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Pulse delay and pulse compression of ultrashort light pulses in tight focusing

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ABSTRACT

The tight focusing of radially polarized ultrashort pulsed laser beam is investigated, based on the Richards–Wolf vector diffraction theory. It is found that pulse delay phenomenon occurs near the focus. This is, near the focus, the ultrashort light pulse slows down. Meanwhile, with the decrease of the velocity the ultrashort pulsed laser beam is compressed in propagation direction. The space compression in propagation direction indicates that, though the pulse duration of the ultrashort light pulse does not change, the spatial pulse length of the ultrashort light pulse is reduced due to the pulse delay phenomenon. It is also shown that the different numerical aperture of the objective exhibits different velocity of the pulse light.

1. Introduction

In recent years, there have been increasing interests in generation and tight focusing of light beams with radially and azimuthally polarized fields due to their unusual properties and potential applications [1–9]. It has been shown that the tight focusing of radially polarized beam has many applications, such as optical trapping, material processing, particle acceleration and high-resolution microscopy etc. [10–13]. On the other hand, ultrashort light pulses have been recently employed in several time-resolved optical measurements with a temporal resolution down to the femtosecond range [14–18]. Meanwhile, the spatial and temporal properties of a focused ultrashort laser pulse may influence the resolution of a produced image [19]. And the polarization of the laser beam also affects both the efficiency and the quality of ultrashort pulse microprocesses [20]. Thus, it is a worthwhile work to explore the conjunction of the radial polarization properties and the ultrashort light pulses. It is known that the focusing of laser beams through a high numerical aperture (*NA*) objective will achieve tighter focal spots which can be used in applications such as microscopy, lithography, optical data storage, optical trapping and plasma physics etc. Therefore, in this study we investigate the focusing properties of radially polarized ultrashort pulsed laser beams through a high numerical aperture objective. Special interests have been paid to the propagation velocity and the pulse width of the focused laser pulses in propagation direction.

2. Theoretical analysis

The electric field of the incident radially polarized Gaussian femtosecond light pulse can be expressed as

$$E(\rho,\ 0,\ t) = E_0\left(\frac{\rho}{w_0}\right)\exp\left[-\left(\frac{\rho}{w_0}\right)^2\right]A(t)\mathbf{e}_r, \tag{1}$$

here E_0 and w_0 are the constant amplitude and the beam size; $\mathbf{e}_r$ is the unit vector of radial polarization. And $A(t)$ is the temporal pulse shape, having Gaussian profile, i.e. [21]

$$A(t) = \exp\left[-\left(\frac{a_g t}{T}\right)^2\right]\exp(-i\omega_0 t). \tag{2}$$

where $a_g = (2\ln 2)^{1/2}$, T is the pulse duration and ω_0 is the central angular frequency. Under sine condition $\rho = f\sin\theta$, we have $(\rho/w_0) = \beta(\sin\theta/\sin\alpha)$, where f is the focal length of the high *NA* objective. As shown in Fig. 1, θ is the numerical-aperture angle, α is the maximum angle of the focus and β is the ratio of the pupil radius to the beam waist of the incident beam [22,23]. Therefore, the pupil apodization function of a single spectral component can be expressed as [16]

$$S(\theta,\ \omega) = \frac{T}{\sqrt{2}a_g}\int_{-\infty}^{\infty}\mathbf{E}(\rho,\ 0,\ t)\exp(i\omega t)dt$$

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$$= \frac{T}{\sqrt{2}a_g}\beta\frac{\sin\theta}{\sin\alpha}\exp\left[-\left(\beta\frac{\sin\theta}{\sin\alpha}\right)^2\right]\exp\left[\frac{-T^2(\omega-\omega_0)^2}{4a_g{}^2}\right]\mathbf{e}_r. \tag{3}$$

It is assumed that the incident femtosecond laser pulse is focused by a circularly symmetric focusing system, as shown in Fig. 1. According to the Debye theory [24], the electric field of a single spectral component of $S(\theta,\omega)$ in the focal region is given by the formula [1,22,24,25]

$$\boldsymbol{E}(r,\varphi,z,\omega) = \begin{bmatrix} E_r(r,\varphi,z,\omega) \\ E_\varphi(r,\varphi,z,\omega) \\ E_z(r,\varphi,z,\omega) \end{bmatrix} = \begin{bmatrix} B\int_0^\alpha \cos^{1/2}\theta \sin(2\theta)S(\theta,\omega)J_1(\frac{\omega}{c}r\sin\theta)e^{-ik(\omega)z\cos\theta}d\theta \\ 0 \\ 2iB\int_0^\alpha \cos^{1/2}\theta\sin^2\theta S(\theta,\omega)J_0(\frac{\omega}{c}r\sin\theta)e^{-ik(\omega)z\cos\theta}d\theta \end{bmatrix}. \tag{4}$$

here r, φ and z are the cylindrical coordinates of an observation point. $c=3\times10^8$ m/s is the velocity of light in vacuum, B is a constant, J_0 and J_1 are the first kind of Bessel functions of order zero and order one, respectively.

Applying a Fourier-transformation, the total electric field of the femtosecond pulse in the focal region can be obtained by the superposition of each spectral component as [16]

$$E_j(r,\varphi,z,t) = \int_0^\infty E_j(r,\varphi,z,\omega)\exp(-i\omega t)d\omega, \quad (j=r,\varphi,z). \tag{5}$$

We get the total intensity near the focus from the formula [8,26,27]

$$I(r,\varphi,z,t) = \sum_{j=r,\varphi,z} I_j(r,\varphi,z,t) = \sum_{j=r,\varphi,z} \left|E_j(r,\varphi,z,t)\right|^2. \tag{6}$$

According to Ref. [28], the centroid of the beam is defined as the weighted average of position by the local intensity. Therefore, by letting $\varphi=0$, we use the beam centroid here to define the average propagation distance of the pulse on z-axis [28]

$$z(\varphi=0,t) = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} zI(r,\varphi,z,t)drdz}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} I(r,\varphi,z,t)drdz}. \tag{7}$$

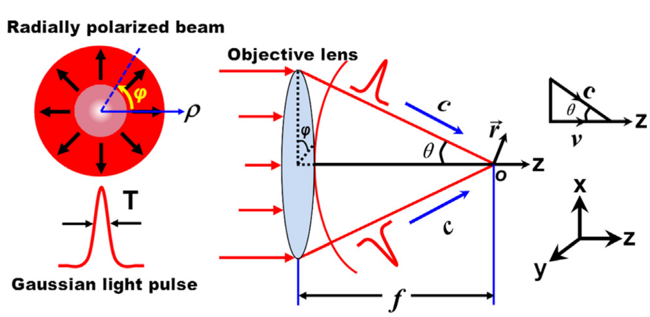

Fig. 1. Tight focusing system.

The velocity of the femtosecond light pulse along the z-axis can be evaluated by the expression [23,29]

$$v(t) = \frac{dz(t)}{dt} \tag{8}$$

Based on the above derived equations, we will investigate the focusing properties of the femtosecond light pulse through a high *NA* objective in the following by some numerical calculations, particularly the propagation velocity and the pulse width of the focused laser pulses in propagation direction.

3. Results and discussions

By use of the preceding formulas, we present some numerical calculations to analyze the focusing properties of radially polarized ultrashort pulsed laser beams through an annular high *NA* objective. Firstly, the intensity distribution of the light pulse in y–z plane near the focus is illustrated by Fig. 2. In particular, the z-component, r-component and total intensity, of the light pulse are shown in Fig. 2(a), (b) and (c) respectively. All the position coordinates are normalized to the central wavelength ($\lambda=249$ nm) of the ultrashort light pulse. And it is noticed that the spot is focused down to the sub-wavelength order.

The propagation evolution of the radially polarized ultrashort pulsed laser beams when it is focused by a high *NA* objective is shown in Fig. 3 (Media a, b, and c). Three video media, of (a), (b) and (c) show the propagation evolutions of the z-component intensity, the r-component intensity and the total intensity near the focus, respectively. It is found that ultrashort light pulse propagates slower when it passes through the focal plane (i.e. $z=0$ plane). As the light pulse continues to propagate its speed increases and its spot becomes larger again. Consequently, being focused by a high *NA* objective the velocity of the light pulse slows near the focus. Furthermore, it is noticed that near the focus the velocity of the z-component is slightly slower than that of the r-component.

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Based on Eq.(8), we can calculate the velocity of the radially polarized ultrashort pulsed laser beams propagating through the focus [28], which is shown in Fig. 4. The pulse velocity is normalized to the velocity of light in vacuum $c=3\times10^8$ m/s. We find that, near the focus v_z (velocity of z-component) is less than v_r (velocity of r-component) and v_{z+r} (velocity of total light pulse), and in the

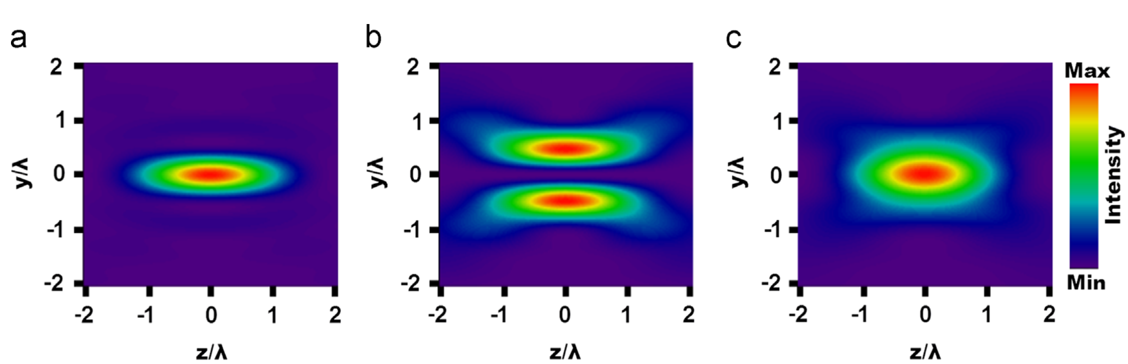

Fig. 2. Contour plots of the intensity distributions near focus: (a) z-component I_z; (b) r-component I_r; and (c) total intensity I_{z+r}. The other parameters are chosen as $\omega_0=7.57\times10^{15}$ s^{-1}, $T=5$ fs, $NA=0.9$, $\beta=1.3$, $t=0$ fs.