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Twin primary rainbows scattering by a liquid-filled capillary

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ABSTRACT

The twin primary rainbows scattered by a liquid-filled capillary are investigated with Debye theory and geometric optics. From the numerical simulations, a critical radius ratio of the core to the coating of the coated cylinder is proposed to judge the existence of the α and β supernumerary bows. When the ratio is less than or around the critical value, the α and β supernumerary bows disappear. On the premise that the α and β supernumerary bows exist, the β rainbow can always be detected. However, the α supernumerary bows sometimes submerge in the other scattering structure so that the *α* rainbow cannot be detected under this condition. Four frequency peaks in the angular frequency spectrum of the twin primary rainbows are investigated numerically. Furthermore the relationship between F_3 (the third frequency peak which is similar to the ripple frequency of the scattering by a homogeneous cylinder) and the external radius of the capillary is obtained. Experiments are carried out to verify the numerical works with capillary filled with deionized water.

1. Introduction

Since the 1980s, rainbow refractometry for droplet/cylinder parameters measurement has been widely investigated owing to its advantages of non-invasion, high precision and real-time performance. The refractive index and size of a droplet (or spray droplets) can be determined by the intensity distribution around the rainbow angle. Roth first used the technique for the parameters measurement of a single homogeneous droplet in 1988 [1–3]. Van Beeck presented a global rainbow refractometry for the size distribution and mean refractive index measurement of the spray droplets in 1999 [4]. However, the refractive index inside most droplets/cylinders in industrial processes is inhomogeneous, e.g. coated particle/cylinder [5–7]. Hiroyuki used the water-filled glass pipes with different internal and external diameters to measure the refractive index of the water by determining the peak location of the scattering intensity [8]. Presby acquired the intensity distribution scattered by different optical fibers, and the relationship between the peak location of β supernumerary bows and the fiber parameters was established [9]. J. A. Lock investigated the rainbow angles and intensity distribution of the twin primary rainbow by the coated spheres with Debye theory and geometric optics under varieties of sizes and refractive indices [10]. From J.A. Lock's work that the coating thickness of the coated cylinder is related to the peak locations of *α* and *β* supernumerary bows, Charles measured the thickness of the water film surrounding a glass cylinder [11,12]. Lock and Laven interpreted a catalog of possible ray paths in a coated sphere/cylinder. The evolution of the α and β primary rainbows was also investigated as the size of core is increased [13,14]. When the core is very small, the scattering diagram of a coated cylinder is similar to that of a homogeneous cylinder with the refractive index of the coating. In some conditions, rays in the core suffer total reflection at the core-coating boundary. Hence, α rays and β rays do not always exist. Even when the α rainbow exists, it may be difficult to detect because it is weaker than other types of scattering.

The paper aims to investigate the existence condition of a twin primary rainbows and the observability of *α* supernumerary bows scattered by a liquid-filled capillary. First, the scattering intensity distributions by a liquid-filled capillary with different parameters are studied numerically through the use of geometric optics and Debye theory. The existence conditions and observability of the twin primary rainbows scattered by a liquid-filled capillary are then investigated. The peaks in the angular frequency spectrum of the twin primary rainbows are explained by calculating the interference of combinations of different rays. Finally, the twin primary rainbows of a liquid-filled capillary are experimentally investigated, and the experimental results of the scattering intensity distributions and angular frequency spectrum by the capillaries are presented and analyzed.

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2. Fundamental theory

2.1. Geometric optics

When a homogeneous cylinder is illuminated by a parallel light beam, the primary rainbow can be observed mainly due to the interference of the rays undergoing one internal reflection [15]. By comparison, for the scattering by a coated cylinder, the internal reflection also happens at the internal surface of the core and coating layers of the cylinder. The rays undergoing one internal reflection at the internal surfaces of the core and coating are named α ray and β ray, respectively. Fig. 1 shows a schematic diagram of the cross-section of a coated cylinder and the typical optical paths of the rays undergoing two kinds of one internal reflection. The coated cylinder consists of two homogeneous layers: the core with the refractive index m_1 and the coating with refractive index m_2. The medium surrounding the coated cylinder is assumed to be air with the refractive index of 1.0. The scattering angles of α ray and β ray are represented by θ^{α} and θ^{β}, respectively. Other five important angles are denoted by $\tau_1 \sim \tau_5$, and the parameter b is the Sine of the incident angle τ_4 in Fig. 1. α and β supernumerary bows are produced by the interferences of α rays and β rays respectively. After the interferences of other rays are superimposed to the α and β supernumerary bows, the twin primary rainbows (α and β primary rainbows) are formed.

From Fig. 1, Eqs. (1)–(3) relevant to the scattering angle of each individual ray can be obtained based on geometrical optics:

$$\tau_2 = \tau_3 + \tau_5 \tag{1}$$

$$\theta^{\alpha} = \pi - 4\tau_1 + 2\tau_4 + 2\tau_5 \tag{2}$$

$$\theta^{\beta} = \pi - 4\tau_1 + 2\tau_4 + 4\tau_5 \tag{3}$$

With Snell law, Eqs. (4–(6) can be obtained

$$\sin(\tau_4) = b \tag{4}$$

$$\sin(\tau_3) = \sin(\tau_4)/m_2 \tag{5}$$

$$\sin(\tau_1) = \sin(\tau_2)\cdot\frac{m_2}{m_1} \tag{6}$$

From Cosine theorem, we can get

$$\cos(\tau_5) = \frac{1}{R}\sin^2(\tau_3) + \sqrt{1 - \frac{b^2}{m_2^2}}\cdot\sqrt{1 - \frac{b^2}{R^2 m_2^2}} \tag{7}$$

with $R = r_1/r_2$.

According to the Eqs. (1)–(7), $\tau_1 \sim \tau_5$ are all the functions of m_1, m_2, R and b. Therefore for a coated cylinder (m_1, m_2 and R are known), a variety of scattering angles (θ^{α}, θ^{β}) of α ray and β ray can be determined under different incident angles.

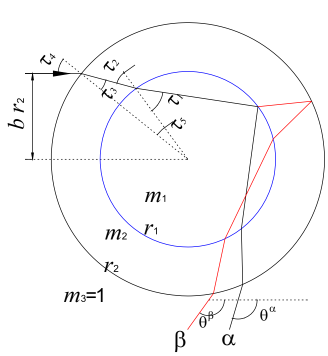

Fig. 1. Schematic of a typical optical path of the rays undergoing one internal reflection in a coated cylinder.

2.2. Debye theory

With geometrical optics each ray can be individually analyzed. Lorenz–Mie theory can accurately compute the scattering intensity of an illuminated coated cylinder [16–18]. Mie coefficients can be decomposed by Debye theory into a series of Debye coefficients so the contribution of each ray to the scattering intensity distribution can then be determined [19]. Debye coefficients of the coated cylinder are expressed as:

$$\left.\begin{array}{l} a_n = \\ b_n = \end{array}\right\} \frac{1}{2}[1 - Q_n^2] \tag{8}$$

where n is the index of infinite series, 1 represents the diffraction, Q_n^2 is the factor taking into account the reflection coefficients and all of the reflected and refracted rays scattered by the coated cylinder. Q_n^2 can be calculated as:

$$Q_n^1 = R_n^{212} + \sum_{p_1=1}^{\infty} T_n^{21} (R_n^{121})^{p_1 - 1} T_n^{12} \tag{9}$$

$$Q_n^2 = R_n^{323} + \sum_{p_2=1}^{\infty} T_n^{32} Q_n^1 T_n^{23} (R_n^{232} Q_n^1)^{p_2 - 1} \tag{10}$$

where Q_n^1 is the factor of a homogeneous cylinder, T_n^{ij} and R_n^{iji} are the reflection coefficients and transmission coefficients. In Eqs. (9) and (10), $p_1 - 1$ and $p_2 - 1$ denote the numbers of the internal reflections at the core and coating layers (R_n^{121} and R_n^{232}) [19,20]. With Debye theory the scattering coefficients of each ray can be expressed, for example the coefficients of β rays are

$$\left.\begin{array}{l} a_n = \\ b_n = \end{array}\right\} T^{32} T^{21} T^{12} R^{232} T^{21} T^{12} T^{23} \tag{11}$$

If the interference of α rays is calculated, the coefficients are expressed as

$$\left.\begin{array}{l} a_n = \\ b_n = \end{array}\right\} T^{32} T^{21} R^{121} T^{12} T^{23} \tag{12}$$

The coefficients of the rays shown in Fig. 2 are

$$\left.\begin{array}{l} a_n = \\ b_n = \end{array}\right\} T^{32} T^{21} T^{12} R^{232} T^{21} T^{12} R^{232} T^{21} T^{12} T^{23} \tag{13}$$

The ray is similar to the β ray in Fig. 1, forming the supernumerary bows of the secondary rainbow.

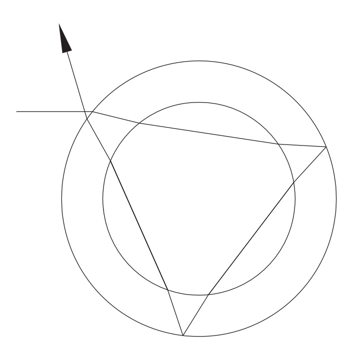

Fig. 2. Light path of the ray $T^{32}T^{21}T^{12}R^{232}T^{21}T^{12}R^{232}T^{21}T^{12}T^{23}$.