{k}_{13}\cdot\mathbf{r})+\cos(\mathbf{k}_{23}\cdot\mathbf{r})\right], \quad (2)$$

where $\mathbf{k}_{12}=k_0\{1,1/\sqrt{3}\}$, $\mathbf{k}_{13}=k_0\{1,-1/\sqrt{3}\}$, $\mathbf{k}_{23}=k_0\{0,2/\sqrt{3}\}$, and the imaginary part of the *PT*-symmetric potential is

$$I(x,y)=\frac{4}{3}W_0\left\{\sin\left[k_0\left(x+\frac{1}{\sqrt{3}}y\right)\right]+\sin\left[k_0\left(x-\frac{1}{\sqrt{3}}y\right)\right]+\sin\left(k_0\frac{2}{\sqrt{3}}y\right)\right\}. \quad (3)$$

here $k_0=2\pi/d$, with lattice spacing d, while V_0 and W_0 are the modulation depths of the conservative and dissipative lattices, respectively. To present generic results, we choose, as appropriate values, $V_0=2/3$, $W_0=0.2$, and $d=\pi$. Figs. 1(a) and (b) show the real and imaginary parts of the *PT*-symmetric triangular potential, respectively. In the experiment, it is possible to optically induce a 2D triangular lattice, e.g., in a biased photorefractive crystal SBN:60 (a strontium barium niobate crystal) by means of three coherently interfering ordinarily polarized broad laser beams, that may be produced by the frequency-doubled Nd:YVO4 cw laser [27,29].

As mentioned above, there is a "phase transition" point in such *PT*-symmetric systems. The continuous spectrum in this case contains Bloch bands filled by delocalized modes in the form of $U(x,y,z)=u(x,y)e^{i\mu z}e^{i(k_x x+k_y y)}$, where k_x and k_y are the Bloch wave numbers in the first Brillouin zone, μ is the propagation constant and, $u(x,y)$ is a periodic function with the same period as the underlying lattice. Substituting $U(x, y, z)$ into the linear version of Eq. (3), one obtains the corresponding eigenvalue equation for μ:

$$Vu+\left(\frac{\partial^2}{\partial x^2}+2ik_x-k_x^2\right)u+\left(\frac{\partial^2}{\partial y^2}+2ik_y-k_y^2\right)u=\mu u \quad (4)$$

This equation can be solved by means of the plane-wave-expansion method. Spectrum $\mu(k)$ of the linear Bloch waves for $W_0=0.2$ has the band structure shown in Fig. 1(c), where the Γ-point is the center of the Brillouin zone, whereas the M-point and X-point are the band edges. It is known that the first and second Bloch band merge together when W_0 approaches the *PT*-symmetry-breaking (phase-transition) point, at which complex eigenvalues μ emerge, although when the *PT*-symmetry condition $V(x, y)=V^*(-x,-y)$ still holds. The phase transition point of the present *PT*-symmetric triangular lattice is at $W_0=1$, which is different from the *PT*-symmetric square lattices, where it is at $W_0=0.5$ [10]. The first bandgap is $0.17\le\mu\le1.8$, while the semi-infinite gap is $\mu\ge2.57$.

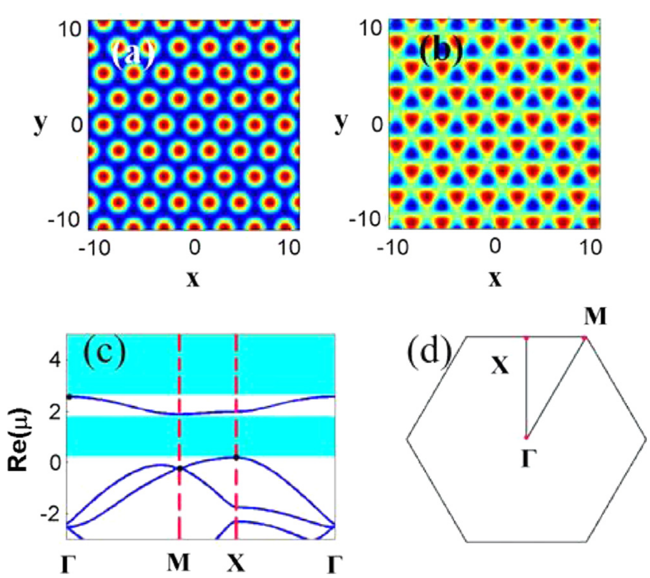

Fig. 1. (a) The real and (b) the imaginary parts of the refractive-index modulation in the *PT*-symmetric triangular photonic lattice. (c) The bandgap structure of the triangular lattice with the *PT*-symmetry for the strength of the imaginary part of potential (2) $W_0=0.2$. (d) The irreducible Brillouin zone.

3. Fundamental solitons

We search for soliton solutions of Eq. (3) in the form of $U(x,y,z)=u(x,y)e^{i\mu z}$, where μ is a real propagation constant and

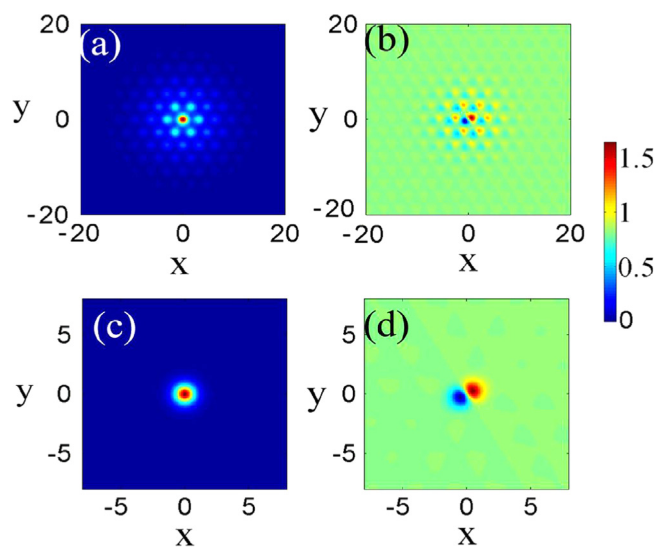

Fig. 2. Profiles of fundamental solitons. (a) and (b): The real and imaginary parts for $\mu=2.61$. (c) and (d): the same for $\mu=5.0$.

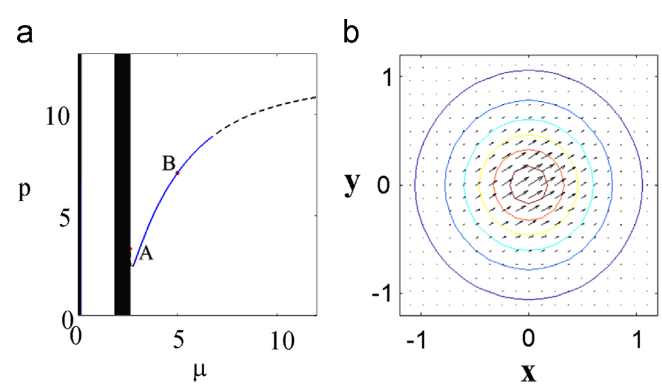

Fig. 3. (a) The power of fundamental solitons versus the propagation constant (black shaded regions are Bloch bands). Solid and dashed segments denote stable and unstable subfamilies, respectively. (b) The transverse distribution of the local power-flux vector in the fundamental *PT*-symmetric soliton solution for $\mu=6$, the contours representing the distribution of the absolute value of the field, $|u(x,y)|$.