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Self-focusing of the high intensity ultra short laser pulse propagating through relativistic magnetized plasma

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ABSTRACT

In this paper, evolution of the spot size of the ultra short intense laser beam propagating in underdense magnetized cold plasma, taking into account the nonlinearity up to third order and the relativistic effect, has been studied. The plasma embedded in a constant external magnetic field that is set in the plane perpendicular to the electric field vector of the laser beam with different directions. The paraxial wave equation in plasma has been used and the source dependent expansion (SDE) method is employed to solve the equation. Using continuity equation and equation of motion for plasma electrons in the electric field of laser beam a set of equations for the evolution of laser beam structure in plasma is found. Results show that imposing the external magnetic field enhances self-focusing property of the laser beam. Taking into account the relativistic effect increases the effect of the external magnetic field on self-focusing of the laser beam. Increasing the angle between the laser beam magnetic field and external magnetic field will decrease the self-focusing property.

1. Introduction

Because of wide range of applications in many important fields such as optical harmonic generation, wake field acceleration, x-ray lasers, and plasma based light sources [1–4], nonlinear interaction of intense laser beam and plasma has attracted attention of researchers.

Intense lasers are great sources of energy in multi-mode media such as plasma capable to excite nonlinear phenomena in this media [5]. In this case several nonlinear phenomena will appear in the interaction region. One of them is self-focusing and defocusing of high intensity laser beam propagating through plasma [6,7]. Gaussian profile of the intense laser beam leads to modifying the plasma refractive index which may lead to self-focusing or defocusing of the laser beam in the propagation path. According to $n=\sqrt{1-4\pi n_0 e^2/m\omega_0^2}$, in which n is the refractive index of plasma, n_0 is the plasma density, m and e are the electron mass and electrical charge respectively, and ω_0 is the laser beam angular frequency, the plasma refractive index is sensitive to variation of density and mass of the plasma electrons. Variation of plasma density and electron mass changes the refractive index of plasma in opposite ways. Increasing plasma density locally leads to decrease plasma refractive index locally while increasing the mass of the electrons increases plasma refractive index. Therefore for enhancing the self-focusing property of the laser beam, we must decrease density of the electrons on the axis of plasma column. Also, increasing the electron mass due to relativistic effect from stronger electric field at the axis of plasma column will increase self-focusing property.

Density and mass of the plasma electrons can be modified by thermal, relativistic and ponderomotive effects [8,9]. Also, any external force can affect density and velocity of the electrons in plasma. So, imposing external magnetic field can modify the density profile of the plasma electrons by changing the electrons velocity and generate a new component for electrons velocity vector. Changing in the electrons density and velocity vector modifies dispersion relation of the laser beam and nonlinear current density of the plasma electrons. Therefore, imposing external magnetic field can modify self-focusing property of the laser beam propagating in plasma. The direction of the external magnetic field vector is an important parameter in the influence of the external magnetic field on self-focusing property. Changing in direction of the external magnetic field can change contribution of the electrons velocity components therefore modifies electrons current density.

Ghorbanalilu has studied the spot size evolution of the laser beam propagating in magnetized plasma [10]. In his work the external magnetic field is taken to be constant, in the laser beam

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propagation direction. He has shown that the axially magnetic field decreases the minimum spot size and increases the focal length significantly. Jha et al. have studied the self-focusing of the laser beam in magnetized plasma but they have assumed that external magnetic field vector was set in the direction of the laser beam magnetic field vector [11]. They have shown that transverse magnetization of plasma enhances the self-focusing property of the laser beam. Xing et al. have studied self-focusing of the laser beam propagating in plasma with the presence of external magnetic field with arbitrary direction but the relativistic effect has been neglected in their work [12].

In the present paper, we consider the effect of external magnetic field on the spot size evolution of the laser beam with Gaussian profile propagating in under dense cold plasma. We take into account relativistic effect and nonlinearity up to third order. In our model the external magnetic field vector is set in the plane that is perpendicular to the laser beam electric field. As an important parameter, the angle between external magnetic field vector and the laser beam magnetic field vector θ is variable. The effects of the direction and strength of external magnetic field on the spot size evolution of laser beam propagating through plasma have been considered. We have investigated the relation between external magnetic field and required critical laser beam power for self-focusing. Also, the effect of incident laser beam intensity on spot size evolution has been studied. It should be noted that, we investigated the evolution of the laser beam in few Rayleigh length propagation distance. In this short propagation distance, the Faraday rotation angle of laser beam polarization is very small. So, we ignored the Faraday rotation effect in our model. Of course, the Faraday rotation effect is an important mechanism in long propagation distance.

This paper is organized as follows. Following the introduction in Section 1, in Section 2 the nonlinear plasma current density as a source of the wave equation governing the propagation laser beam in magnetized plasma has been calculated. In Section 3 the equation governing the evolution of the laser beam spot size in length of the propagation is obtained and critical power of the laser beam is defined. Section 4 is devoted to results and discussion and finally, conclusion is presented in Section 5.

2. Basic equations

The start point is the wave equation governing the propagation of the laser beam in plasma, given by

$$\left(\nabla^2-\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}=\frac{4\pi}{c^2}\frac{\partial \mathbf{J}}{\partial t} \tag{1}$$

where $\mathbf{E}$ is the electric field vector, $\mathbf{J}=-en\mathbf{v}$ is the plasma electron current density vector and e, n, and $\mathbf{v}$ are electron charge, density and velocity, respectively. We assume that the laser beam propagates along the z direction with linearly polarization. In this case, the laser beam electric field $\mathbf{E}(r,z,t)$ along the x direction is given by

$$\mathbf{E}(r,z,t)=\frac{1}{2}E_0(r,z,t)e^{i(k_0 z-\omega_0 t)}\mathbf{i}+c.c. \tag{2}$$

in which $E_0(r,z,t)$ is the complex amplitude of laser beam, k_0 is the carrier wave number, ω_0 is the carrier frequency, and $c.c.$ denotes the complex conjugate. Relativistic interaction between laser beam and plasma electrons is governed by the motion and the continuity equations

$$\frac{d(\gamma\mathbf{v})}{dt}=-\frac{e}{m}\mathbf{E}-\frac{e}{cm}\mathbf{v}\times(\mathbf{B}+\mathbf{B}_0) \tag{3}$$

and

$$\frac{\partial n}{\partial t}+\nabla.(n\mathbf{v})=0 \tag{4}$$

here $\gamma=(1+v^2/c^2)^{1/2}$ is the relativistic factor, m is the electron mass, $\mathbf{B}$ is the magnetic field vector of laser beam, and $\mathbf{B}_0$ is the external magnetic field vector that is set in the z–y plane as follows:

$$\mathbf{B}_0=B_0(\sin\theta\mathbf{k}+\cos\theta\mathbf{j}) \tag{5}$$

where θ is the angle between $\mathbf{B}_0$ and y-axis. At this point, we use the perturbational technique to obtain the coupled equations for velocity components

$$\frac{\partial v_x^{(1)}}{\partial t}=-\frac{e}{m}E-\omega_c v_y^{(1)}\sin\theta+\omega_c v_z^{(1)}\cos\theta \tag{6a}$$

$$\frac{\partial v_y^{(1)}}{\partial t}=\omega_c v_x^{(1)}\sin\theta \tag{6b}$$

$$\frac{\partial v_z^{(1)}}{\partial t}=-\omega_c v_x^{(1)}\cos\theta \tag{6c}$$

in which $\omega_c=eB_0/mc$ is the cyclotron frequency of the plasma electrons. Taking velocity as $v^{(1)}=v^{(1)}(r,z)e^{i(k_0 z-\omega_0)}+c.c.$ one can obtain the solutions of the first order approximated velocity components as

$$v_x^{(1)}=\frac{\omega_0^2 c}{2i(\omega_0^2-\omega_c^2)}ae^{i(k_0 z-\omega_0 t)}+c.c \quad ., \tag{7a}$$

$$v_y^{(1)}=\frac{\omega_c\omega_0 c\sin\theta}{2(\omega_0^2-\omega_c^2)}ae^{i(k_0 z-\omega_0 t)}+c.c \quad ., \tag{7b}$$

$$v_z^{(1)}=-\frac{\omega_c\omega_0 c\cos\theta}{2(\omega_0^2-\omega_c^2)}ae^{i(k_0 z-\omega_0 t)}+c.c \quad ., \tag{7c}$$

where $a=eE_0/mc\omega_0$ is the normalized laser beam field amplitude. Eqs. (7) reveal that presence of the external magnetic field increases the transverse quiver velocity. Eq. (7a) also leads to generation of a new transverse and longitudinal component of velocities of Eqs. (7b) and (7c). This fluctuation in velocity vector of electrons changes the density distribution and mass of electron and results in modification of the refractive index of plasma. Using the second-order expansion of Eq. (3), we find the following set of equations of velocity components:

$$\frac{\partial v_x^{(2)}}{\partial t}+(\mathbf{v}^{(1)}\cdot\nabla)v_x^{(1)}=\frac{ek_0}{m\omega_0}v_z^{(1)}E-\omega_c\sin\theta v_y^{(2)}+\omega_c\cos\theta v_z^{(2)} \tag{8a}$$

$$\frac{\partial v_y^{(2)}}{\partial t}+(\mathbf{v}^{(1)}\cdot\nabla)v_y^{(1)}=\omega_c\sin\theta v_x^{(2)} \tag{8b}$$

$$\frac{\partial v_z^{(2)}}{\partial t}+(\mathbf{v}^{(1)}\cdot\nabla)v_z^{(1)}=-\frac{ek_0}{m\omega_0}v_x^{(1)}E-\omega_c\cos\theta v_x^{(2)} \tag{8c}$$

Using the first-order velocities, the solutions of the above set of equations are

$$v_x^{(2)}=\frac{k_0c^2\omega_c\omega_0^2(\omega_0^2-4\omega_c^2)\cos\theta}{4i(\omega_0^2-\omega_c^2)^2(4\omega_0^2-\omega_c^2)}a^2e^{2i(k_0 z-\omega_0 t)}+c.c. \tag{9a}$$

$$v_y^{(2)}=-\frac{3k_0c^2\omega_c^2\omega_0(\omega_0^2+\omega_c^2)\cos\theta\sin\theta}{8(\omega_0^2-\omega_c^2)^2(4\omega_0^2-\omega_c^2)}a^2e^{2i(k_0 z-\omega_0 t)}+c.c. \tag{9b}$$

$$v_z^{(2)}=\frac{k_0c^2\omega_0(-4\omega_0^4+5\omega_c^2\omega_0^2-\omega_c^4+3\omega_c^2(\omega_0^2+\omega_c^2)\cos^2\theta)}{8(\omega_0^2-\omega_c^2)^2(4\omega_0^2-\omega_c^2)}a^2e^{2i(k_0 z-\omega_0 t)}+c.c. \tag{9c}$$

Eqs. (9) show that the second-order transverse components of velocity is generated due to external magnetic field. Furthermore,