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Building patterns by traveling dipoles and vortices in two-dimensional periodic dissipative media

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ABSTRACT

We analyze pattern-formation scenarios in the two-dimensional (2D) complex Ginzburg–Landau (CGL) equation with the cubic–quintic (CQ) nonlinearity and a cellular potential. The equation models laser cavities with built-in gratings, which stabilize 2D patterns. The pattern-building process is initiated by kicking a compound mode, in the form of a dipole, quadrupole, or vortex which is composed of four local peaks. The hopping motion of the kicked mode through the cellular structure leads to the generation of various extended patterns pinned by the structure. In the ring-shaped system, the persisting freely moving dipole hits the stationary pattern from the opposite side, giving rise to several dynamical regimes, including periodic elastic collisions, i.e., persistent cycles of elastic collisions between the moving and quiescent dissipative solitons, and transient regimes featuring several collisions which end up by absorption of one soliton by the other. Still another noteworthy result is the transformation of a strongly kicked unstable vortex into a stably moving four-peaked cluster.

1. Introduction

The fundamental principle behind the creation of dissipative solitons is that their stability relies upon the simultaneous balance of conservative and dissipative ingredients in the underlying system [1]. These are the diffraction and self-focusing nonlinearity in the conservative part of the system, and the linear and non-linear loss and gain terms in the dissipative part. Well-known physical realizations of such systems are offered by lasing [2,3] and plasmonic [4] cavities, the respective models being based on the complex Ginzburg–Landau (CGL) equations with the cubic–quintic (CQ) set of gain and loss terms, combined with the background linear loss [3]. This combination is well known to maintain stable localized modes [5]. The CGL equations constitute a generic class of dissipative pattern-formation models [6], which find many other applications, including bosonic condensates of quasi-particles in solid-state media [7], reaction–diffusion systems [8], and superconductivity [9].

Originally, the CGL equation of the CQ type was introduced [5] as a model for the creation of stable two-dimensional (2D) localized modes. Following this work, similar models were derived or proposed as phenomenological ones in various settings. Many 1D and 2D localized states, i.e., dissipative solitons, have been found as solutions of such equations [10–15].

An essential ingredient of advanced laser cavities is a transverse periodic grating, which can be fabricated by means of available technologies [16]. In addition to the permanent gratings, virtual photonic lattices may be induced in photorefractive crystals as interference patterns by pairs of pump beams with the ordinary polarization, which illuminate the crystal along the axes x and y, while the probe beam with the extraordinary polarization is launched along z [17]. A 2D cavity model with the grating was introduced in Ref. [18]. It is based on the CQ-CGL equation including the cellular (lattice) potential, which represents the grating. In fact, the laser cavity equipped with the grating may be considered as a photonic crystal built in the active medium. Periodic potentials also occur in models of passive optical systems, which are driven by external beams and operate in the temporal domain, unlike the active systems which act in the spatial domain [19–21].

Localized vortices, alias vortex solitons, are an important species of self-trapped modes in 2D settings. In uniform media, dissipative vortex solitons cannot be stable without the presence of a diffusion term, in the framework of the CGL equation (see, e.g., Ref. [12]). However, this term is absent in models of waveguiding

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systems (it may sometimes be present in temporal-domain optical models [22]). Compound vortices, built as complexes of four peaks pinned to the lattice potential, may be stable in models including the grating in the absence of the diffusion [18]. Using this possibility, stable 2D [23] and 3D [24] vortical solitons have been found in the framework of CGL equations including trapping potentials.

In a majority of previous works, the studies of various 2D localized patterns have been focused on their stabilization by means of the lattice potentials. Another relevant issue is the mobility of 2D dissipative solitons in the presence of the underlying lattice (dissipative solitons may move without friction only if the diffusion term is absent, therefore the mobility is a relevant issue for the diffusion-free models of laser cavities). Localized modes can be set in motion by the application of a kick to them, which, in the context of the laser-cavity models, implies launching a tilted beam into the system. Recently, the mobility of kicked 2D fundamental solitons in the CQ-CGL equation with the cellular potential was studied in Ref. [25]. It has been demonstrated that the kicked soliton, hopping through the periodic structure, leaves in its wake various patterns in the form of single- or multi-peak states trapped by the periodic potential. In the case of periodic boundary conditions (b.c.), which correspond to an annular system, the free soliton completes the round trip and hits the pattern that it has originally created. Depending on parameters, the free soliton may be absorbed by the pinned mode (immediately, or after several – up to five – cycles of quasi-elastic collisions), or the result may be a regime of periodic elastic collisions, which features periodic cycles of passage of the moving soliton through the quiescent one.

A natural extension of the analysis performed in Ref. [25] is the study of the mobility of kicked soliton complexes, such as dipoles, quadrupoles, and compound vortices, and various scenarios of the dynamical pattern formation initiated by such moving complex modes, in the framework of the 2D CQ-CGL equation with the lattice potential. This is the subject of the present work. In fact, such configurations are truly two-dimensional ones, while the dynamical regimes for kicked fundamental solitons, studied in Ref. [25], actually represent quasi-1D settings. The model is formulated in Section 2, which is followed by the presentation of systematic numerical results for dipoles, quadrupoles, and vortices of two types, onsite- and offsite-centered ones (alias "rhombuses" and "squares") in Sections 3, 4, and 5, respectively. The paper is concluded in Section 6.

An essential finding is that the interaction of a freely moving dipole with pinned patterns, originally created by the same kicked dipole, gives rise to new outcomes under the periodic b.c. In particular, the quiescent dipole can be absorbed (cleared) by the moving one, which may have obvious applications to the design of all-optical data-processing schemes, where one may need to install or remove a blocking soliton. Also noteworthy is the transformation of an unstable vortex by a strong kick into a stable moving four-soliton cluster.

2. The cubic–quintic complex Ginzburg–Landau model with the cellular potential

The CQ-CGL equation with a periodic potential is written as

$$\frac{\partial u}{\partial Z}=\left[-\delta+\frac{i}{2}\nabla_{\perp}^{2}+(i+\epsilon)|u|^{2}-(i\nu+\mu)|u|^{4}+iV(X,Y)\right]u. \quad (1)$$

It describes the evolution of the amplitude of electromagnetic field $u(X,Y,Z)$ along propagation direction Z, with transverse Laplacian $\nabla_{\perp}^{2}=\partial^{2}/\partial X^{2}+\partial/\partial Y^{2}$. Parameter δ is the linear-loss coefficient, ϵ is the cubic gain, μ the quintic loss, and ν the quintic self-defocusing coefficient (it accounts for the saturation of the Kerr effect if $\nu>0$).

The 2D periodic potential with amplitude V_0 is taken in the usual form, $V(X,Y)=V_0[\cos(2X)+\cos(2Y)]$, where the normalization of the field and coordinates is chosen so as to make the normalized period equal to π, which is always possible. The total power of the field is also defined as usual:

$$P=\iint|u(X,Y)|^{2}\,dX\,dY. \quad (2)$$

We solved CGL equation (1) by means of the fourth-order Runge–Kutta algorithm in the Z-direction, and five-point finite-difference scheme for the computation of the transverse Laplacian $\nabla_{\perp}^{2}$. Periodic boundary conditions (b.c.) were used for the study of kicked dipoles and quadrupoles, and absorbing b.c. for kicked vortices. In the latter case, the absorbing b.c. are implemented by adding a surrounding linear-absorption strip to the computation box. The absorption coefficient varies quadratically with X and Y from zero at the internal border of the strip to a value large enough to induce complete absorption of any outgoing pulse, at its external border. This smooth variation, if the width of the strip is not too small, allows one to suppress any reflection from the absorption strip.

Values of coefficients chosen for numerical simulations are $\delta=0.4$, $\epsilon=1.85$, $\mu=1$, $\nu=0.1$, and $V_0=-1$. This choice corresponds to a set of parameters for which the initial static configurations for the dipoles, quadrupoles, and vortices are stable (in-phase bound states of two dissipative solitons are also possible, but, unlike the dipoles, with the phase shift of π between the bound solitons, they are unstable). The kick is applied to them in the usual way, by adding the linear phase profile to the initial field:

$$u_0(X,Y)\rightarrow u_0(X,Y)\exp(i\mathbf{k}_0\cdot\mathbf{r}), \quad (3)$$

where $\mathbf{r}\equiv\{X,Y\}$. The key parameters are length k_0 of kick vector $\mathbf{k}_0$, and angle θ which it makes with the X-axis:

$$\mathbf{k}_0=(k_0\cos\theta, k_0\sin\theta). \quad (4)$$

In the laser setup the kick corresponds to a small deviation of the propagation direction of the beam from the Z-axis. If K_0 is the full wave number and φ is the deviation angle, the length of the transverse wave vector in physical units is $K_0\sin\varphi$, which corresponds to k_0 in the normalized form. Below, we investigate the influence of kick parameters k_0 and θ, defined as per Eq. (4), on a variety of multi-soliton complexes, which are created by moving dipoles, quadrupoles, or vortices (of both onsite- and offsite-centered types) in the 2D CGL medium with the cellular potential.

3. The pattern formation by kicked dipoles

3.1. Generation of multi-dipole patterns by a dipole moving in the transverse direction

In this section we consider the simplest soliton complex in the form of a stable vertical dipole, which consists of a pair of solitons aligned along the Y-axis and mutually locked with phase difference π, which is shown in Fig. 1. The same color code as in Fig. 1 (a) is used in all figures showing amplitude distributions throughout the paper. First, the dipole is set in motion by the application of the kick in the horizontal (X) direction (i.e., transverse to the dipole's axis), as per Eqs. (3) and (4) with $\theta=0$.

As shown in Fig. 2, the moving dipole multiplies into a set of secondary ones, similar to the outcome of the evolution of the kicked fundamental soliton [25]. Each newly created dipole features the fixed phase shift π between two constituent solitons, and the entire pattern, established as the result of the evolution, is robust. The particular configuration displayed in Fig. 2 is a chain of five trapped dipoles, and a free one, which has wrapped up the motion and reappears from the left edge, moving to the right, due