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Self-trapped elliptical super-Gaussian beam in cubic–quintic media

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ABSTRACT

We find self-trapped propagation of elliptical super-Gaussian beam in cubic–quintic nonlinear media. The soliton beam preserves its shape and size during propagation in Kerr media. Both defocusing and focusing quintic nonlinearities are considered. In a cubic (focusing)-quintic (defocusing) media breather like beam propagation with intriguing beam width oscillation is observed. The influence of beam ellipticity, super-Gaussian nature and quintic nonlinearity on self-trapping has been studied. A formula for critical power for self-focusing has been derived and it readily agrees with the results obtained by variational method. In Kerr and focusing quintic media beam collapse occurs quicker for higher order super-Gaussian beam. The critical power of self-focusing in defocusing (focusing) quintic medium prominently increases (decreases) with increasing strength of quintic nonlinearity. This variation rate is greater for higher order super Gaussian beam. A beam with greater ellipticity requires larger power for self-trapping.

1. Introduction

Nonlinear optical beam propagation and consequent formation of spatial optical soliton has been a fascinating topic of both fundamental and applied research in nonlinear optics [1–4]. This area of research is highly active due to its versatility and potential application in optical communication, all-optical device fabrication and many other fields. Spatial optical solitons are optical beams confined in directions transverse to the propagation axis. In other words, they are optical beams of invariable or periodically fluctuating cross section. Owing to their additional degrees of freedom along transverse dimension spatial solitons are much more versatile over their temporal counterpart. Beyond optics, spatial solitons are familiar in mathematics (in fact, the idea of soliton was conceived from integrable system), mechanical engineering, fluid dynamics, thermodynamics, pattern formation, biology, etc. Even spatial solitons are available in nature. For example, the tsunami waves, sand dunes, some strange cloud formation, namely, 'morning glory' cloud or 'rolling cloud' [5]. However, the mechanism of such soliton formation is very complicated and topic of fundamental multidisciplinary research. Our topic of interest, i.e., optical spatial soliton, arises due to counter balance of self-diffraction with nonlinearity induced self-focusing in conservative media. They have been studied to address different aspects in various media that ranges from turbulent atmosphere to media with higher order nonlinearity. Also, a wide variety of beam profile, starting from simple hyperbolic secant to complicated decentered elliptical Hermite–Gaussian beam (DEHGB) has been adopted [6–11]. Generally, Kerr media supports $(1+1)$ D stable soliton of hyperbolic secant profile that encouraged the early stage research to adopt the aforesaid profile or more convenient Gaussian profile. However, soliton of similar forms can be expected for higher dimension too. Attempts have been made to explore the possibility of soliton formation with more complex beam profiles. Sinusoidal-Gaussian and hyperbolic–sinusoidal-Gaussian beam propagation has been studied in quadratic nonlinear media [6]. The propagation properties of cosh-Gaussian beams [7], Hermite cosh-Gaussian beams [8], elegant Hermite–cosh-Gaussian beams [9], Hermite cosine-Gaussian beams [10] and off-axial Hermite–cosh-Gaussian beams [11] have been widely studied. Self-focusing of necklace beams [12], self-trapped vector waves [13] and self-trapping, compression and decompression of Bessel beams has been studied in Kerr media [14]. A sinh-Gaussian beam propagating through Kerr medium eventually converts into sin-Gaussian type beam at low and moderate initial power [15]. Also transformation from cosh-Gaussian to flat-top beam has been achieved in cubic–quintic nonlinear media [16].

Laguerre–Gaussian (LG) beam Propagation has been studied in a local cubic–quintic nonlinear medium, using the variational approach [17]. Generation and propagation of two-dimensional vortex solitons, has been studied in presence of nonlocal focusing nonlinearity, where the nonlocality stabilizes the dynamics of an otherwise unstable vortex beam [18]. Stable spatiotemporal spinning solitons with internal vorticity in a bimodal system has also

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been revealed [19]. Propagation of Lorentz and Lorentz–Gauss beams through an apertured fractional Fourier transform optical system has been examined [20]. Report is available on stable propagation of $(2+1)$-dimensional spinning ring solitons in cubic–quintic medium [21]. A family of stable ring-profile vortex solitons can exist in defocusing cubic nonlinear media with an imprinted Bessel optical lattice [22].

Considerable work has been done on non-circular beam profiles too. In a pioneer paper by Cornolti et al. the nonlinear dynamics of elliptic Gaussian beam is demonstrated through a semi-analytical treatment in the framework of WKB, paraxial approximation [23]. Thereafter the propagation of elliptical Gaussian beams has been studied through circular aperture [24], saturating nonlinear medium [25], misaligned optical system [26] and even turbulent atmosphere [27]. The dynamics of decentered elliptical Hermite–Gaussian (DEHGB) passing through a nonsymmetrical paraxial optical system has been studied [28]. Hollow elliptical Gaussian beam propagation through aligned and misaligned paraxial optical systems has also been investigated [29]. Elliptical cosh-Gaussian beam has attracted considerable attention [30,31]. Propagation of such beams has been studied in axially non symmetrical ABCD optical system [30] and misaligned system [31]. Flat top beam is yet another profile of interest. Flat-top Gaussian beams can be considered as sum of Laguerre–Gaussian beams of successive order. Flat-top Gaussian beams have been studied in turbulent media and scintillation index have been calculated [32]. Stable soliton of super-Gaussian shape can be achieved in cubic–quintic nonlinear media with proper choice of parameters [33]. Doughnut-shaped super-Gaussian beams [34] and axially symmetric flattened Gaussian beams [35] have also been reported. Recent study show the generation of two-dimensional matter-wave gap solitons trapped in an elliptically deformed concentric lattice potential, within the framework of the Gross–Pitaevskii equation with self-attraction or self-repulsion [36]. In the attractive case families of elliptic annular solitons and double solitons are observed for a fixed eccentricity of the lattice. Whereas, the repulsive case can only yields the family of double solitons. Yet another recent work investigates families of fundamental and dipole-mode solitons in an optical media with elliptically diffusive nonlinearity [37]. Using the variational approximation and numerical method impact of anisotropic nonlocality on the arrest of the collapse and stabilization of dipole-mode solitons in two-dimensional models has been determined.

Apart from Kerr or cubic nonlinearity, beam propagation has been studied in media with higher order nonlinearity. Introduction of such nonlinearity significantly modifies the beam propagation, more importantly, may give rise to completely new phenomena. As an additional advantage beam stability is achieved with higher order nonlinearity. The study of beam propagation and induced focusing in cubic–quintic media revealed that circular Gaussian beam transforms to elliptical Gaussian beam during propagation. Also addition of quintic nonlinearity leads to a kind of stable composite soliton formation, which is not present in Kerr media [38]. Also, the presence of quintic nonlinearity helps in stable and quasi stable spatio-temporal soliton formation, while Kerr media can create only quasi stable variety [39]. In quintic nonlinear media a very weak probe beam can be guided and focused with a pump beam even if its power is well below threshold value of focusing of the pump beam alone [40]. Stable self-trapping of super-Gaussian laser beams was found in cubic–quintic nonlinear media [41]. Spatial high-power cylindrically symmetric soliton beam is achieved in cubic–quintic nonlinear media stabilized by self-induced multi photon ionization [42]. Propagation dynamics of mixture of TEM00 and TEM01 modes and stability analysis have been done in both cubic–quintic [43] and saturating media [44]. Both cases witnessed better stability in higher order nonlinearity. Although stationary self-trapped propagation of elliptic Gaussian laser beam is forbidden in saturable media, a virtual threshold power for self-focusing has been defined [45]. Slightly above the virtual threshold power the elliptic Gaussian beam focuses and below it defocusing is observed. Self-tapped propagation of elliptic Gaussian beams in an elliptic-core nonlinear fiber is shown in presence of absorption or gain [46]. In this system the elliptic Gaussian beam may transform to circular Gaussian one during propagation. On the other hand conversion of circular Gaussian laser beams into elliptic Gaussian laser beams is observed due to induced focusing in cubic–quintic nonlinear media [47].

Even though a wide variety of beam has been studied, propagation of elliptical super-Gaussian beam, to the best of our knowledge, has not been explored in higher order nonlinear medium. Therefore, we propose to investigate the propagation of elliptical super-Gaussian beam in a cubic–quintic nonlinear media. This beam can be viewed as a generic type as it is reduced to circular super-Gaussian, elliptic Gaussian and more fundamental circular Gaussian beam with proper choices of beam parameters. This implies that rectangular, square, elliptical and even circular spot can be derived from the single profile. Additionally, being super-Gaussian this profile offers enhanced beam power and uniform energy distribution over the spot. This uniform power distribution in wide area and versatility in spot shape make it interesting for application even beyond optical communication.

2. Mathematical formulation

We consider the propagation of elliptical super Gaussian beam in cubic–quintic nonlinear media. The beam propagation equation is a nonlinear Schrodinger equation (NLSE) of the following form [38]:

$$-i\frac{\partial A}{\partial z}+\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}+n_1|A|^2A+sn_2|A|^4A=0, \tag{1}$$

where, A is the slowly varying amplitude of the optical field of the beam. The first term describes the evolution of the field along propagation direction z. Second and third terms represent diffraction along two orthogonal spatial dimensions, i.e., along x-axis and y-axis respectively. The fourth and fifth terms correspond to the cubic/Kerr and quintic nonlinearities respectively. $s=\text{sign}(n_2)=\pm 1$. The choice of $s=-1$ corresponds to defocusing quintic nonlinearity that tries to balance the cubic nonlinearity thus gives rise to a competing nonlinearity in the system. On the other hand, $s=+1$ designates self-focusing nonlinearity and only enhances the cubic nonlinearity by a small fraction. The medium and operating frequency decide the value of s. We take elliptical super-Gaussian beam profile of the following form [33,38]:

$$A(x,y,z)=\varphi_0(z)e^{i\theta(z)}e^{-(x/ra)^{2m}-(y/rb)^{2m}}e^{i(\alpha x^2+\beta y^2)}e^{i(Vx+Uy)}, \tag{2}$$

where $\varphi_0(z)$ is amplitude, $ra(z)$, $rb(z)$ are the width parameters of beam along x and y-axes respectively. $\alpha(z)$, $\beta(z)$ are respectively the chirps along x and y-axes. $U(z)$ is the tilt angle along the x-axis and $V(z)$ is the same along y-axis. m is the super Gaussian parameter responsible for beam flatness. Many exact analytical methods can only yield the exact solution of the NLSE, which is hyperbolic secant (sech). Instead, we choose elliptical super-Gaussian beam profile in the present case. The reason is manifold. Firstly, since our objective is to study the self-trapped propagation of elliptical super-Gaussian beam, the same must be taken as a trial function. Moreover, marked difference in beam propagation is observed with this type of beam. Secondly and more importantly, both the super-Gaussian parameter m and the beam ellipticity provide a control on diffraction, self-focusing and self-trapping of the beam. Thus by varying the beam geometry self-trapping of the