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Astigmatism error modification for absolute shape reconstruction using Fourier transform method

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ABSTRACT

A method is proposed to modify astigmatism errors in absolute shape reconstruction of optical plane using Fourier transform method. If a transmission and reflection flat are used in an absolute test, two translation measurements lead to obtain the absolute shapes by making use of the characteristic relationship between the differential and original shapes in spatial frequency domain. However, because the translation device cannot guarantee the test and reference flats rigidly parallel to each other after the translations, a tilt error exists in the obtained differential data, which caused power and astigmatism errors in the reconstructed shapes. In order to modify the astigmatism errors, a rotation measurement is added. Based on the rotation invariability of the form of Zernike polynomial in circular domain, the astigmatism terms are calculated by solving polynomial coefficient equations related to the rotation differential data, and subsequently the astigmatism terms including error are modified. Computer simulation proves the validity of the proposed method.

1. Introduction

Interferometric test for surface flatness of optical elements is constantly gaining importance. The tested result of a common-path interferometer, such as a Fizeau interferometer, includes the common shape information of the reference and test surfaces, which is reliable as long as the reference shape's deviation from flatness may be negligible compared to the test one. However, if the accuracies of the two surfaces are near or comparable, it is necessary to calibrate the shape of the reference surface.

Absolute tests are usually used to calibrate the shape of the reference surface, whose substance is to separate it from the shape of the test surface. The best known of the absolute tests is three-flat test [1–12]. Three-flat test needs three flats, two of them as transmission flats, which should have the same diameter, approximate wedge magnitude. When such a second transmission flat for an interferometer is not available, three-flat test cannot be performed.

Another kind method of absolute test is two-flat test method, whose advantage compared to three-flat test is no demand of a second transmission flat. Among two-flat test methods, the typical one is Zernike polynomial fitting method [13–15], in which reconstruction accuracy is related to the employed polynomial number. Generally more employed terms means more considerable accuracy, but more calculation amount. A method based on Fourier transform is proposed by Frank Morin in 2006 [16], in which four translation measurements are performed, and in the shape reconstruction process the characteristic relationship between the differential shape and original shape is utilized in spatial frequency domain. In this method, power and astigmatism errors occur in the reconstructed results because additional tilt errors are introduced into the translation measurement data.

In fact, the absolute shapes of the reference and test surfaces can be obtained with only two translation measurements by using Fourier transform method, just as done in wavefront reconstruction form shear or differential phase maps [17–19]. However, the same as Frank Morin's method, power and astigmatism errors exist in the reconstructed result. In this paper, an astigmatism error modification method is proposed. In Section 2, the principle of a two-translation-measurement method based on Fourier transform is described. In Section 3, the reason of generating astigmatism error is analyzed carefully, and the composition of astigmatism errors is disclosed. A method is presented to modify astigmatism errors. In Section 4, a simulation experiment is carried out to validate the proposed method.

2. Two-flat absolute test principle based on Fourier transform

Frank Morin first proposed a two-flat absolute test method based on Fourier transform, in which four translation measurements are performed [16]. In fact, the shapes of the reference and

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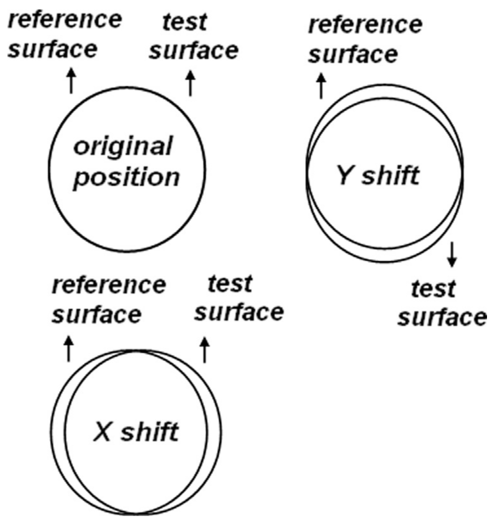

Fig. 1. Measurement process demonstration.

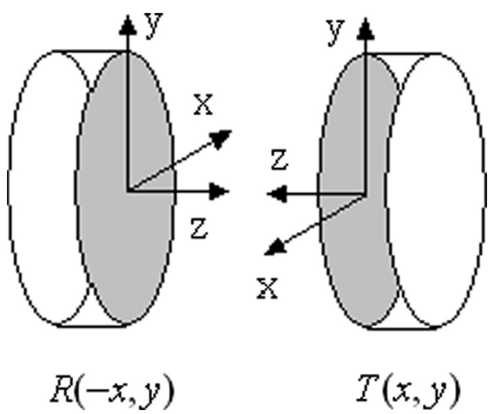

Fig. 2. Measurement coordinate system.

test surface can be obtained using two translation measurements, similar to wavefront reconstruction form shear or differential phase maps [17–19]. Fig. 1 shows the measurement process demonstration. The detailed principle is as follows.

If an optical plane is measured by a Fizeau interferometer, the measurement data is the comparison result between the shapes of the reference and test surface, which can be expressed as follows:

$$W_0(x,y)=R(-x,y)+T(x,y) \tag{1}$$

where R and T respectively denote the shapes of the reference and test surface. The measurement coordinate system is shown in Fig. 2. If the test flat is translated for a distance of s in X positive direction, the measurement result can be expressed as follows:

$$W_1(x,y)=R(-x,y)+T(x-s,y) \tag{2}$$

From Eq. (1) and Eq. (2), the differential shape of the test surface in X direction can be obtained:

$$D_x(x,y)=T(x,y)-T(x-s,y) \tag{3}$$

Therefore the following equation can be deduced:

$$FT(T)=\frac{1}{[1-\exp(-i2\pi us)]}FT(D_x)=\frac{[1-\exp(i2\pi us)]}{4\sin^2(\pi us)}FT(D_x) \tag{4}$$

where u denotes the spatial frequency in X direction, and $FT(\cdot)$ carrying out Fourier transform. Namely a characteristic relationship between the differential shape and original shape exists in spatial frequency domain. If a translation of the test flat is performed in Y direction, and the translation distance is also s, the following equation can be obtained:

$$FT(T)=\frac{1}{[1-\exp(-i2\pi vs)]}FT(D_y)=\frac{[1-\exp(i2\pi vs)]}{4\sin^2(\pi vs)}FT(D_y) \tag{5}$$

where v denotes the spatial frequency in Y direction, and D_y the differential shape of the test surface in Y direction. From Eq. (4) and Eq. (5), it can be deduced:

$$FT(T)=\frac{[1-\exp(i2\pi us)]FT(D_x)+[1-\exp(i2\pi vs)]FT(D_y)}{4[\sin^2(\pi us)+\sin^2(\pi vs)]} \tag{6}$$

It can be seen from Eq. (6) that if us and vs are both integers, the corresponding spectrum components cannot be recovered, and they have to be valued at 0. If inverse Fourier transform is performed on Eq.(6), the shape of the test surface can be obtained[19]:

$$T(x,y)=\iint\frac{[\exp(-i2\pi us)-1]FT(D_x)+[\exp(-i2\pi vs)-1]FT(D_y)}{4[\sin^2(\pi us)+\sin^2(\pi vs)]}\times\exp[i2\pi(ux+vy)]dudv \tag{7}$$

Therefore the shape of the test surface can be recovered except the frequency spectrum components corresponding to periods of s/n, $n=1,2,\ldots$. When s is small, the lost components are trivial, which can be ignored. Subsequently, the shape of the reference surface can be obtained from Eq. (1).

3. Astigmatism error modification method

3.1. Astigmatism error analysis

In fact, it is difficult to perform the translations without any tilt contributions. Because the translation device cannot guarantee the test and reference flats rigidly parallel to each other after the translations, an additional tilt is introduced to the measured data, at least at the tenths of nanometers level, whose magnitude is not known to us. As a result, the obtained differential data in X and Y directions both have tilt errors, which cause power and astigmatism errors in the reconstructed shape [14]. Computer stimulation in Section 4 shows that if the reconstructed shape is regarded as the summation of a series of Zernike polynomials, the reconstruction error includes not only power, primary astigmatism, but also high-order astigmatisms of the terms of $r^{4k}\sin(4k\theta)$, $k=1,2,3,\ldots$, where r and θ respectively denote the radial variable and angle variable in polar coordinates.

3.2. Astigmatism error modification

As known, astigmatism terms are related to angle variable in polar coordinates.

Therefore they can be obtained by adding rotation measurements. In fact, only one additional rotation measurement is needed to obtain the real magnitude of the error astigmatism terms. And the detailed principle is as follows.

If a measurement is carried out after the test flat is rotated by the angle of φ in the clockwise direction, the difference between the data measured before and after rotation can be expressed as in polar coordinates:

$$E(r,\theta)=T(r,\theta)-[T(r,\theta)]^{\phi} \tag{8}$$

where $[\cdot]^{\varphi}$ denotes the operation of rotating some data by the angle of φ in the clockwise direction. Fig. 3 shows the demonstration of the added rotation measurement. The shape of the test surface can be regarded as the summation of a series of Zernike polynomials:

$$T(r,\theta)=\sum_{n,l}J_n^l[z_n^l\cos(l\theta)+z_n^{-l}\sin(l\theta)] \tag{9}$$