Optics Communications 332 (2014) 31–35

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

Analysis of chaotic behavior in an optical microresonator

M. Vahedi [a], A.R. Bahrampour [b], H.R. Safari [a,*]

[a] Department of Physics, Iran University of Science and Technology, Tehran, Iran
[b] Department of Physics, Sharif University of Technology, Tehran, Iran

ARTICLE INFO

Article history:
Received 22 March 2014
Received in revised form
7 June 2014
Accepted 22 June 2014
Available online 5 July 2014

Keywords:
Microresonator
Chaos
Fabry-Perot cavity
Quality factor
Threshold power

ABSTRACT

In this paper, for the first time, chaotic behavior of a classical moving-mirror Fabry–Perot cavity is obtained by finding numerical solution of a system of delay differential equations (previously obtained by a phenomenological approach (T. Carmon, M. C. Cross, K. J. Vahala, Phys. Rev. Lett. 98 (2007) 167203)). Fourier transform of the electromagnetic power for different values of pump power is calculated. By increasing the power, a period-doubling route to chaos is observed.

Since the quality factor of the cavity has an important role in the chaotic behavior, variation of Lyapunov exponent and threshold power for the onset of chaos versus quality factor are investigated. A near linear dependence of the threshold power (measured in miliwatts) to quality factor is obtained. These results may be exploited in experiments on microresonators to determine the degree and the domain of the chaotic behavior of the system.

1. Introduction

Optical microresonators are important devices for being building blocks for integrated photonic circuits [1] and also for their potential use in quantum optics applications [2,3]. While their unprecedented Q-factors make nonlinear processes to occur in very low thresholds, their small sizes make them suitable for integrated applications. Also, their ability to store photons for a long time proposes them as a good candidate for single-photon quantum optics applications [4,5].

A manifestation of the nonlinear interactions in optical microresonators is their optomechanical interactions. Interaction between optical field of the cavity and its mechanical structure forces cavity boundaries to reform and hence to alter optical power in turn. Thus, energy flows back and forth between optical and mechanical degrees of freedom. Optomechanical effects in microresonators have devoted great focus of the scientific community for their application in developing a laser cooling method for macroscopic mechanical oscillators [6], their effect on photoluminescence intensity and the blinking statistics of nearby quantum dots [7] and for other applications like RF generators [8].

One of the interesting consequences of nonlinear interactions in optical microresonators is chaotic behavior of the optical power stored in or transferred with them. Quantifying this chaotic behavior is of special importance in the design of devices that use optical microresonators. As experimentally shown in [9], chaos is an intrinsic property of the optical microresonators, independent of their shapes. So, in all applications involving microresonators, having a good knowledge of chaos becomes an important issue. On the other hand, photonic implementation of chaos-based communications for achieving high-bandwidth secure transmission of data has attracted much attention recently [10–12]. The basic idea, here, is to encode the messages over chaotic carriers. Compatibility of this kind of secure communication with commercial fiber-optic systems has recently been shown [13]. Optical microresonators would also be good candidates for encoding messages in these communication systems.

Carmon et al had shown that chaotic behavior of the microresonators could be obtained by a set of first order ordinary differential equations [9]. These equations were obtained using a phenomenological reasoning. In the present work, by considering a simple model for microresonators, we discuss the delay differential equations that govern the dynamics of the optical resonator (derived from a multi reflection method, devised previously by our group [14]) and solve them numerically to find the time evolution of the output optical power and mechanical vibration of the moving mirror and finally the onset of chaos. Occurrence of chaos is checked by taking Fourier transform of the optical power and also by calculating Lyapunov exponent. So, we show that the delay differential equations demonstrate a complete set of dynamic regimes (periodic and bistable [14], and chaotic in this work). Also, the Lyapunov exponent (degree of the chaotic behavior) variation versus quality factor is investigated. Finally, threshold power for the onset of chaos is plotted versus cavity quality factor.

* Corresponding author. Tel.: +98 2173225889.
E-mail address: asiahsafari@yahoo.com (H.R. Safari).

http://dx.doi.org/10.1016/j.optcom.2014.06.045
0030-4018/

2. Theoretical model

A simplified model for describing the physics of an optical microresonator is sketched in Fig. 1. An input light with slowly varying amplitude $s(t)$ is coupled into a Fabry–Perot (FP) cavity through a semi-transparent mirror. Here, we suppose classical electromagnetic fields and ignore quantum optical effects of the electromagnetic field such as vacuum enhanced nonlinear absorption and dispersion, described elsewhere [15,16]. The spring attached to one of the mirrors simplifies all of the mechanical degrees of freedom of the microresonator's structure.

The equation of motion for the position of the moving-mirror relative to the equilibrium point, $q(t)$, is as follows [17]:

$$m\ddot{q}+\Gamma_{\rm m}\dot{q}+k_{\rm m}q=F_{\rm RP} \tag{1}$$

where m, $\Gamma_{\rm m}$ and $k_{\rm m}$ are mass, mechanical damping and spring constants, respectively. The radiation pressure force is $F_{\rm RP}=\varepsilon_0 n_0^2\delta|a^{(+)}(z'(t),t)|^2$, where $a^{(+)}(z'(t),t)$ is the amplitude of the right going electric field at the surface of the moving mirror, n_0 is the refractive index of the cavity medium and δ is the effective cross sectional area of light on the mirror and $z'(t)=L+q(t)$. The amplitude of the right going electric field at the surface of the moving mirror can be obtained by the standard multi reflection method as follows [14]:

$$a^{(+)}(z'(t),t)=\chi_1 e^{-ikL}\sum_{n=0}^{\infty}R_0^n e^{-2ikQ_n(t)}s(t-(2n+1)\tau) \tag{2}$$

where $s(t)$ is the envelope of the input beam, χ_1 is the transition coefficient of the input mirror and Q_n is defined by the following relation:

$$Q_n(t)=\sum_{l=1}^{n}q(t-2l\tau) \tag{3}$$

τ is the cavity round trip time and R_0 is the cavity round trip complex attenuation for the input frequency ω_p, $R_0=r_1r_2e^{-2\alpha L}e^{-2ikL}$, which is determined by the input wave number ($k=n_0\omega_{\rm p}/c$) and cavity parameters r_1, r_2 and α (reflection coefficients of the mirrors and damping coefficient).

3. Numerical method and results

In the case of a FP with fixed mirrors, $q(t)=0$, for a constant-amplitude input wave $s(t)=s_0$, $a^{(+)}(L,\ t)$ has a simple form. From Eq. (2) we have

$$a^{(+)}(L,\ t)=\chi_1 s_0 e^{-ikL}\sum_{n=0}^{\infty}R_0^n=\chi_1 s_0 e^{-ikL}\frac{1}{1-R_0} \tag{4}$$

Let the mirrors to be fixed for a long time. So our initial conditions would be as follows:

$$a^{(+)}(L,\ t)=\chi_1 s_0 e^{-ikL}\frac{1}{1-R_0}\ \text{for } t<0 \tag{5}$$

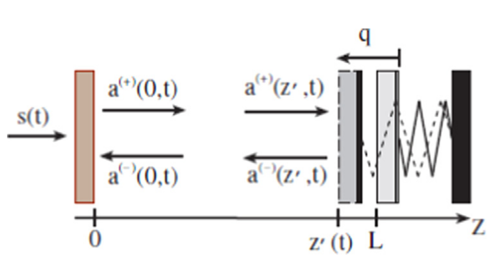

Fig. 1. A schematic diagram of the moving mirror FP cavity.

For one moving mirror FP system shown in Fig. 1, employing Eq. (2), a recursion relation for $a^{(+)}(L,\ t)$ is obtained

$$a^{(+)}(L,\ t)=e^{-ikL}e^{-\alpha L}\chi_1 s(t-\tau)+R_0e^{-2ikq(t-2\tau)}a^{(+)}(L,\ t-2\tau) \tag{6}$$

Using a step by step calculation method, we go forward in time, i.e., when $a^{(+)}(0,t)$ is known, by Eq. (6), the sequence $a^{(+)}(L,\ 2n\tau)$ ($n=1,\ 2,\ \ldots$) could be obtained. The values of $a^{(+)}(L,\ t)$ on the intervals $(2n\tau,\ 2(n+1)\tau)$ are obtained from propagation equation. So, at each instant of time, from Eq. 6, we find new $a^{(+)}(L,\ t)$. Inserting this amount in Eq. 1 gives new $q(t)$.

In summary, the dynamic equations of motion for a constant input field, introducing the conjugate momentum of the moving mirror, $p(t)$, into the equations, become as follows:

$$\dot{q}(t)=p(t)$$

$$\dot{p}(t)=-ap(t)-bq(t)+c|a^{(+)}(L,\ t)|^2 \tag{7}$$

$$a^{(+)}(L,\ t)=e^{-ikL}e^{-\alpha L}\chi_1 s(t-\tau)+R_0e^{-2ikq(t-2\tau)}a^{(+)}(L,\ t-2\tau)$$

where $a=1.4\times10^6$ Hz, $b=1.2\times10^{17}$ Hz^2, $c=9779$ sec$/m$, and $k=2\pi/1.5\times10^{-6}$ m^{-1} are chosen in accordance to Ref. [9]. In this way, $\chi_1=0.99$, $e^{-\alpha L}\simeq1$, and $|R_0|\simeq1$. Length of the cavity, L, is chosen for a typical microresonator, i.e. $3\pi\times10^{-6}$ m.

The system of delay differential Eq. (7) is solved by iteration using a forth order Runge–Kutta method. Results of time evolution of the cavity-confined optical power for different input powers are presented in the left panels of Fig. 2. In the right panels, Fourier transforms of the corresponding optical powers are shown. The phase space diagrams are shown in Fig. 3. Also shown in this figure is the time evolution of natural logarithm of power difference of two very similar initial conditions.

As is evident, for input power equal to 28.7 mW, a transition to chaotic regime occurs. The behavior is consistent with bifurcation analysis: increasing of the input power causes a transition from a stable-mode operation to periodic, two-periodic and finally chaotic dynamics.

So far, the important result is that chaotic behavior of the microresonator is a consequence of the delay response of the cavity mechanical structure to the cavity optical field. This phenomenon was not obvious in the previous demonstration of the chaotic behavior of the microresonators [9]. So, the origin of the chaotic behavior could be better understood.

For illustrating dependence of chaotic behavior on quality factor of the cavity, variation of Lyapunov exponent versus quality factor for a constant input power is shown in Fig. 4. Lyapunov exponent is the slope of the rising part of the left panels of Fig. 3. By increasing the quality factor, for a fixed input power, the amount of field that couples into and out of the cavity decreases. For very low quality factors, field enhancement inside the cavity increases, while, for higher quality factors, it decreases (i.e. competitive effects of coupling and damping) [18]. So, in the chaotic regime, for ordinary quality factors, we witness the decreasing part of the diagram and Lyapunov exponent (as a measure of degree of the chaotic behavior of the system) decreases by increasing the quality factor.

The minimum power for transition from periodic regime of operation to chaotic behavior is called threshold power. Variation of threshold power versus cavity quality factor is shown in Fig. 5. As it is expected from previous discussion, by increasing the cavity quality factor, the threshold power increases due to a decrease in the field enhancement. A power relation of $P_{\rm th}({\rm mW})=10^{-5}Q^{0.9}$ exists, a near linear dependence is observed between threshold power and quality factor.